11R.3 HOW GOOD ARE D_0 ESTIMATES USING POLARIMETRIC VARIABLES?

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1. INTRODUCTION

One of the major applications of polarimetric radar is improvement in rainfall estimation by estimating raindrop size distributions (DSDs). However, these are difficult to verify at the radar resolution volume. In this study, we use profiler DSD estimates, which have reasonably well understood error characteristics (e.g. Schafer et al. 2002), to evaluate the accuracies of the polarimetric estimations.

2. DATA SETS

In this study, the C-band polarimetric scanning radar (C-Pol) operating near Darwin, Australia, is used to estimate the median raindrop diameter over two vertically-pointing profiling radars located 24 km away (May et al. 2001). The C-Pol scanning radar estimates the reflectivity at both the vertical and horizontal polarizations, ZH and ZV, differential reflectivity, Z_{DR}, specific differential phase, K_{DP}, and the correlation between the vertical and horizontal polarized reflectivities at zero time lag, $\rho_{HV}(0)$ (Keenan et al. 1998).

The two profiling radars estimate the vertical air motion and the raindrop size distribution from about 1.7 km to just below the melting layer at 4 km. One profiler operates at 50 MHz and is sensitive to Bragg scattering from turbulent air motions. The other profiler operates at 920 MHz and is sensitive to Rayleigh scattering from hydrometeors. The DSD is estimated at each range gate by first shifting the observed 920-MHz Doppler velocity spectra by the 50-MHz profiler estimated vertical air motion. The spectral broadening of the resulting terminal fall speed spectra is removed using the convolution modeling method (Rajopadhyaya et al. 1999, Schafer et al. 2002) and the deconvolution modeling method (Lucas et al. 2004 and Schafer et al. 2002). The median raindrop diameter is estimated from the retrieved DSD.

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3. UNCERTAINTIES IN D_0 – POLARIMETRIC RADAR

The simplest algorithms for D_0 estimation relate the D_0 to Z_{DR} via a power law based on regressions and simulations (e.g. papers by Jameson 1991, Brandes et al. 2004a and 2004b, Bringi et al. 2002, and Vivekanandan et al. 2004). These are often of the form: $D_0 = AZ_{DR}^B$. For example, using surface disdrometer and scanning S-band polarimeteric radar observations in Florida Bringi et al. (2002) derived the Z_{DR} - D_0 relation for ZH less than 35 dBZ

$$D_0 = 1.81 Z_{DR}^{0.486} \text{ mm.} \quad (1)$$

It is straightforward to estimate the statistical error in D_0 using this parametric form when given uncertainties in the ZH and ZV measurements. For typical spectral widths and weather radar time series, we can assume that the standard deviation of the horizontal and vertical polarized reflectivities are equal and approximately 1 dB (i.e., std(ZH) = std(ZV) ~ 1dB). Note that this does not include errors associated with spatial variability or attenuation, both of which are ignored for the moment.

The uncertainty in Z_{DR} is NOT simply the sum of the variances as would be the case if ZH and ZV were independent. Rather we need to consider the correlation of the errors. To first order, this correlation can be estimated from the correlation of the ZH and ZV time series themselves, for which we have an estimate: $\rho_{HV}(0)$. During rain, the value of $\rho_{HV}(0)$ is typically about 0.97.

Estimating the variance of the Z_{DR} =(ZH – ZV) is simply given by:

$$\begin{aligned} \mathrm{var}(Z_{DR}) &= \mathrm{var}(ZH) + \mathrm{var}(ZV) - 2\,\mathrm{cov}(ZH,ZV) \\ &\sim \mathrm{var}(ZH) + \mathrm{var}(ZV) - \\ &\qquad 2\rho_{HV}(0)(\mathrm{var}(ZH)\,\mathrm{var}(ZV))^{1/2} \end{aligned} \quad (2)$$

For var(ZH) ~1 and $\rho_{HV}(0) \sim 0.97$ (as is seen in the data for rain) this gives var[Z_{DR}] ~ 0.06 dB^2

which implies the standard deviation is approximately std(Z_{DR}) ~ 0.25dB.

