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THE IMPACT OF THE PLANETARY SCALE ON THE DECAY OF BLOCKING AND THE USE OF PHASE DIAGRAMS AND LYAPUNOV EXPONENTS AS A DIAGNOSTIC

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1. INTRODUCTION

In recent years, several studies have partitioned atmospheric quantities into planetary and synoptic scale components in order to examine either the relative role of each scale and their interactions, or to examine the nature of the interactions themselves (e.g., Tsou and Smith, 1990; Tracton, 1990; Lupo and Smith, 1995b; Lupo, 1997; Marques and Rao, 1999; Colucci, 2001; Burkhardt and Lupo, 2005; Lupo et al., 2005). The above list represents studies in both the Northern and Southern Hemispheres. In addition to these studies, and from those of earlier pioneers (e.g., Kalnay-Rivas and Merkine, 1981; Frederiksen, 1982; Shutts 1983; Mullen 1986, 1987), a consistent picture emerges that the synoptic-scale plays an important role in the lifecycle of blocking events (necessary, but not sufficient condition). Many of the studies represented above show that the magnitude of the synoptic-scale forcing is large compared to that of the planetary-scale forcing.

However, others have shown that the planetary-scale is very influential in the lifecycle of blocking events (e.g., Haines and Holland, 1998; Colucci and Baumhefner, 1998). While the studies referenced in the above paragraph do not downplay the role of the planetary-scale, they do focus more on the role of synoptic-scale contributions. In their model study, Haines and Holland suggest that blocking regimes will break down when there is a substantial change in the planetary-scale flow regime. Colucci and Baumhefner, 1998) focus on the role of planetary-scale deformation as providing for a pre-conditioned environment for the formation of blocking events (Colucci, 2003, personal communication). These two studies together support the notion that while the planetary-scale may not itself lead to block formation and maintenance, but this scale can provide for a favorable environment in the interaction with the synoptic-scale environment. Thus, a substantial change in the large-scale flow regime would not support blocking and these events would decay fairly quickly.

The goal of this work is to demonstrate that abrupt changes in the planetary-scale environment can lead to the rapid decay of blocking, and that it may be possible to use phase diagrams to identify when these changes occur and correlate them with the decay of three Southern Hemisphere blocking events studies in Burkhardt and Lupo (2005) and Lupo et al. (2005). Additionally, this work will look at the utility of Lyapunov exponents (Lyapunov, 1966) as a diagnostic tool in blocking studies.

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2. METHODS AND ANALYSES

2.1. Analyses

The data set used here was the National Center for Environmental Prediction (NCEP) and National Center for Atmospheric Research (NCAR) gridded re-analyses (Kalnay et al., 1996). These data were archived at NCAR and obtained from the mass-store facility in Boulder, CO. These re-analyses were the 2.5^{o} by 2.5^{o} latitude-longitude analyses available on 17 mandatory levels from 1000 to 10 hPa at 6-h intervals. These analyses include the standard atmospheric variables geopotential height, temperature, relative humidity, vertical motion, u and v wind components and surface information. The mandatory level data were interpolated quadratically in ln [p] to 50 hPa level-increments, and these more closely resemble raw sounding information (Lupo and Bosart, 1999).

2.2. Methods

The blocking criterion of Lupo and Smith (1995a) was used here to determine the onset and termination times for the blocking events studied here, and these are presented in Table 1. Details regarding this criterion and it's application can be found in the references in Section 1. A climatological (Table 1), synoptic, and dynamic description of the three events using Potential Vorticity (PV) diagnostics can be found in Burkhardt and Lupo (2005) and Lupo et al. (2005). The diagnostic techniques used there are described in Lupo and Bosart (1999). Briefly, PV framework was used as the analysis and map display tool, and PV was calculated on 300 hPa surfaces since these PV fields are similar to those calculated on an isentropic surface (e.g., Lupo and Bosart 1999).

This study will focus primarily on the use of phase diagrams, and in particular using these to demonstrate that changes in the planetary-scale flow regimes can be correlated to the onset and, more importantly, the decay of blocking events. The techniques used here to extract planetary-scale variability have been used to extract interannual variability from a one-dimensional time series recently by Mokhov et al. (2000, 2004) (and references therein) or Federov et al. (2003) and will be briefly presented here with modifications. Also, an analysis using phase plots was performed briefly by

Bengtssen (1981) in an attempt to understand the behavior of the planetary-scale flow associated with a case of blocking.

