P9R.6 POLARIMETRIC WEATHER RADAR BASE MOMENT ALGORITHM VALIDATION

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1 INTRODUCTION

Polarimetric weather radars measure very small differences in received signals between channels. From these very small differences, the polarimetric moments, differential reflectivity (Z_{DR}), differential propagation phase (ϕ_{DP}), correlation coefficient (ρ_{HV}), and linear depolarization ratio (L_{DR}) are estimated. For accurate estimation of these quantities, system calibration must be very precise and the algorithms used to generate the base moment data must minimize error. In order to assess the system calibration and validate the base moment algorithms, a series of procedures were developed using commercial waveform generator equipment and common RF and IF componentry. This paper describes the validation procedures and results.

2 EQUIPMENT CONFIGURATION

The equipment required for the evaluation of all the base moments includes three IF waveform generators that can be phase locked together. The first generator provides the burst. The other two generators provide the signals for the horizontal and vertical channel. Additionally, equipment required to up-convert the Intermediate Frequency (IF) signal to the Radio Frequency (RF) signal as well as the cabling required for connection. Finally, to obtain quantitative data from the tests, a spectrum analyzer, and oscilloscope, and a software product such as EEC's EDGE are required. Table I summarizes the required equipment.

Figure 1 shows the System Test Configuration. The first waveform generator generates the IF burst. Its phase drives the phase locking circuitry of other two waveform generators. The other two waveform generators generate the IF signals to be up-converted to RF for injection into the horizontal and vertical receiver chains respectively. Before we can evaluate the moments, first we need to evaluate the test configuration ensuring we have the coherency required for moment validation and to establish calibrated IF power levels for input to the signal processor.

Once the system is configured for the validation assessment, the test system must be calibrated for coherency and power measurements.

Table I Required Test Equipment

Quantity	Description
1	RF Test Signal Generator
3	80 MHz Waveform Generator (Agilent Model 33250A or equivalent)
1	4 Channel Oscilloscope (100 MHz or greater)
1	Spectrum Analyser (0 to 6 GHz)
1	Signal Processor (EDRP-9 or equivalent)
1	Software (EDGE or equivalent)
1	IF to RF Test Assembly
3	RG58, BNC to SMA cables, 15 feet in length
3	RG58, BNC to SMA cables, 24 inches in length
2	RG58, BNC to SMA cables, 12 inches in length
3	BNC T Adaptor

2.1 Coherency Measurement

The first test we will perform is to determine the coherency (detected phase variation of the signal with respect to the burst) of the test configuration. This is extremely important as many of the base data moments, such as velocity, spectrum width, and differential phase are greatly dependent upon minute variations in phase.

The evaluation procedure is as follows:

1) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

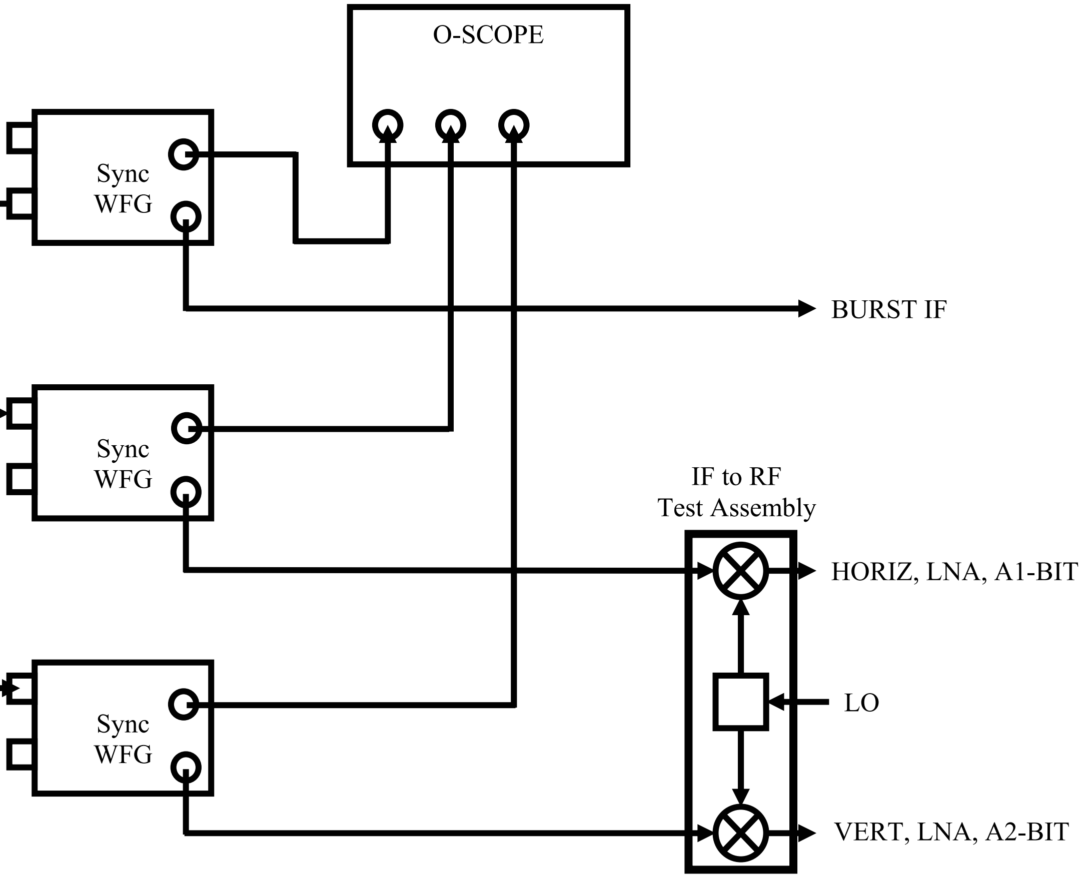

Figure 1 System Test Configuration

2) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

3) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	70
Filter	1
Noise	1

4) Bring up a display to view data. An A-Scope display is preferred.
   a) Select the Horizontal mode.
   b) Select Uncorrected Reflectivity Moment (U)
   c) Record the reflectivity at 80 km.
   d) Select Corrected Reflectivity Moment (Z)
   e) Record the reflectivity at 80 km.
   f) Subtract Z from U. This is the coherency of the horizontal channel.
   g) Select the Vertical mode.

h) Select Uncorrected Reflectivity Moment (U)

i) Record the reflectivity at 80 km.

j) Select Corrected Reflectivity Moment (Z)

k) Record the reflectivity at 80 km.

l) Subtract Z from U. This is the coherency of the vertical channel.

It is important to note that the coherency determined above will be the maximum coherency that can be tested on the system with this equipment. The system itself may be capable of greater coherency and hence greater clutter rejection, but the test equipment can limit that evaluation.

2.2 *Power Calibration*

Throughout the receiver chain there are several components. Signal transfer through the cables and components leads to signal loss. Ideally, one can perform an analysis and estimate the loss from all the components based upon their properties at the frequencies of interest. However, a small miscalculation can result in hours of checking and rechecking to ensure all loss factors are accounted. Alternatively, one can input a signal level into one end and adjust it until the measured value at the other end is at a known value. This is the technique used here. The resulting data will be required throughout the remainder of the test procedures.

1) Attach Spectrum Analyzer to Horizontal IF amplifier (A3) output

2) Turn on Test Equipment.

3) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

4) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

5) Set the Spectrum Analyzer as summarized below.

Setting	Value
Center Frequency	60.00 MHz
Span	10 MHz
Noise	1

7) On Horizontal waveform generator:
    a) Adjust the amplitude so that the 60 MHz peak on the Spectrum Analyzer reads 0.0 dBm. The record the Waveform Generator output in the table below.

    b) Adjust the amplitude so that the 60 MHz peak on the Spectrum Analyzer reads -40.0 dBm. The record the Waveform Generator output in the table below.

8) Disconnect the Spectrum Analyzer from the Horizontal IF amplifier (A3) and connect the Horizontal IF amplifier to the signal processor.

9) Attach Spectrum Analyzer to Vertical IF amplifier (A4) output.

10) On Vertical waveform generator:
    a) Adjust the amplitude so that the 60 MHz peak on the Spectrum Analyzer reads 0.0 dBm. The record the Waveform Generator output in the table below.

    b) Adjust the amplitude so that the 60 MHz peak on the Spectrum Analyzer reads -40.0 dBm. The record the Waveform Generator output in the table below.

11) Disconnect the Spectrum Analyzer from the Vertical IF amplifier (A4) and connect the Vertical IF amplifier to the signal processor.

2.3 *Signal Processor Phase Calibration*

In the calibration procedure, the phase difference between the channels is accounted and corrected. The signals are injected at the antenna port as the waveguide lengths in each channel will be slightly different and thus corrected. However, in this test, we are not injecting at the antenna port, but rather at the front end of the receiver. As such, the phase calibration procedure must be performed after the equipment is set up.

