1. INTRODUCTION

Studies of precipitation efficiency (PE), which is generally defined as the ratio of total precipitation to the total amount of available moisture, have been conducted for more than 50 years. In that time, many factors influencing PE value have been studied. Some of them have been widely accepted while the influence of others is still a matter for debate.

Knowing some of those factors, it has been generalized that PE values vary with seasons and rainfall systems (convection vs. stratiform) without establishing any observational experiment to support them. As such, this study was conducted in order to bolster both aforementioned assertions. Hourly rainfall at the Missouri Climate Center’s Sanborn Field station in Columbia, Missouri, was monitored for one year to represent a continental mid-latitude location, with data collected from October 2003 through September 2004. PE values for these events were calculated using a modified water budget method adopted from Palmén and Newton (1969). This study is unique, as it applies a fairly robust approach to PE calculation to many precipitation systems for a single location throughout the course of one year.

The results indicate that the average PE value varied with season and rainfall system, thus supporting the previous statements concerning PE. The highest value is calculated during summer and the lowest in winter, while convective rainfall events have higher PE values than the stratiform ones. Additionally, PE is compared to environmental factors to determine if any relationship exists.

2. BACKGROUND

With such a long history spanning more than five decades, PE studies have ranged from the very simple to the very complex. Different types of approaches have been proposed to define PE to fit the needs of different researchers.

In the early 1950s, Braham (1952) used data obtained from the Thunderstorm Project to determine the water and energy budget of a thunderstorm. The surface observation networks and multiple soundings taken provided sufficient data to enable Braham (1952) to calculate an average airflow budget based upon mean properties of the atmosphere. Although his study was not directly about an estimation of PE, it can be seen as the first step on how one can use a water budget to calculate PE.

Sellers (1965) defined $PE$ as a ratio of the mean daily precipitation that falls to the average precipitable water of a given location. This method has not attracted much interest since it involves a climatological approach. It does not take into account the cause of water removal from the atmosphere, but rather gives an average view of transports over a discrete time period.

Later, another definition of PE was introduced by Newton (1966) as the ratio of the surface precipitation to the vapor flux through the updraft. Instead of studying an individual thunderstorm cell as in the effort of Braham (1952), he studied the squall line. The calculation of PE for his squall line event gave a value of approximately 50%. Using the same approach, he calculated the PE for Braham’s (1952) storm and the value was approximately 10%. From these values, he concluded that the squall line is five times more efficient at removing moisture from the atmosphere than the air mass storm described by Braham (1952). Newton (1966) theorized that higher efficiencies were achieved by:

- Minimizing the amount of cloud contact with unsaturated air, thereby preventing evaporative losses from dry-air entrainment,
- The longer lifetimes of squall lines versus air mass thunderstorms,
- Greater pressure perturbation associated with storms of more dynamical nature.

Of course, there are many other works that deal with PE, including several in a growing recent body of work where numerical simulations of individual cumulonimbus clouds have been examined. However, they lie outside the scope of the present paper.
3. METHOD

3.1 Event Selection

Columbia, Missouri (MO), was chosen as the focus area for this study, with data taken from the Missouri Climate Center’s Sanborn Field station. This site constitutes a continental, mid-latitude location. Heavy rainfall is not uncommon over this area, and can lead to the occurrence of flash floods. Columbia’s annual mean rainfall accumulation is slightly less than 1000 mm (39.4 in), which is based on an average from 1837 – 2003 (National Weather Service [NWS] 2005). Most of the total rainfall comes in the spring and summer season with mean values of 294 mm (11.56 in) and 297 mm (11.69 in), respectively. This is followed by a fall season with mean value of 244 mm (9.6 in) and a winter season, with a mean value of 144 mm (5.66 in). With such a seasonal rainfall distribution, it is worthwhile to study PE over this area to represent a typical continental mid-latitude region.

For the purpose of this study, certain criteria were set so that only the more significant precipitation events would be sampled. We chose events of continuous rainfall of 4 hours or more, so that rainfall systems that passed across this area would be sampled more effectively. In addition, the total rainfall must have been at least 6.4 mm (0.25 in) in each event, which was essentially a sum of stable non-convective and convective (when present) precipitation. With those criteria, 31 events were identified in a period from October 1, 2003 to September 30, 2004, as shown in Table I. Given our calculation method (discussed below), it is assumed that all of those selected precipitation systems were in a steady-state condition throughout the event period when they passed over Sanborn Field. Snow events were ignored for this particular study because the Sanborn Field observational station did not have a capability to measure reliably the liquid equivalent of such events on an hourly time scale basis.