Table 1. The characteristics of the two blocking events chosen for study here (*for BI see* Weidenman etal. 2002).

Event	Dates (Start / Termination)	Days	Block Intensity (BI)
1	23 July – 2 August 1986	10.5	3.64
2	3 – 16 August 1986	13.5	4.06
3	13 – 24 July 2001	11.0	3.33

The techniques used in these references are based on standard dynamic analysis techniques for physical systems (e.g., Lorenz, 1963). Here, a time series of 500 hPa planetary-scale height fields for the months of July and August 1986, and July 2001 was used. In particular, the planetary-scale height fields were averaged over a 40^o degree latitude by 60^o degree longitude box within the blocking sector to produce one number for each time period. This process is analogous to the procedure used by Hansen (1986) in deriving the wave amplitude index, with the exception that we filtered the fields first then averaged them within a box. They averaged the entire mid-latitude height field into a band then filtered to obtain a single number for the time period.

A second-order, two-dimensional Shapiro (1970) filter was used on the variables in the data set in order to separate the planetary-scale wavelengths from the synoptic-scale wavelengths. Applying this filter results in a response function, which retains 2%, 44%, 80% of the signal for waves having a wavelength of 3000, 4500, and 6000 km at 45 degrees N (or S) latitude, respectively. More details regarding the use of the filtering procedure can be found in Lupo and Smith (1995b).

The basis for this analysis is derived by constructing simple phase plots of the first derivative of the time series versus the time series itself. If ideally, the function represented by the cyclic time series is sinusoidal (or approximately sinusoidal),

$$X(t) = A(t)\sin[\omega(t) + \phi(t)] \quad (1)$$

where X(t) represents a time series of some variable, A(t) the amplitude, and ω(t) the frequency, and ϕ(t) the initial phase, then X(t) represents a general solution to a Sturm-Liouville problem (oscillator equation) of form;

$$\ddot{X} + \omega^2 X = 0 \quad (2)$$

A simple two-dimensional phase plot of the first derivative versus the time series itself will yield a circular set of trajectories about some mean state (see Fig. 1b). In Fig. (1), the monthly mean temperatures from 1955 – 2003 for Columbia, MO, were, as expected, strongly influenced by the annual cycle (Fig. 1a) and represent an (approximately) harmonic process. The first derivative of monthly mean temperatures (Fig. 1b) was calculated using second order finite differencing, and higher order finite differencing (e.g., 4th order, not shown here) yielded similar, but not necessarily more robust results.

The use of these phase plots is not difficult to analyse and may suggest that the trajectories derived from a time series may be stable (unstable) as represented by their spiraling inward (outward) or toward some solution which could represent a limit cycle or fixed point (e.g., Zdunkowski and Bott, 2003, Ch. M7). A trajectory which spirals may suggest that the mean state is moving over time. A stable system would represent the behavior of a damped oscillator.

Historically, the planetary-scale flow in both hemispheres has been assumed to behave as, or treated like, an oscillating pendulum (e.g., Lorenz, 1963; Hansen, 1986; Nese et al., 1987; Hansen and Sutera, 1988). More recently, this type of physical behavior has been discussed by Lynch (2003) who also extended the analogy to describe the large-scale flow as a swinging "spring" in describing the behavior of Rossby wave triads.

If the planetary-scale flow were steady-state and geostrophically balanced over a long period of time, the phase plot would approximate the harmonic behavior like that seen in Fig. 1. However, it is apparent that the planetary-scale flow may have more than one stable state (e.g., Charney and DeVore, 1979; Yoden, 1985; Nese et al., 1987; Nitsche, et al. 1994). Whether there are two such states or more is not the focus of this work. Here the focus is identifying a change in flow regime character and associating this with the growth or decay of blocking events.

A source of diagnostic tools for blocking onset and decay can be found in the mathematical theory of stability in infinite dimensions. The basic premise there is that atmospheric blocking should be thought of as a quasi-stationary atmospheric circulation whose state is best analyzed by its stability characteristics, as we've stated above.

