AXISYMMETRIC INTRUSIONS

Josh Nault* and Bruce R. Sutherland
University of Alberta, Edmonton, Canada

1 INTRODUCTION

Fluid of one density propagating horizontally into an ambient fluid of different, or varying, density is called a gravity current. These are manifest in a wide range of circumstances which are surveyed by Simpson (1997). Examples include the spread of oil slicks and river plumes near the ocean surface and the advance of cold air along the ground from a thunderstorm outflow or sea breeze front. Gravity currents can also propagate within an ambient fluid whose density varies with height. For example, these are visible as rope clouds that propagate radially away from a thunderstorm along the tropopause. Here we exclusively call these "intrusions" or "intrusive gravity currents" to distinguish them from gravity currents that move along the upper or lower boundaries of the ambient.

Compared with gravity current research, the dynamics of intrusions are less well understood. Most laboratory experiments have examined vertically symmetric circumstances amenable to theoretical analysis. These include studies in a uniformly stratified fluid (Wu 1969; Amen and Maxworthy 1980) and in a two-layer fluid with equal upper and lower-layer depths (Britter and Simpson 1981; Lowe, Linden and Rottman 2002; Mehta, Sutherland and Kyba 2002) of intrusions that have density equal to the average ambient density and so propagate at mid-depth.

Here we examine the dynamics of a radially spreading axisymmetric intrusions. As in corresponding rectilinear intrusion experiments (Sutherland, Kyba and Flynn 2004), the experiments reveal the surprising result that axisymmetric intrusions propagate at constant radial velocity far beyond the distance at which axisymmetric gravity currents decelerate.

*Corresponding author address: Josh Nault, University of Alberta, Department of Mathematical and Statistical Sciences, Edmonton, AB, Canada T6G 2G1; e-mail: jnault@math.ualberta.ca.

2 THEORY

Shortly after its release from the cylinder a gravity current should spread at the steady state speed predicted by Benjamin (1968). Generally, the speed is given by

$$U_{\mathrm{gc}} = \sqrt{\frac{\phi}{2}\left(1-\frac{\phi}{2}\right)}\sqrt{g' h_T} \qquad (1)$$

in which $g' = g(\rho_\ell - \rho_0)/\rho_0$ is the reduced gravity based on the difference in density of the lock-fluid and the ambient, h_T is the full depth of the ambient, and ϕ is the ratio of the depth of dense fluid in the lock compared to h_T. For full-depth, lock release experiments, of the kind we report upon here, $\phi = 1$.

In theory, and in contrast with rectilinear gravity currents, the constant-velocity regime for axisymmetric gravity currents should exist only for a short time: once the current spreads by a significant fraction of its radius, R, the head height continually decreases as the current spreads radially outward. Surprisingly, experiments show the current spreads at constant velocity as far as 3 radii, from the lock (Hallworth et al. 1996). Only after this distance does the current decelerate at a rate predicted by self-similarity theory.

A partial explanation for this is given by the following self-similarity argument. We imagine the current spreads maintaining a cylindrical shape with its height, h, decreasing and its radius, r, increasing so that the total volume is always the constant $\pi R^2 h_T$. Setting $\phi = h/h_T =$

R^2/r^2 in (1) and writing $U_{\rm gc} \equiv dr/dt$, we have a differential equation for $r(t)$.

A simple power series approximation about $t = 0$ gives

$$r(t) = R + \frac{\sqrt{g' h_T}}{2} t - \frac{\sqrt{g' h_T}}{3R^2} t^3 + O(t^4). \quad (2)$$

Because the coefficient of t^2 vanishes, we might expect the current to propagate at constant speed for a longer time before decelerating.

By symmetry we expect this result will extend to predict the speed of an axisymmetric intrusion in a two-layer fluid with vertical symmetry, meaning that the upper and lower-layer fluids have equal depth and the lock fluid is the average density of the ambient.

3 EXPERIMENTS

3.1 Set-up

Experiments are performed in a cylindrical acrylic tank that is 30.0 cm high with inner diameter 90.7 cm. Attached to the tank platform is a 2 meter vertical frame; supporting a camera and lock release mechanism.

In gravity current experiments the tank was filled with fresh water to a depth h_T = 10 cm. An acrylic cylinder ("the lock") of height 22.8 cm and radius $R = 6.0$ cm was inserted in the centre of the tank and salt was added to establish lock-fluid of density ρ_ℓ. Dye was also added to visualise the current.