1) Turn on Test Equipment.

2) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

3) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	35
Filter	0
Noise	1
SQI	10
Radar Mode	Horizontal LDR

5) Bring up a display to view data. An A-Scope display is preferred.

6) Set the software to display as follows:

Moment	Uncorrected Reflectivity
Mode	Horizontal
Cursor	80 km in range

7) On Horizontal waveform generator set the amplitude so that the output into the signal processor is -40.0dBm. (Level recorded in last section).

8) On Vertical waveform generator set the amplitude so that the output into the signal processor is -40.0dBm. (Level recorded in last section).

9) On the control workstation, execute the EDRP-9 Phase Utility by typing `edrp9phase` at the prompt.

10) Allow the program to acquire data for two minutes.

11) At the completion of the data acquisition time, depress the correct phase button on the utility display.

Verify that both the Horizontal and Vertical Phase Measurements as displayed by the utility reposition to zero (0) degrees indicating that the correction factor has been applied.

3 BASE MOMENT VALIDATION

Once the test equipment is configured and calibrated, the assessment procedures may begin. In this section, the validation procedures and their theoretical basis are described for each moment separately. These moments include the reflectivity factor, radial velocity, spectrum width, differential reflectivity, differential propagation phase, cross-polar correlation coefficient, and linear depolarization ratio.

3.1 Reflectivity Factor (Z)

The reflectivity factor is a radar independent characterization of the target return. The reflectivity factor is proportional to power received by the radar system scaled by the radar constant, the square of the range, and the atmospheric attenuation of the signal. Mathematically, this is written as,

$$z = C_r r^2 P_r L_a \, , \tag{1}$$

where z is in linear units (mm^6/m^3), C_r is the radar constant, r^2 is the range to the target, L_a is the atmospheric loss, and P_r is the received power.

The scaling by the radar constant is the factor that causes the reflectivity factor to be radar independent. The radar constant is dependent upon the physical parameters of the radar system., i.e.

$$C_r = \frac{2.69\times10^{16}\lambda^2 L_t}{P_T \tau\theta\phi G^2} \tag{2}$$

where

λ ≡ the radar's wavelength in cm,
L_t ≡ the dimensionless transmit loss factor, i.e. factor identifying the amount of power loss from the transmitter to the antenna,
τ ≡ the transmitted pulsewidth in μs,

θ ≡ the horizontal beamwidth in deg,
ϕ ≡ the vertical beamwidth in deg,
G ≡ the dimensionless antenna gain,
P_T ≡ the transmit power in kW.

Since the reflectivity factor can vary over several orders of magnitude (powers of 10), the reflectivity factor is normally referred in logarithmic units, dBZ. Mathematically, this is written as,

$$Z = C + 20\log(r) + \alpha r + P_r \,, \tag{3}$$

where C is the radar constant in dB, α is the atmospheric attenuation factor in dB/km, and P_r is the received power in dBm.

The signal received by the antenna is absorbed by a detector and converted into an electrical signal. This electrical signal is amplified and downconverted into an intermediate frequency (IF) signal. This IF signal is also amplified and digitized. The Digitized IF is digitally downconverted to baseband with an in-phase (i) component and a quadrature (q) component. It is from numerous samples of this digitized signal located at the same range (determined by sampling the signal at the same time relative to pulse transmission). If we call the signal $s(t)$, and the sampled signal s_n, we obtain a set of samples for each range, $s(r) = \{s_0, s_1, \cdots, s_{N-1}\}$. The estimated power for that range bin is given by,

$$P(r) = \gamma \left\{ \frac{1}{N} \sum_{n=0}^{N-1} s_n^* s_n \right\}, \tag{4}$$

where N is the number of pulses integrated, γ is the factor converting the A/D value to power, and the * represents complex conjugation.

The expected values for the reflectivity come from the radar range equation (Eqn. 3). From the radar range equation, it is clear that the estimate of the reflectivity error is given by the power estimate error. The reflectivity variance bounds are based upon the system specifications and are tested in the calibration procedure.

The validation procedures are as follows,

1) Turn on Test Equipment.

2) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

3) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	35
Filter	0
Noise	1
SQI	10
Radar Mode	Horizontal LDR

5) Bring up a display to view data. An A-Scope display is preferred.