This point of view leads, in analogy to finite dimensional dynamical systems, see Walters (1982), to examining "local Lyapunov exponents" for the barotropic vorticity equations (with prescribed forcing). For an initial vorticity field ω_o and time T = nΔt, these Lyapunov exponents are in principle given by;

$$\lambda_i(\omega_o, T) = \frac{1}{2n}\log \nu_i \quad (3)$$

for ν_i the putative eigenvalues of;

$$M^*M, M = \prod_{k=-n}^{k=n} B(k\Delta t) \quad (4)$$

B(t) the linearization operator of the barotropic equations at ω(t). A numerical implementation of this principle (via a Crank-Nicholson scheme) in Dymnikov et. al. (1992) using data for a three-year period after applying a 15-day filter (planetary-scale) on domains D over the North Atlantic and Western Europe shows a strong correlation between the sum of the positive Lyapunov exponents, eigenvalues of the linearization of barotropic flows and the domain integrated enstrophy;

$$\sum \lambda_+ \approx \int_D |\omega|^2(x, y)dxdy \quad (5)$$

To the extent that the average time for ω-trajectories to diverge decreases as the sum of the positive Lyapunov exponents increases, e.g., Theorem 5.1 of Pesin (1977), we

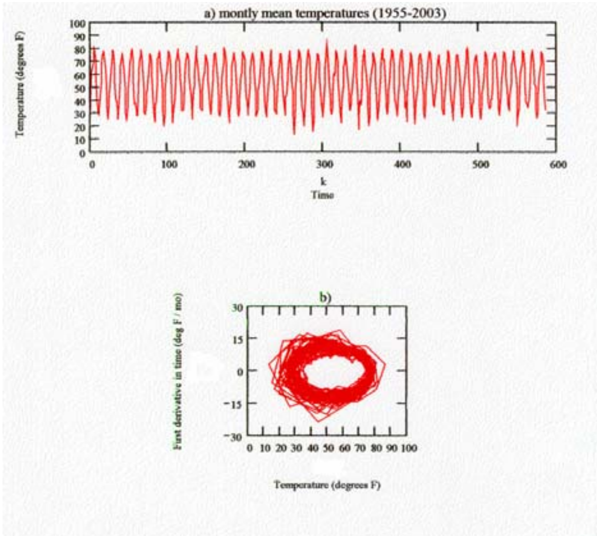

Figure 1. A monthly time series of a) monthly average temperatures (^{o}F) versus time, and b) the first derivative of temperature (^{o}F mo^{-1}) versus temperature (^{o}F) for Columbia, MO (COU), 1955 - 2003.

get a preliminary indication of how physical quantities, such as enstrophy, can be viewed as forecasting indicators.

Of course, the rigorous treatment of atmospheric flow as an infinite dimensional dynamical system requires the development of the theory of Lyapunov exponents in infinite dimensions, e.g., Ruelle (1982), Constantin and Foias (1985), Dymnikov and Kazantsev (1993). It cannot be emphasized enough that these relations refer to "Lyapunov exponents" of the infinite dimensional system on the space of possible atmospheric flows as opposed to the classical Lyapunov exponents of a single steady flow (i.e. a finite dimensional system) on a (hemi-)sphere as in Cohen and Schultz (2005) and references therein.

Given the relation of positive exponents with eigenvalues of the linearization, we present next a simplified version of this approach for steady flows. For steady barotropic flow on the full sphere, with Rayleigh friction but without orography, let;

$$C_1 = \max|\nabla\Psi| \qquad C_2 = \max|\nabla\Omega_a| \qquad (6)$$

for Ψ the stream function of a stationary flow and

$$\Omega_a = \nabla^2\Psi + 2\sin\phi \quad (7).$$

From Skiba (2002), the eigenvalue problem for the linearization of the flow can be written as;

$$\Delta u + Mu = \lambda u \qquad (8)$$

for $\left|< Mu, u >\right| \le C\|u\| \; |||u|||$, for C = max{$C_1$,$C_2$/2} for two different norms || · ||, ||| · |||. Then the argument in Prodi (1962, pp. 385–387), applies to give that the (discrete) spectrum is inside the parabola;

$$\Re(\lambda) = -\frac{1}{4C^2}\left(\Im\lambda\right)^2 + C^2 \quad (9)$$

In other words, all the (finitely many) unstable modes of the stationary flow are all included on the part of the complex plane between the imaginary axis and the parabola given in Eq. (9) for C = max{C_1, C_2/2}. Therefore, if either the maximum absolute value of $\nabla\Omega_a$ or the maximum value of $\nabla\Psi$ increases the flow is becoming increasingly unstable. Conversely, decreasing gradients of both Ω_a and Ψ lead to increasingly more stable atmospheric regimes.