In intrusion experiments a bottom layer of salt-water with density ρ_0 was filled to a depth of 5 cm. Fresh water of density ρ_f was then added through a sponge float on the surface. Using this method a 5 cm layer of fresh water was added to the tank, thus the total fluid depth was $h_T = 10$ cm. The same cylindrical lock was then lowered into the center of the tank and the fluid within it mixed so that the density, ρ_ℓ, of fluid in the lock was given by the arithmetic mean of the top and bottom layer densities.

For both gravity current and intrusion experiments, the lock was attached to a string and pulley system to allow quick and axis-parallel removal.

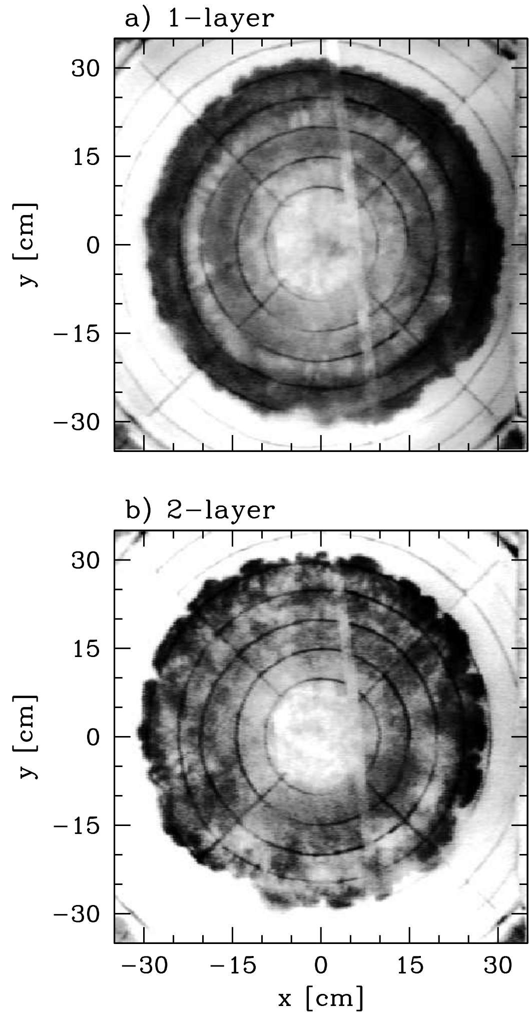

Figure 1: Top view taken 4.5 s after lock release for a) a bottom-propagating gravity current and b) a vertically-symmetric intrusion in a two-layer fluid. Darker regions of dye indicated where the head height is thicker. In both experiments $h_T = 10$ cm and $\rho_\ell - \rho_0 = 0.0533\,\mathrm{g/cm^3}$. In the intrusion experiment $\epsilon = \Delta = 0$ so that $h_1 = h_0$ and $\rho_1 - \rho_\ell = \rho_\ell - \rho_0$.

In a range of gravity current experiments, ρ_ℓ was varied. In a range of intrusion experiments, the bottom-layer density was varied and the lock-fluid's density density was determined from the average density of the upper- and lower-layer fluids.

Experiments were recorded using a COHU CCD camera mounted to the vertical frame. Images were analyzed using the software DigImage (Dalziel 1993). A grid of concentric circles spaced by 5 cm intersecting perpendicular lines were drawn on the bottom of the tank to allow translation of the digital images into real world coordinates. Using the software package, gravity current propagation rates are found. Furthermore, surface particle tracers allowed examination of the surface flow properties.

3.2 Results

The snapshot in figure 1b shows an over-head view of the intrusion 4.5 s after lock release. Compared with the corresponding gravity current experiment shown in figure 1a, it is clear that the intrusion head is not so well defined. The structure of the front is corrugated and the radial extent of the lee of the head is not easily identified.

The time series in figure 2a and b show the radial extent of the gravity current and intrusion heads, respectively, as they progress in time. The images are constructed by stacking slices across the tank diameter of images taken from the experiments at successive times. The resulting images show the radial expansion of the front starting from time $t = 0$ when the lock is first extracted.

The gravity current decelerates shortly after the lock is extracted as evident from the parabolic shape to the front position in time. In contrast, the intrusion front moves radially at approximately constant speed and it maintains this speed upon reflection, intersecting at the center of the tank and spreading radially outward once more.