Given an estimate of the uncertainty of Z_{DR}, the uncertainty of the D_0 estimate is found by propagation of errors. Using the regression formulation shown in (1) as an example, the uncertainty of D_0 can be determined using

$$\mathrm{var}(D_0) = \mathrm{var}(Z_{DR})\left(\frac{dD_0}{dZ_{DR}}\right)^2 \qquad (3)$$
$$\mathrm{var}(D_0) = 0.06 * (1.81 * 0.486(Z_{DR}^{-0.514}))^2$$

Therefore, std(D_0) ~ 0.2/ Z_{DR} mm. Now, if we average N independent estimates then this uncertainty of course reduces by $\sqrt{N}$.

More complex retrievals involving more measured polarimeteric observations will have more complex uncertainty relationships, but in general, including more variables will always increase the random statistical errors. Note that this analysis DOES NOT include errors due to sampling, attenuation and perhaps most importantly the uncertainty in the regression relationship itself. The variances of all of these error sources contribute to the total variance of the desired estimated quantity, i.e. D_0.

4. UNCERTAINTIES IN D_0 – PROFILING RADAR

As we are all aware, estimating the raindrop size distribution from vertically pointing Doppler radars is a complex issue dependent on a range of factors including the vertical air motion, the spectral broadening of the recorded Doppler velocity spectra, and the underlying assumption that the DSDs are stationary during the radar dwell time. The uncertainties of these factors all contribute to the uncertainties of the estimated DSD. In this study, we don't want to quantify the uncertainties of each factor, but rely on the results of previous studies using simulated data to characterize the "correct" statistical properties of the DSDs. One of the most comprehensive of these studies is Shafer et al. (2002).

From the results of Schafer et al. we can see several trends. For example as the clear air spectral width increases, the errors in the estimated DSD increase quite dramatically. However, there is also a dependence on the underlying DSD. To first order, this follows the median volume diameter, with larger errors associated with D_0 smaller than about 0.7 –1 mm and errors increasing again as the D_0 becomes very large. These limits arise because for small D_0 the raindrop fall speeds collapse into a narrow range near the clear air peak, making for difficult retrievals. For very large D_0, a problem arises as the fall speeds of the large drops asymptote to a constant value as the raindrop diameter increases.

From Figures 2 through 4 of Schafer et al. (2002), we can estimate the standard deviation of D_0 as a function of the air motion spectral width, $\sigma(\omega)$, and the value of D_0. Table 1 shows these relations.

Table 1. Standard deviation of profiler estimated D_0 as functions of the air motion spectral width and the value of D_0.

	$\sigma(\omega) = 0.5$	$\sigma(\omega) = 1.5$	$\sigma(\omega) = 3.0$
D_0=1 mm	0.1 mm	0.2 mm	0.3 mm
D_0=2 mm	0.15 mm	0.3 mm	0.5 mm

5. CONSIDERATIONS FOR REAL DATA

In addition to the purely statistical variations discussed above we will have differences associated with the different spatiotemporal sampling of the two radar systems. In the time domain, the scanning radar makes essentially instantaneous observations as it performs RHIs over the profiler compared with the 45 second dwell time required by the profiler. In the space domain, the scanning radar sample volume has a vertical extent of about 500 meters while the profiler has a 100 meter vertical resolution. Since the statistical errors discussed above are uncorrelated with the spatiotemporal sampling errors, their variances can be combined to yield D_0 uncertainties derived from polarimeteric and profiling radars to be at least 0.2 mm.

6. EXAMPLE OF REAL OBSERVATIONS

Figure 1 shows profiler and scanning C-Pol observations collected during one rain event during the 2002-2003 wet season. The top panel shows the profiler reflectivity at its original 100 m vertical and 1 minute temporal resolution. These observations were reduced to the 500 m vertical and 10 minute temporal resolution of the C-Pol scanning radar and are shown in the second panel. The C-Pol reflectivity, ZH, and differential reflectivity, Z_{DR}, at their original resolution are shown in the next two panels. The bottom panel shows the profiler estimated median raindrop diameter, D_0. The profiler raindrop size distribution was estimated using the dual-frequency method described in Rajopadhyaya et al. (1999) with the 50-MHz profiler vertical air motion estimate used to transform the observed Doppler velocity spectra into raindrop terminal fall speed spectra. The DSD was projected onto a Gamma functional form constrained with the $\Lambda-\mu$ relationship described by Zhang et al. (2003).