In this form, (9) relates the maxima of the gradients *anywhere* to instability *somewhere*. A local, boundary-value version, relating maxima of gradients within a certain area to atmospheric instability in the same area, will be presented elsewhere.

In the context of atmospheric blocking, the result can be illustrated by using the following observation from Lupo and Smith (1995a): Before blocking, there always exists upstream of the block, a ridge accompanied by an amplifying short wave within 0.5 ridge wavelength. Given that the ridge and the short wave are associated with increasing gradients, the Lupo-Smith observation can be understood as the creation of suffcient instabilties to trigger the transition from zonal flow to blocking. Furthermore, initial data observations, Lupo and Smith (1995a), Wiedenmann et. al. (2002), indicate a strong correlation between the Block Intensity (BI) and the average (unintegrated) anti-cyclonic vorticity in the neighborhood of the block.

3. DYNAMIC ANALYSIS

3.1 Case study: July 2001

The blocking event chosen for study was a southeast (SE) Pacific region event that occurred as an isolated case during July 2001. This will be compared to that of two events which occurred within an active blocking episode. Synoptic and dynamic analysis suggests the event behaved like other events studied, except than in the Southern Hemisphere, wave-wave scale interactions tend generate blocking as the result of superposition more often than a synergistic interaction (Burkhardt and Lupo, 2005; Lupo et al. 2005).

The plot shown in Fig. 2a displays the mean planetary-scale height fields within the boxes described in section 2.2 for July 2003. Note that during July, the average heights within the box fall until just before block onset (half-day number 25). The heights rise until the block reaches a maximum and then fall again during and after decay. They suggest changes in the behavior of the large-scale flow regime.

Figure 3 shows a phase plot of this month where the mean planetary-scale heights are displayed on the abscissa and their first derivative on the ordinate. The plots show that the behavior of the large-scale flow is complex and that there are periods of time when the flow is relatively stable or unstable.

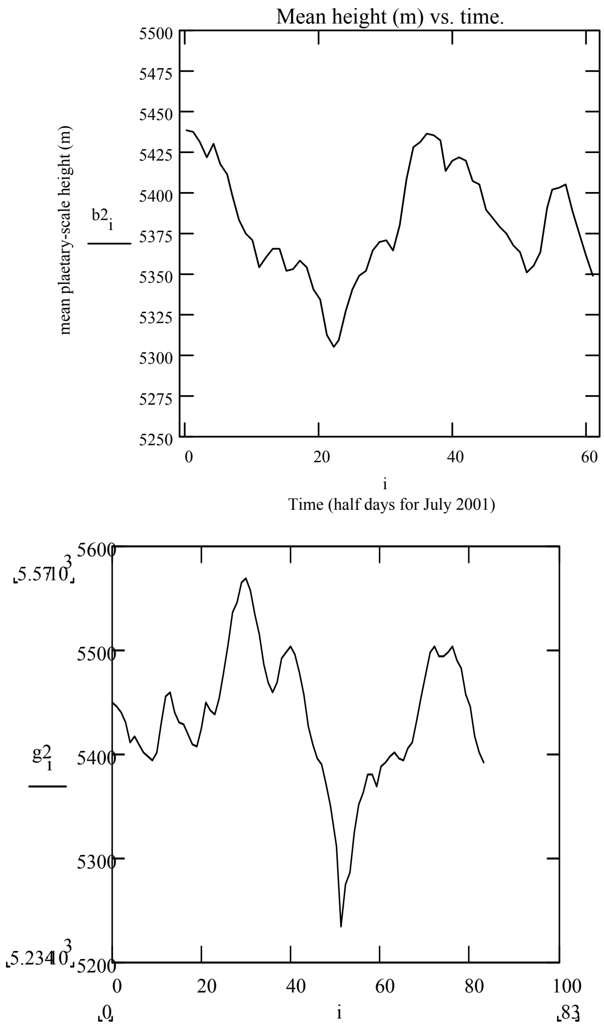

Figure2. The mean twice-daily height values for a) July 2001, and b) 20 July to 31 August 1986.

If, however, we break up these trajectories into parts and plot them as different types in order to examine the behavior of the flow during various times during July 2001, a clearer picture emerges. In Fig. 4a, the month is broken up into equal 10-day periods, and within each period, we can infer different types of behavior. Clearly, the first part of the month (solid) evolves into a stable period, while the second two consecutive 10-day periods (dotted and dashed, respectively), exhibit more unstable behavior.