Why an axisymmetric intrusion should propagate at constant speed well beyond 3 lock-radii remains somewhat a mystery. One might suppose that the no-slip boundary must play an important role in the evolution of the bottom propagating current, but this cannot be correct. The Reynolds number is too large and in experiments of axisymmetric buoyant gravity currents spreading along the surface (not shown here), they likewise decelerate.

Geometrical considerations help explain why both bottom-propagating and intrusive gravity currents are expected to decelerate: their head height should decrease as their circumference expands and therefore the horizontal pressure difference between the current and ambient must decrease.

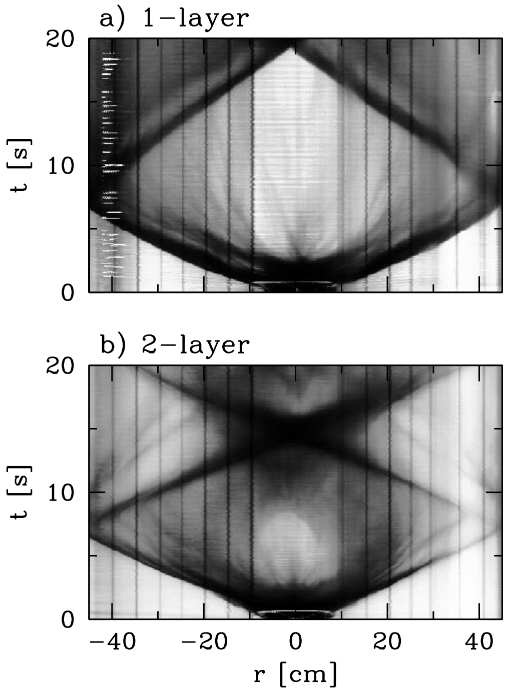

Figure 2: Time series taken vertically across the tank diameter for a) a bottom-propagating gravity current and b) an intrusion in a two-layer fluid. In both experiments $h_T = 10\,\mathrm{cm}$ and $\rho_\ell - \rho_0 = 0.0533\,\mathrm{g/cm^3}$. In the intrusion experiment $h_1 = h_0$ and $\rho_1 - \rho_\ell = \rho_\ell - \rho_0$. The time series in a) and b) are taken from the experiments for which snapshots are shown in figure 1a) and b), respectively.

It is generally not understood why a bottom-propagating gravity current propagates as far as approximately 3 radii before decelerating, but it is even more puzzling why this transition distance should be greater for an intrusion.

Two partial explanations present themselves. First, as is evident in figure 1, the front of the intrusive gravity current is azimuthally unstable. The instability appears to occur as a consequence of the interaction of the expanding front with the stratified interface. During the initial expansion, the ambient can be treated as an approximately two-layer fluid. However, as the front continues to expand the vertical extent of the intrusion head becomes comparable with the interfacial thickness and this is when instability occurs. It is beyond the scope of this paper to examine the dynamics governing the instability mechanism. However, corroboration is provided by supplemental experiments (not illustrated here) in which a bottom propagating axisymmetric gravity current is observed to be azimuthally unstable if the ambient fluid is stratified near the bottom. (The density profile is established by repeating bottom-propagating gravity current experiments one after the other, so building up a dense lower layer.)

Given the intrusion is azimuthally unstable, this helps overcome the apparent contradiction to the deceleration argument posed by geometric constraints: as the intrusion spreads along rays, it does not spread azimuthally and therefore the head height need not decrease.

A second partial explanation is that mixing may play a more important role for axisymmetric gravity currents on this scale than previously supposed. Thus a bottom propagating gravity current decelerates not only because the head height decreases but also because the density difference between the current and ambient decreases as a result of significant entrainment. On the other hand, an intrusion may entrain fluid both from the upper and lower layer, thereby maintaining the same density difference between the intrusion and each of the two layers while also retarding the decrease in height of the intrusion head.

Evidence for substantial mixing is provided by experiments performed in a 40 cm by 40 cm square tank in which the intrusion is released from a quarter-cylinder situated at one corner of the tank. Thus we are able to visualize a side-view cross-section of the propagating intrusion uncontaminated by the effects of parallax. Snapshots from such an experiment, shown in Figure 3, illustrate that the intrusion structure is elongated with no clear bulbous head, as one would see in a rectilinear tank. However, the vertical extent of the intrusion gradually decreases as it advances radially and so, although the density may not change due to entrainment, the head height nonetheless decreases and deceleration on the basis of classic gravity current theory would be anticipated.