6) Set the software to display as follows:

Moment	Uncorrected Reflectivity
Mode	Horizontal
Cursor	80 km in range

7) On Horizontal waveform generator set the amplitude so that the output into the signal processor is 0.0dBm. (Level recorded in last section). Record the Reflectivity level.

8) Adjust the amplitude in 1 dB steps and verify on the display that the resultant reflectivity increments changes by 1 dBZ in the same direction as the amplitude.

9) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	35
Filter	0
Noise	1
SQI	10
Radar Mode	Vertical LDR

10) Bring up a display to view data. An A-Scope display is preferred.

11) Set the software to display as follows:

Moment	Uncorrected Reflectivity
Mode	Vertical
Cursor	80 km in range

12) On Vertical waveform generator set the amplitude so that the output into the signal processor is 0.0dBm. (Level recorded in last section). Record the Reflectivity level.

13) Adjust the amplitude in 1 dB steps and verify on the display that the resultant reflectivity increments changes by 1 dBZ in the same direction as the amplitude.

3.2 Radial Velocity (V)

The second meteorological moment the Doppler velocity. This moment is related to the first autocorrelation lag. The first lag is not real value, rather it is an imaginary number. The argument of the first autocorrelation lag represents the phase of the detected signal. The phase varies from pulse to pulse by with the Doppler frequency. Thus, the Doppler shift is measured as a phase shift.

The phase shift is given by,

$$\psi = \tan^{-1}\left[\frac{\operatorname{Im}\hat{R}(1)}{\operatorname{Re}\hat{R}(1)}\right]. \tag{5}$$

where $\hat{R}(1)$ is the first lag of the autocorrelation function. The n^{th} lag autocorrelation function is given by,

$$\hat{R}(n) = \frac{1}{N-n}\sum_{i=1}^{N-n} s_i^* s_{i+n} .$$

The velocity corresponding to the phase shift obtained in Eqn. 5 is,

$$v = \frac{\lambda\psi}{2\pi T_s}, \tag{6}$$

where λ is the wavelength and T_s is the PRT.

Estimating the error in the radial velocity measurement involves estimating the error of a multivariate function, *f*(*x*,*y*). This error is given by,

$$\sigma_f = \sqrt{\left(\sigma_x \frac{\partial f(x,y)}{\partial x}\right)^2 + \left(\sigma_y \frac{\partial f(x,y)}{\partial y}\right)^2} \tag{7}$$

where the following variables *x* and *y* are,

$$y = \operatorname{Im}\left(\hat{R}(1)\right),$$

$$\sigma_y = \operatorname{Im}\left(\sigma(1)\right),$$

$$x = \operatorname{Re}\left(\hat{R}(1)\right),$$

and

$$\sigma_x = \operatorname{Re}\left(\sigma(1)\right).$$

Then, we have

$$v = \left[\frac{\lambda}{2\pi T_s}\right]\tan^{-1}\left(\frac{y}{x}\right) \tag{8}$$

and

$$\sigma_v = \left[\frac{\lambda}{2\pi T_s}\right]\left\{\left[\sigma_x \frac{\partial}{\partial x}\tan^{-1}\left(\frac{y}{x}\right)\right]^2 + \left[\sigma_y \frac{\partial}{\partial y}\tan^{-1}\left(\frac{y}{x}\right)\right]^2\right\}^{1/2} \tag{9}$$

The derivative of the arctangent is given by,

$$\frac{d}{du}\tan^{-1} u = \frac{1}{1+u^2} .$$

Thus,

$$\frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right) = \left[\frac{-y}{x^2+y^2}\right] \frac{d}{dy}\tan^{-1}\left(\frac{y}{x}\right) = \left[\frac{x}{x^2+y^2}\right].$$

So, the error in the velocity measurement becomes,

$$\sigma_v = \left[\frac{\lambda}{2\pi T_s}\right]\left[\frac{\sqrt{\sigma_x^2 y^2 + \sigma_y^2 x^2}}{x^2+y^2}\right]. \tag{10}$$

Substituting our values,

$$y = \operatorname{Im}\left(\hat{R}(1)\right),$$

$$\sigma_y = \operatorname{Im}\left(\sigma(1)\right),$$

$$x = \mathrm{Re}\left(\hat{R}(1)\right),$$

and

$$\sigma_x = \mathrm{Re}\left(\sigma(1)\right),$$

back into the relation, we have

$$\sigma_v = \left[\frac{\lambda}{2\pi T_s}\right]\frac{1}{\left|\hat{R}(1)\right|^2}\left\{\left[\mathrm{Re}\left(\sigma(1)\right)\mathrm{Im}\left(\hat{R}(1)\right)\right]^2 + \left[\mathrm{Im}\left(\sigma(1)\right)\mathrm{Re}\left(\hat{R}(1)\right)\right]^2\right\}^{1/2} . \quad (11)$$

This expression is the error estimate in its true form and impossible to calculate with most signal processors. Therefore, the only alternative is to obtain an estimate from other moments and set parameters.