If the month is broken up differently, such that the solid trajectory represents the time until block onset, it can be seen that the flow becomes unstable before block onset and during the development phase of the blocking event. This type of behavior in the flow within the blocking region has been noted by many (e.g., Frederiksen, 1982) and thus this result

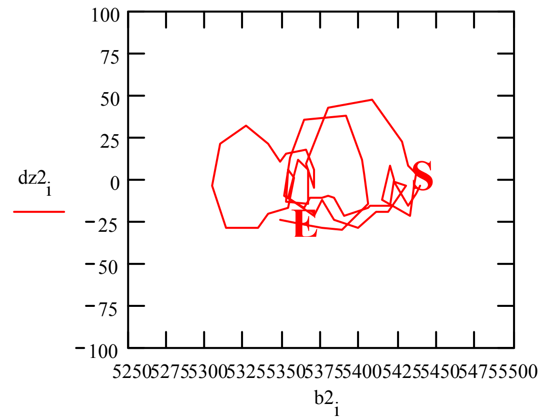

Figure 3. A phase diagram of mean 500 hPa height (m) (abscissa) and the first derivative of mean height with respect to time (m day^{-1}) (ordinate) for a stationary box (130^{o} W to 170^{o} W and 30^{o} S to 60^{o} S) in the mid-latitude Southern Hemisphere flow. The start and end points of the trajectory are marked S and E, respectively.

should not be surprising. The lifecycle of the block is represented by a trajectory that just barely establishes a circular path, but then become unstable again, and at termination the start of a new closed path is established. After block termination a new spiral path emerges and makes one circuit around moving outward suggesting a different, but unstable, state for the planetary-scale flow regime

3.2 Case Study: July-August 1986

Figure 2b, like Fig. 2a shows a plot of the mean planetary-scale heights in the blocked region after the middle of July and though the entire month of August. The heights initially remain steady through the end of July and then rise quickly in the beginning of August (approximately time period 25), as the second block emerges. Burkhardt and Lupo (2005) showed that the heights in the blocked region are higher for the second blocking event. This corresponds to a stronger second event (Table 1). Then, the heights fall precipitously as the blocking episode (two simultaneous blocks – see Burkhardt and Lupo, 2005) comes to an aprupt end around 18 August, 1986, and corresponds to a phase shift in the planetary-scale forcing (Burkhardt and Lupo, 2005).

The phase plot during this period (Fig. 6) also can be used to examine various periods of a blocking episode in which three events occurred between 20 July and 18 August in the Pacific Ocean basin. Two events occurred back-to-back in the Southeast Pacific and were described by Burkhardt and Lupo (2005), and a third event occurred in the Southwest Pacific ocean during mid-August. In Fig. 6, the first part of the trajectory, (solid - during the onset time and lifecyle of block 1 suggests a stable flow as the trajectory moves inward. The flow moves to a slightly different state as block 2 is established and during its lifecycle the trajectory suggests a stable state (dotted). However, late in the lifecycle of block 2 and after termination (dashed) the flow becomes unstable again and two events, which were occurring simultaneously over the Southwest and Southeast Pacific Ocean basin come to an abrupt end within a 24 hour period.