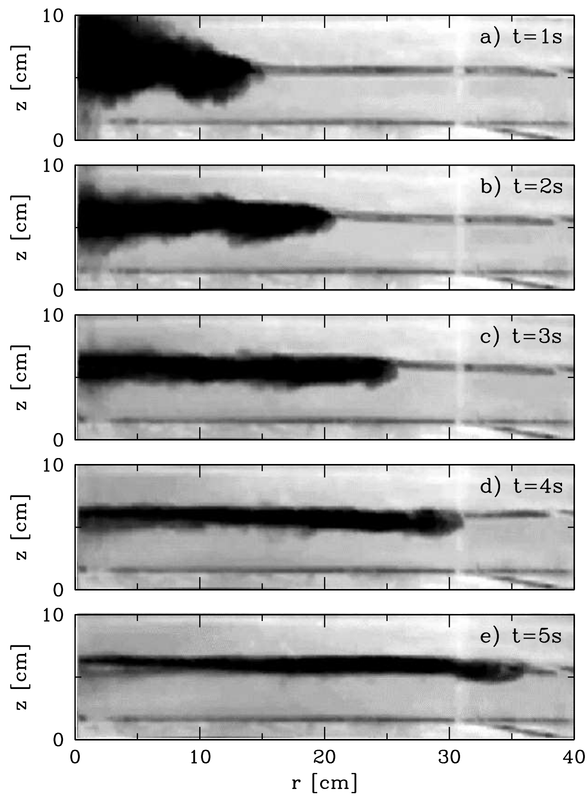

Figure 3: Successive sideview snapshots of an intrusive gravity current spreading radially from the corner of a square tank.

Both hypotheses are not entirely satisfactory because they do not explain why the current should maintain constant speed from its initial release to long times during which instability develops or entrainment becomes significant.

4 CONCLUSIONS

Through laboratory experiments with axisymmetric intrusions we found that the intrusion head propagates outward at approximately constant velocity, whereas theory suggests that the radial advance should slow with increasing time. Two possible explanations for the failure of the theory to describe accurately the velocity of the head. First, the head of the intrusion appears to spread as rays rather than an axisymmetric structure; this partially explains why the deceleration forced by geometric constraints is avoided. Second, it appears that mixing may play a more significant role in axisymmetric intrusions than previously assumed.

This work demonstrates that one cannot simply invoke symmetry to extend bottom propagating gravity current theory to theory for intrusions. The experimental results here and those that examine vertical asymmetry and rotational effects (Sutherland, Nault and Blackburn 2005) provide a challenge for theoretical modellers.

Acknowledgements

This research was supported by the Canadian Foundation for Climate and Atmospheric Science (CFCAS).

References

Amen, R. and Maxworthy, T. 1980. The gravitational collapse of a mixed region into a linearly stratified solution. *J. Fluid Mech.*, 96:65–80.

Benjamin, T. B. 1968. Gravity currents and related phenomena. *J. Fluid Mech.*, 31:209–248.

Britter, R. E. and Simpson, J. E. 1981. A note on the structure of the head of an intrusive gravity current. *J. Fluid Mech.*, 112:459–466.

Dalziel, S. B. 1993. Rayleigh-Taylor instability: experiments with image analysis. *Dyn. Atmos. Oceans*, 20:127–153.

Hallworth, M. A., Huppert, H. E., Phillips, J. C., and Sparks, R. S. J. 1996. Entrainment into two-dimensional and axisymmetric turbulent gravity currents. *J. Fluid Mech.*, 308:289–311.

Lowe, R. J., Linden, P. F., and Rottman, J. W. 2002. A laboratory study of the velocity structure in an intrusive gravity current. *J. Fluid Mech.*, 456:33–48.

Mehta, A., Sutherland, B. R., and Kyba, P. J. 2002. Interfacial gravity currents: Part II - wave excitation. *Phys. Fluids*, 14:3558–3569.

Simpson, J. E. 1997. *Gravity Currents*. Cambridge University Press, Cambridge, England, 2nd edition.

Sutherland, B. R., Kyba, P. J., and Flynn, M. R. 2004. Interfacial gravity currents in two-layer fluids. *J. Fluid Mech.*, 514:327–353.

Sutherland, B. R., Nault, J., and Blackburn, L. 2005. Axisymmetric intrusive gravity currents. *Phys. Fluids*, submitted.

Wu, J. 1969. Mixed region collapse with internal wave generation in a density stratified medium. *J. Fluid Mech.*, 35:531–544.