From Doviak and Zrnic, (Doviak, 1993), we find that the error estimate for radial velocity is,

$$\sigma_v = \left[\frac{W\lambda f_{PRF}}{8N\sqrt{\pi}}\right]^{1/2}, \quad (12)$$

assuming that the signal to noise ratio is high and the spectrum width, W, is narrow (small). It is important to note that this error is dependent upon the PRF, the spectrum width, and the number of samples.

The expected values of the radial velocity are the Doppler velocities calculated from the corresponding frequency shift. Since we are discussing electromagnetic, we must look at the relativistic Doppler shift (Einstein, 1905),

$$f' = f\sqrt{\frac{1-\frac{V}{c}}{1+\frac{V}{c}}}, \quad (13)$$

where,

f' is the detected frequency,

f is the transmitted frequency,

c is the speed of light,

V is the relative speed between the radar and target.

Solving Eqn. 13 for the speed V, we have,

$$V = \Delta f\lambda, \quad (14)$$

where λ is the wavelength of the transmitted radiation. Using Eqn. 13 to estimate the expected velocity gives an answer that is incorrect. For example, suppose the radar system has a wavelength of 5.0 cm and the frequency difference (Δf) is 400 Hz. Eqn. 14 gives the detected speed of 20 m/s, however the target would be moving at a speed of 10 m/s. Is the physics wrong?

The answer is no, the physics is not wrong. Eqn. 13 is correct for a sensor that is listening to radiation emitted from a target with a radial speed V relative to the listener. That is the case after the scattering has occurred. However, before the scattering the radar is the source of the radiation and the target (raindrops) are the listeners. So, Eqn. 15 is applied to that initial phase. In otherwords, the target (rain drops) hear a frequency and scatter the microwave energy with a frequency given by Eqn 15. The radar then detects the microwaves with the frequency altered from what the targets detected, i.e.

$$f'' = f'\sqrt{\frac{1-\frac{V}{c}}{1+\frac{V}{c}}}, \quad (15)$$

In terms of the frequency transmitted by the radar, Eqn. 15 becomes,

$$f'' = f'\sqrt{\frac{1-\frac{V}{c}}{1+\frac{V}{c}}} = f\left(\frac{1-\frac{V}{c}}{1+\frac{V}{c}}\right). \quad (16)$$

Solving Eqn. 16 for V the radial speed of the target and reducing, we obtain,

$$V = \frac{1}{2}\Delta f\lambda . \quad (17)$$

Eqn 17 gives the radial velocity in terms of the Doppler frequency shift, Δf

A radial velocity variance bounds of ±1.0 m/s is based upon the specification for the Evansville New Generation project (Stagliano, 2003). With this specification, we see that the spectrum width and number of samples for a PRF of 1180 Hz would be,

$$N = \left[\frac{\lambda f_{PRF}}{8\sigma_v^2\sqrt{\pi}}\right]W . \quad (18)$$

Eqn. 18 is linear with a slope inversely proportional to the variance of the measurements.

The evaluation procedures are as follows.

1) Turn on Test Equipment.

2) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

3) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	35
Filter	0
Noise	1
SQI	10

5) Bring up a display to view data. (In EDGE, the A-Scope display is preferred)

6) Set the software to display as follows:

Moment	Velocity
Mode	Horizontal
Cursor	80 km in range

7) Record the velocity level.

8) Set the software to display as follows:

Moment	Velocity
Mode	Vertical
Cursor	80 km in range

9) Record the velocity level in the table below.

10) Repeat for the waveform generator frequencies listed in the table.

3.3 Spectrum Width (W)

The current test confiuration provides CW signals with constant phase. As such, we cannot evaluate the spectrum width at this time. However, the successful completion of the test configuration was the proof of concept for a new test signal generator.