3.2 Calculating PE

The efficiency of a precipitation system, wherein atmospheric moisture is converted to precipitation, is often described as the ratio of total precipitation to total available moisture. Thus, two components have to be determined before we can calculate the PE. Total precipitation can be obtained directly from any surface weather station observation located within the study area, while the total available moisture has to be calculated. By assuming a steady-state system, we can convert time series data into space, and apply the expressions that follow.

In this study, we employ a total water balance equation introduced in Palmén and Newton (1969) to determine the total available moisture for a single site, which is given as:

\[ P = E - \frac{1}{g_p} \left( \int_p^{p_o} \frac{\partial q}{\partial t} dp - \int_p^{p_o} \nabla q \cdot V dp \right) \]  \hspace{1cm} (1)

Term A is surface precipitation, term B is surface evaporation and term C is the moisture availled to a column through local processes or moisture flux divergence. During a rainfall event, the relative humidity would reach 100%. As the atmosphere becomes saturated, term B will become very small and is thus neglected in these calculations. Moreover, with such short periods of time involved, there is no significant difference in the calculation by ignoring term B. To calculate the total available moisture of term C, the surface pressure, \( P_o \), will be taken as 1000 mb, which is a typical station pressure for Columbia (located approximately 880 feet above sea level), while the upper level pressure, \( P \), is taken as the standard 300 mb since the atmosphere is relatively dry above that level. Term C is divided into two components, which are the local change of specific humidity, \( \frac{\partial q}{\partial t} \), and the horizontal moisture flux divergence, \( \nabla q \cdot V \), integrated from between the 1000 mb and 300 mb pressure levels.

Equation 1 describes the relationship between moisture distributions in a column of air and the vertical motions associated with it, which was first developed by Fulks (1935) and later revamped by Bannon (1948). When the equation is solved, a ratio is taken with the total precipitation as in Equation 2. This produces a value of \( \frac{P}{X} \), where \( X \) theoretically should arrive between 0 and 1. \( X \) subsequently is multiplied by 100 to give PE value in percentage.

\[ \frac{P}{\frac{1}{g} \left( \int_p^{p_o} \frac{\partial q}{\partial t} dp + \int_p^{p_o} \nabla q \cdot V dp \right)} = X \]  \hspace{1cm} (2)

This equation is considered reliable because there are good agreements obtained between computations and the observed surface precipitation based on the earlier study such as in Palmén and Holopainen (1962). Over long time scales, Equation 3.1 will demonstrate conformity to the concept of conservation of water mass. For this study, with each event lasting for only a few hours, one would not expect to see a balance between the left hand side and the right hand side of Equation 3.1 because of the short period process involved.
3.3 Data

For this study, we have already established the use of surface rainfall data measured at the Sanborn Field automated weather station for the precipitation contribution in equation (2). In addition, we also make use of the initial field from the Rapid Update Cycle (RUC) model (hereafter RUC20), to get atmospheric variables at different pressure levels. The RUC20 (Benjamin et al. 2004) is a mesoscale meteorological numerical model, featuring 50 layers in the vertical and 20 km horizontal grid spacing. It is designed to improve the accuracy and timeliness of current weather analyses and short-range weather forecasts, particularly for the aviation community and the general public. Grids from the RUC20 hourly initializations were used to establish term $C$ in equation (1) or the denominator in (2).

4. RESULTS

4.1 PE Values

Following the methods explained in detail in section 3, PE values were calculated using equation (2). These values range from a minimum of -78%, which took place on 4 August 2004, to a maximum of 236%, which occurred on 12 July 2004.

Physically, as mentioned in Section 3, PE values over the lifetime of a precipitation system should range between 0% and 100%. Upon deeper inspection, those cases studied herein that lie outside the stated range failed to meet our assumption of a steady state precipitation system. Indeed, Marwitz (1972) stated that high efficiency (~100% and more) is probably a reflection of the degree of accuracy of the available moisture and rainfall rate. Also, a precipitating system tends to have PE greater than 100% when the value is estimated during the system’s dissipating stage. It rains out while at the same time the water influx is approaching zero, which can produce a $PE$ value up to $\infty$. This statement is best visualized with the conceptual diagram of Market et al. (2003), which is their Fig. 1. Newton (1963, 1966), Foote and Fankhauser (1973), Gamache and Houze (1983), and Ferrier et al. (1996) all have shown that $PE$ values could be larger than 100% depending on the stage of development of the precipitation system.