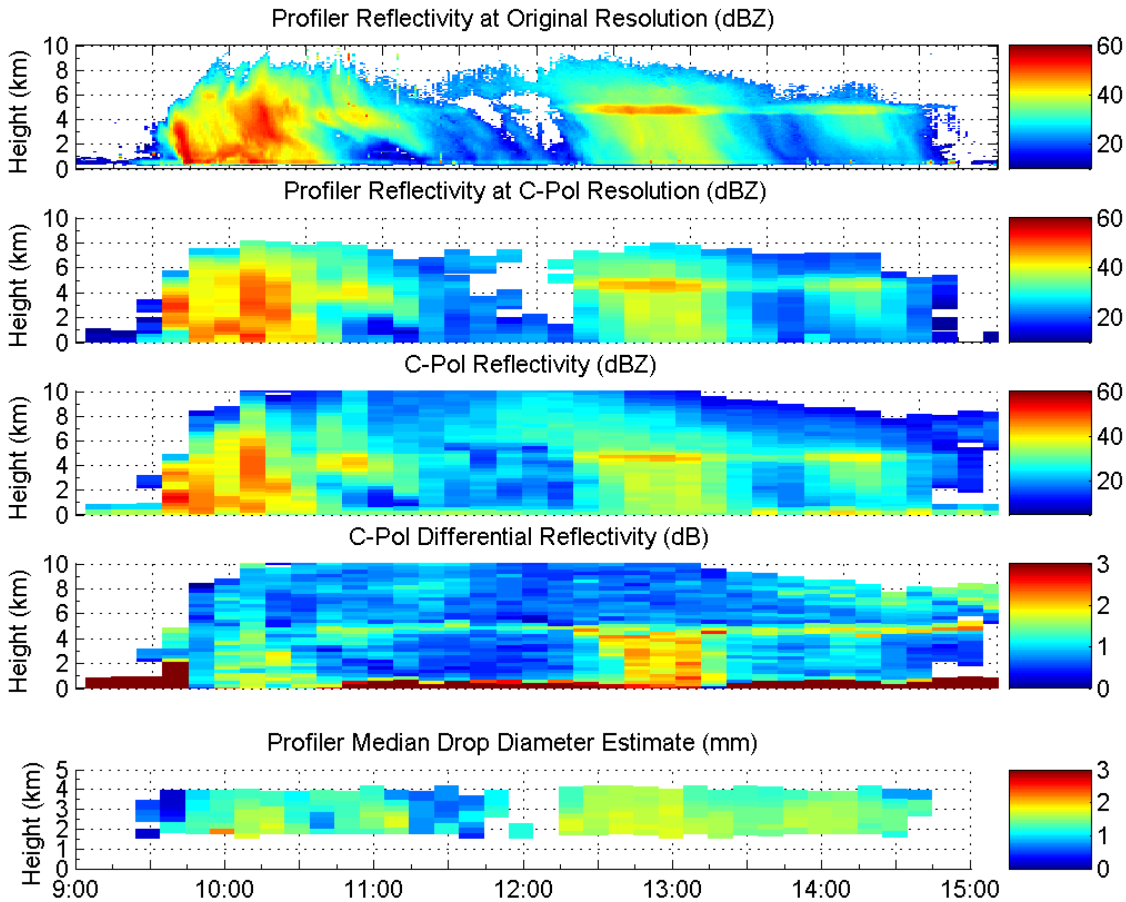

Figure 1. Time-height cross-sections of profiler and C-Pol observations over the profiler site. (a) Profiler reflectivity at the original 100 m vertical and 1 minute time resolution, (b) profiler reflectivity reduced to the C-Pol 500 m vertical and 10 minute resolution, (c) C-Pol reflectivity, ZH, (d) C-Pol differential reflectivity, Z_{DR}, and (d) profiler estimated median drop diameter, D_0.

7. CONCLUDING REMARKS

Regressions of the form $D_0 = AZ_{DR}^B$ are the simplest algorithms to estimate D_0 from polarimeteric scanning radar measurements and the uncertainty of D_0 can be estimated by propagating the errors of Z_{DR} through the derived power law regression. Using model simulations, D_0 uncertainties from profiler retrievals are found to be functions of the spectral broadening within the radar pulse volume and the actual DSDs being observed. These statistical analyzes indicate that D_0 uncertainties derived from polarimeteric and profiling radars are at least 0.2 mm. We are currently analyzing the profiler retrieved D_0 and C-Pol measured Z_{DR} estimates from several rain events near Darwin, Australia, to acquire the radar observations needed to support or contest this statistical analysis. The statistics from these radar observations will be presented at the conference.

8. REFERENCES

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