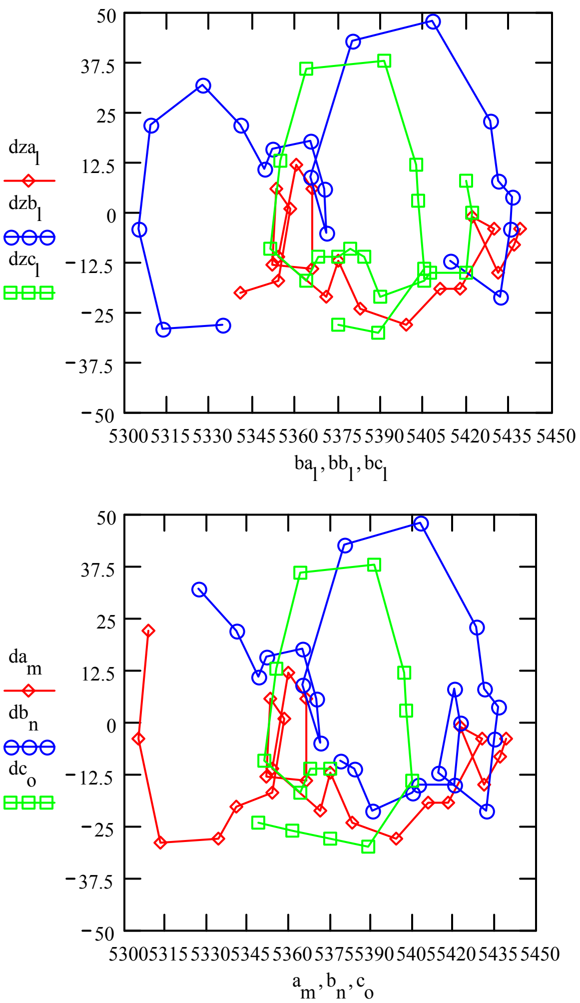

Figure 4. Phase plots for July 2001, where the abscissa is central height within the blocking region and the ordinate is the change in height with respect to time. In a) the red, blue, and green trajectories represent the first, second, and third 10 day period in July 2001, respectively. In b) the month is broken up according to the period before block onset (red), during the block lifecycle (blue), and after block termination (green).

Calculations of the Lyapunov exponents following eq. (5) from Dymnikov et al. (1992) for July, 2001 (Fig. 7a) demonstrates a correspondence between this value and Fig. 2a. From Fig. 7a, the area averaged enstrophy reaches a minimum shortly after block onset and is at a relative minimum during the lifecycle of this blocking event. The enstrophy is calculated in (5) using the planetary-scale heights used to construct Fig. 2. This is also consistent with the view that, in a quasi-barotropic flow, and the planetary-scale flow should be strongly barotropic, that the blocking state represents a minimum state of enstrophy (and entropy – see e.g., Dymnikov and Filatov (1995)). Since these correlate to the positive Lyapunov exponents and are relatively small during blocking events, this implies that once the blocking event established itself, the large-scale flow is relatively more predictable. Examining values of fluid trapping (not shown here) indicate that negative values of fluid trapping (again implying more predictability, or a more stable condition) grow in concert with the intensity of the blocking event. Additionally, it would appear that stronger negative values dominate the blocking region during its lifetime. Also, the timing for the changes in the behavior of the flow implied in Fig. 7a correspond well to the timing of those in Figs. 2a and 4a. Fig. 7b implies similar changes in behavior of the planetary-scale flow for the July-August 1986 blocking events as well.

Additionally, calculations of C_1 (the maximum of del(Ψ)) and C_2 (the maximum of del(Ω)) from Eq. (6) – (9), are shown in Figs. 8a and 8b for July, 2001. Recall, from section 2 that in theory, the calculations of C_1 and C_2 should be at a maximum when the flow is unstable, which implies planetary-scale flow regime change. At the time of block onset (decay), or 10 - 13 July (22 – 24 July), there is a dominant peak in the value of C1 (Fig. 8a). There is a similar peak in the value of C2 for block onset, but the block decay peak in the value of C2 is not as pronounced.

3.3 Discussion

In both blocking episode cases, it is demonstrated that the trajectories on a simple phase plot can be used to examine the nature of the planetary-scale flow regime during a period which encompasses the blocking events studies here. In the July 2001 case, the flow moved to a different unstable state during and after termination of the blocking event. During the July and August 1986 episodes, clearly there are two separate stable states representing the successive blocking events described in Table 1, and then two simultaneously occurring events came to an abrupt end during an unstable period in mid-to-late August. Thus, changes in the nature of the planetary-scale flow can be correlated with block decay

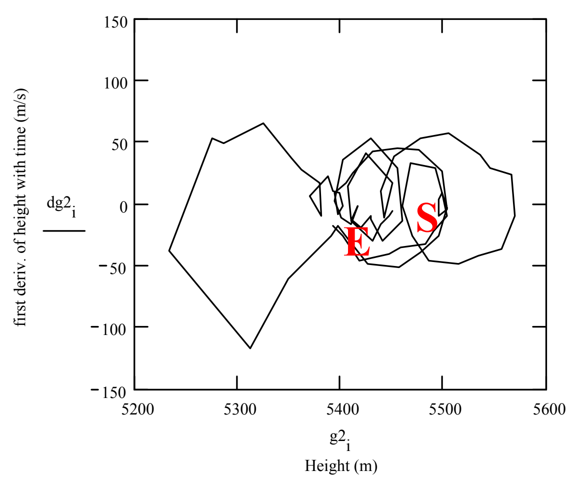

Figure 5. As in Fig. 3, except for the 1986 blocking events.

supporting the general implications of the work of Tsou and Smith (1990), Haines and Holland (1998), Colucci and Baumhefner (1998), and that is that the planetary-scale provides an important contribution to blocking lifecycles by providing a favorable environment for the blocking event to occur, in spite of the large contributions by the synoptic and interaction components of the forcing.