None of the earlier known research has discussed $PE$ estimation that has a value of less than 0%. In this study, a value of -78% has been encountered in the $PE$ calculation, which appeared on 4 August 2004. Further investigation suggests that the system was precipitating out as well as undergoing strong moisture advection away from the system (likely through the anvil), experiencing a net loss in moisture during the whole event. This process leads to a negative total available moisture value when calculated using term (3) in Equation 2. Since this system collapsed as it passed through the study area, it did not fulfill the assumption we stated earlier, that the system should be in a steady state throughout the event period whenever it is within the study area. Thus, this event is ignored in the ensuing analyses.

4.2 Seasonal Variations

For this study, the four annual seasons were identified according to their meteorological definition, whereby summer encompasses the months of June, July and August; fall is September, October and November; winter is December, January and February; and spring is March, April and May. Most of the highest seasonal rainfall for Columbia, MO, was recorded during summer, followed by spring, fall and winter. Land heating along with ample atmospheric moisture supply enhances the development of convective systems with high amounts of rainfall.

The variation of $PE$ values, as plotted in Fig. 1, indicate that they tended to be higher from the beginning of March until the end of August and decreased in the middle of September to the end of February. This means that the values started to increase when the spring season began in March and become higher in June as the summer season commenced. The values decreased at the end of summer and continued to decrease as the winter arrived. The calculations of the average values of $PE$, according to their respective season, indicate that the highest value was recorded in the period of summer with 69% efficiency, followed by spring, fall and winter with 51%, 43% and 12% efficiencies respectively as in Table I. This behavior mirrors that of the season precipitation totals discussed earlier.

![Figure 1. Variation of PE values (%) with date of PE event (mm/dd/yyyy).](image)

From the definition, the $PE$ value depends on the moisture that is available in the atmosphere. The moisture enters the atmosphere through evaporation processes; thus, air near large bodies of water, such as oceans, have the highest levels of moisture. Furthermore, evaporation rates are a function of temperature where the higher the temperature the more water evaporates (Stull...
Table I: PE averages according to season.

<table>
<thead>
<tr>
<th>Month</th>
<th>No. of Events</th>
<th>Season</th>
<th>Average PE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec, Jan, Feb</td>
<td>6</td>
<td>Winter</td>
<td>12</td>
</tr>
<tr>
<td>Mar, Apr, May</td>
<td>9</td>
<td>Spring</td>
<td>51</td>
</tr>
<tr>
<td>Jun, Jul, Aug</td>
<td>8</td>
<td>Summer</td>
<td>69</td>
</tr>
<tr>
<td>Sept Oct, Nov</td>
<td>7</td>
<td>Fall</td>
<td>43</td>
</tr>
</tbody>
</table>

Due to an increase of land heating as the summer approaches, more energy is absorbed by the earth. This would lead to higher temperature which results in more evaporation (and evapotranspiration), which produces more moisture and rainfall during the warmer season.

Moisture content in the atmosphere can be quantified through the measurement of relative humidity and precipitable water. The higher the relative humidity and precipitable water value, the more the available moisture in the atmosphere. The seasonal variation of relative humidity is correlated well to rainfall, which is greater during spring, summer and fall, in this study area while precipitable water values (Ross and Elliott 1996) are typically greater through the warm season and lower in the cold season. The half-hourly precipitable water data recorded by University of the Missouri-Columbia (UMC) SuomiNet system also showed that the values tended to be higher during warm season and lower in cold season. With the highest relative humidity and precipitable water available in summer, it is obvious that more moisture is produced, which can generate the highest PE value at that time. These relations appear to support the statement by Doswell et al. (1996) that PE value is affected by relative humidity and precipitable water.

Meanwhile, by setting the months from April to September as warm season, and October to March as cold season, a descriptive statistical report from the box plot by seasons in Fig. 2 shows that the 75th percentile of PE value for cold season system lays slightly above the 25th percentile of PE value for warm system. In addition, a Mann-Whitney statistical test between those two seasons gives approximately a 97% confidence level for the two-tail probability test, which is considerably significant. Both of these statistical reports strongly suggest that the PE value is higher during the warm season and lower in the cold season.

4.3 System Variations (Stratiform vs. Convective)

Rainfall systems can be divided into two major types: convective and stratiform. For this study, hourly surface observation reports for Columbia, MO, were examined in an effort to differentiate between types of systems for all the events in Table I. Additionally, NEXRAD radar images were also being used to support the conclusion. If thunderstorm activity was reported in any of the hourly observations during the event period, the system would be classified as convective; otherwise, it would be categorized as stratiform. Of 30 study events, 19 were identified as convective while the other 11 were stratiform as listed in Table I. A summary of the rainfall system type is presented in Table II.

Table II: Summary of rainfall system types.