Additionally, supporting evidence for the change in planetary-scale flow regimes comes from examining the Lyapunov exponent (flow stability) calculations. Fluid trapping values and the area integrated enstrophy values (Fig. 7) (Dynmikov et al. 1992) fall to a minimum during the lifetime of the block in the blocked region in agreement with what would be expected for each blocking event. Finally, values of C_1 and C_2 (Fig. 8) imply the planetary-scale flow became unstable around the time of block onset and decay for the July 2001 case. It is possible that the planetary-scale flow at these two times moved from one (geostrophically) stable state to another, and the corresponding behavior of the other metrics shown in Figs. 2, 3, and 4 corroborate this interpretation. Thus, the methodologies shown here, which are relatively easy to generate, have some value as a diagnostic tool for atmospheric phenomena. They may even have value as a metric for predictability, however, more study is need in order to adequately demonstrate such value.

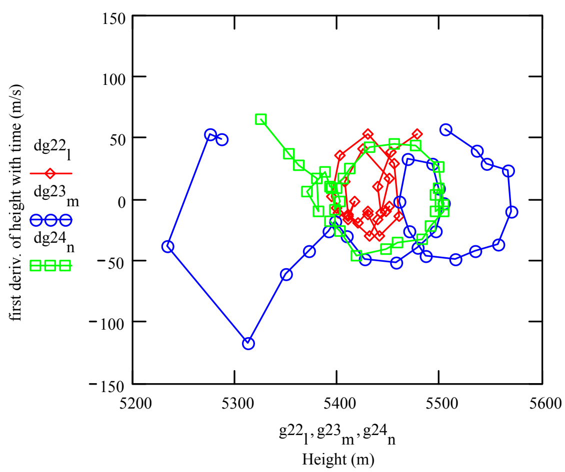

Figure 6. As in Figure 4b, except for the 1986 events. The red shows the pre-block and block 1 lifecycle, the blue shows the block 2 lifecycle, and the green is the flow after termination of the blocking events.

That the 1986 and 2001 blocking events came to an abrupt end demonstrates the influence the planetary-scale may have in the decay of blocking events. That there were simultaneously occurring events in 1986 is a fortuitous event because these results here do not necessarily then, contradict the conclusions of those who have pointed out an important role by the synoptic-scale in block lifecycles. In the study of simultaneously occurring events, Lupo (1997) shows dynamically, that the formation and intensification of two

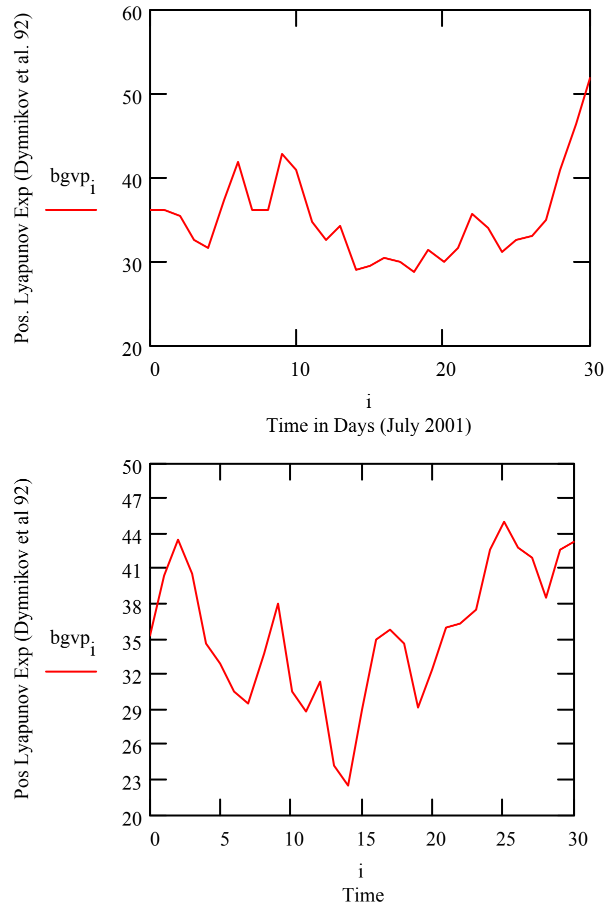

Figure 7. A calculation of area averaged enstrophy using Eq. (5) for the blocking episodes of a) July 2001 beginning (ending) with 1 (31) July, and b) July – August, 1986 beginning (ending) with 20 July (20 August).

simultaneously occurring events were governed by local, synoptic-scale events. Lejenas and Okland (1983) earlier use statistics to make a similar point. However, decay can occur when there is no longer active synoptic scale support for the events, when the synoptic-scale impacts negatively on the blocking events, or when the planetary-scale flow regime changes character. It is also suggested that either of the former two scenarios could act in concert with changes in the planetary-scale flow.

4. SUMMARY AND CONCLUSIONS

Three South Pacific cold season blocking events were studied here using the NCAR NCEP re-analyses and the PV paradigm as the diagnostic tool. Phase diagrams are used here to correlate major changes in the character of the planetary-scale flow regime with the decay of these blocking events whose climatological, synoptic, and dynamic character were examined by Burkhardt and Lupo (2005) and Lupo et al., (2005). Bengtsson (1981) was the first to use these diagrams to examine the behavior of an individual case. Finally, stability theory (Lyapunov exponents) was also investigated in order to evaluate their use as simple diagnostic tools.

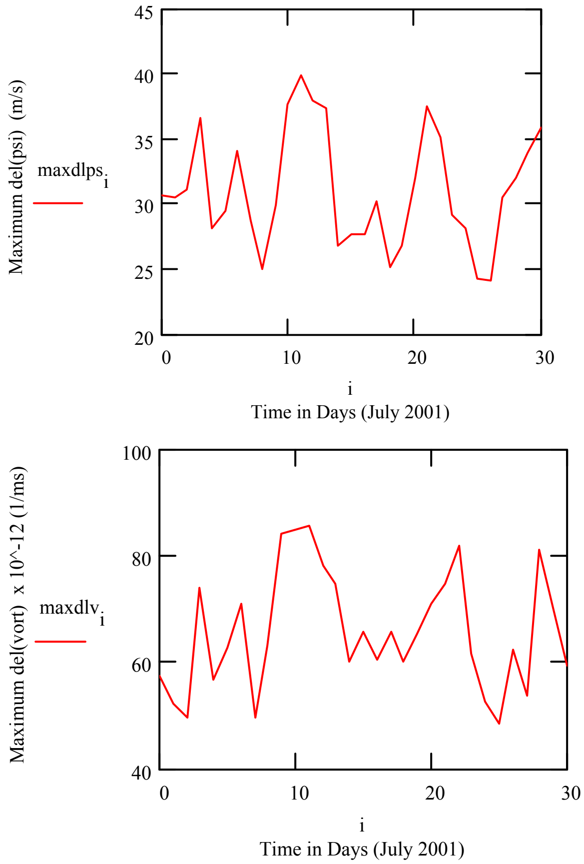

Figure 8. A calculation of a) C_1 – the maximum of the absolute value of del(Ψ), and b) C_2 – the maximum of the absolute value of del(Ω) for July, 2001.

This dynamic analysis produced a couple of key results. First, it appeared that phase diagrams can be used to study the behavior of Southern Hemisphere blocking events and that the decay periods of blocking events seem to match well with changes in the nature of the planetary-scale flow regimes. Secondly, these results are consistent with the implications of Tsou and Smith (1990), Haines and Holland (1998), and Colucci and Baumhener (1998) whose results lead them to speculate that the large-scale flow provides a favorable background environment for the persistence of blocking. Haines and Holland (1998) suggest blocking regimes may persist as long as the large-scale flow remains balanced and does not become unstable and break down or transition to a new state. Then, the importance of the planetary-scale in preconditioning or providing a favorable background is confirmed for the SH, even if the individual scale-partitioned PV tendencies are small.

The final key result was that there seem to be four reasonable scenarios which govern block decay mechanisms; a) the decay of blocking events involving the lack of synoptic-scale support (e.g., Lupo and Smith, 1995b; Lupo, 1997; Lupo et al., 2005), b) active synoptic-scale decay (e.g., Lupo and Bosart, 1999; Burkhardt and Lupo, 2005), or c) and d) either mechanism in concert with an abrupt change in the character of the planetary-scale flow (Burkhardt and Lupo, 2005).

5. ACKNOWLEDGMENTS

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