<table>
<thead>
<tr>
<th>Rainfall System</th>
<th>N</th>
<th>Sum</th>
<th>Fall</th>
<th>Win</th>
<th>Spr</th>
<th>Mean PE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convective</td>
<td>19</td>
<td>6</td>
<td>7</td>
<td>1</td>
<td>5</td>
<td>56</td>
</tr>
<tr>
<td>Stratiform</td>
<td>11</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>29</td>
</tr>
</tbody>
</table>

The average PE value for respective systems indicates that it was higher for convective systems than the stratiform. Convective systems are likely to occur in regions of high atmospheric moisture, warm surface temperature, maximum ambient instability, best lift and in slow moving systems (Djuric 1994); most of these factors occur during the warmer season. Results in Table II justify those relations that most of the convective cloud systems form when the warm season advances. With warm environmental conditions and a strong convective system, the precipitation system would be more efficient, thus producing higher PE values, as discussed in Li et al. (2002) in their modeling study. Moreover, the ability of updrafts and downdrafts to collect water droplets in convective cloud systems makes PE values higher (Ferrier et al. 1996). Besides that, clouds formed during the warmer season tend to have greater depth, which can promote higher moisture content of air and enhance the collision-coalescence process. This would discharge more rainfall that leads to a greater PE value estimation.
of these factors that could generate higher PE values do not exist in stratiform cloud systems.

Stratiform clouds, by definition, are clouds of extensive horizontal development, as contrasted to the vertically developed convective types, and stratiform precipitation results from such clouds. Houze (1993) stated that precipitation in stratiform clouds grow primarily by descent through a widespread updraft whose magnitude is less than 1 m s\(^{-1}\). The growth occurs primarily by continued condensation and deposition (i.e., the diffusion of water vapor onto droplets or ice crystals, respectively). Having weak vertical velocity, the droplet size distribution of this cloud type tends to be narrow (Houze 1997). Purely stratiform precipitation generally results from mid-latitude frontal systems, convergence into lows, or upslope flow, all situations in which the lower troposphere is stably stratified.

With smaller chances to form a greater droplet size through the collision-coalescence process within its shallow cloud depth, the stratiform cloud generally produces little or no precipitation. What little might fall consists of minute particles, such as drizzle, which places most of the stratiform systems outside the criteria set for this study: that is, to have more than 6.4 mm total rainfall during each event. Without having to depend on a warm environment and substantial moisture, most stratiform systems are likely to form during the cold season, typically with low PE values, which appears to support the results as shown in Tables I and II.

Meanwhile, a descriptive statistical report from the box plot by rainfall system types in Fig. 3 shows that the 75\(^{th}\) percentile of PE value for stratiform system lays near the 25\(^{th}\) percentile of PE values for convective systems. In addition, a Mann-Whitney test between those two data sets gives about 97\% confidence level for the two-tail probability test, which is considerably significant. These statistical reports strongly suggest that the PE value is generally higher for a convective system than the stratiform one.

5. CONCLUSIONS

The goal of this study was to support the assertions that PE values vary with season and cloud type. To accomplish this goal, a PE calculation method was presented based on the water budget equation introduced by Palmén and Newton (1969) to estimate the PE value for each rainfall event during one year of study. Over the long term period, the equation would turn out to be in balance. Yet for this study, PE was calculated during each rainfall event, which lasted for only a few hours time, and a value of between 0\% and 100\% was expected. However, PE values ranging from -78\% to 238\% were calculated from 31 rainfall events that have been identified through one year of observations. This large variation is due to the application of the steady-state approach to all systems used for this study.

Even though the estimated PE values were of a wide range, the study did show that the average values varied with season and cloud system. They were higher during warm season and lower during cold season, where the summer generated the highest value with 69\% efficiency follow by 51\% in spring, 43\% in fall and 12\% in the winter seasons. Analysis of the type of cloud system demonstrated that the average PE value was higher for convective clouds with 59\% efficiency, and lower for the stratiform type, with 29\% efficiency. With these results, we can now validate and use them to support the theoretical claim by earlier studies that PE values vary with season and cloud type.

Thus, two broad conclusions are offered. First, convective precipitation systems possess greater mean PE profiles because of the presence of relatively vigorous updrafts and downdrafts to promote collision-coalescence. This finding concurs with the earlier work of Ferrier et al. (1996). Secondly, the association of higher PE with clouds of a greater warm cloud depth (cloud temperature > -10°C) is a function of the ambient environment, which will (as a rule) always be warmer in the warm season. This is the primary reason that Market et al. (2003) found no correlation between the PE of mid-latitude, continental, warm-season mesoscale convective system PE and the depth of the warm cloud. In short, there are many short-term controls on PE, but warm cloud depth is not among them.

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6. REFERENCES