3.4 Differential Reflectivity (Z_{DR})

The first and simplest polarimetric moment is the differential reflectivity. The differential reflectivity is the ratio of the vertical reflectivity factor to the horizontal reflectivity factor in linear units or the difference in logarithmic units. In terms of the logarithmic units,

$$Z_{DR} = Z_h - Z_v . \tag{19}$$

The error in the differential reflectivity measure will be,

$$\begin{aligned} \sigma_{Z_{DR}} &= \sqrt{\sigma_{Z_h}^2 + \sigma_{Z_v}^2} \\ &= \frac{10}{\ln 10}\sqrt{\frac{\sigma_{hh}^2(0)}{\hat{R}_{hh}^2(0)} + \frac{\sigma_{vv}^2(0)}{\hat{R}_{vv}^2(0)}} \, . \end{aligned} \tag{20}$$

This error cannot be measured directly by the signal processor. Thus, we must infer the error in the differential reflectivity through other means.

The expected values for the differential reflectivity are simply the difference in the reflectivity values between the horizontal and vertical channels.

The differential reflectivity variance boundary of ±0.1 dB is intimately ties to the maximum error allowed in Z_{DR} for adequate precipitation measurements (Illingworth, 2003).

The evaluation procedures are as follows.

1) Turn on Test Equipment.

2) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

3) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	35
Filter	0
Noise	1
SQI	10
ZDR Offset	0.0
Radar Mode	Sim ZDR

5) Bring up a display to view data. (In EDGE, the A-Scope display is preferred)

6) Set the software to display as follows:

Moment	Horizontal Z
Mode	Hor. LDR
Cursor	80 km in range

7) On Horizontal waveform generator set the amplitude so that the input to the signal processor is 0.0 dBm. Record the Reflectivity level.

8) Set the software to display as follows:

Moment	Vertical Z
Mode	Vert LDR
Cursor	80 km in range

9) On Vertical waveform generator set the amplitude so that the input to the signal processor is 0.0 dBm. Record the Reflectivity level.

10) Set the software to display as follows:

Moment	Differential Reflectivity
Mode	Sim ZDR
Cursor	80 km in range

11) Record the Differential Reflectivity (ZDR) level. Since both the horizontal and vertical input values are the same, this value is the ZDR Offset for the signal processor. (Note: This is the offset for the signal processor only, a full system calibration will provide the system ZDR offset).

12) In the control software, set the ZDR Offset to the value determined in Step 11.

13) On Vertical waveform generator set the amplitude so that the input to the signal processor is -10.0 dBm.

14) Set the Horizontal waveform generator so that the input to the signal processor is at -18.0 dBm. Record the horizontal reflectivity (Z_h), vertical reflectivity, (Z_v), and differential reflectivity (Z_{DR}).

15) Increase the amplitude of the horizontal waveform generator by 1.0 dBm. Record the values. Repeat until the input to the signal processor has reached -2.0 dBm.

16) Return the amplitude of the horizontal waveform generator so that the input to the signal processor is -10.0 dBm.

17) Set the Vertical waveform generator so that the input to the signal processor is at -18.0 dBm. Record the horizontal reflectivity (Z_h), vertical reflectivity, (Z_v), and differential reflectivity (Z_{DR}).

18) Increase the amplitude of the vertical waveform generator by 1.0 dBm. Record the values. Repeat until the input to the signal processor has reached -2.0 dBm.

19) Return the amplitude of the vertical waveform generator so that the input to the signal processor is -10.0 dBm.

3.5 Differential Propagation Phase (Φ_{DP})

Raindrops are not spherical in nature. Rather they are ellipsoidal, elongated along the horizontal. Due to this, horizontally polarized radiation travels slightly slower than vertically oriented radiation. The net result is a variation in the detected phase between the horizontal and vertically oriented radiation.

The actual estimation depends upon whether the system is an alternating or simultaneous polarimetric system, as we are concerned with simultaneously polarized systems the following discussion will be so focused.

For a simultaneous polarimetric system, only a single cross-polar correlation is required,

$$\Phi_{DP} = \arg\left(\hat{R}_{hv}(0)\right). \tag{21}$$

The estimated measurement error with the differential propagation phase is a multivariate function. Defining,

$$y = \mathrm{Im}\left(\hat{R}_{hv}(0)\right),$$

$$\sigma_y = \mathrm{Im}\left(\sigma_{R_{hv}}(0)\right),$$

$$x = \mathrm{Re}\left(\hat{R}_{hv}(0)\right),$$

and

$$\sigma_x = \mathrm{Re}\left(\sigma_{R_{hv}}(0)\right),$$

we get,

$$\sigma_{\Phi_{DP}} = \left[\frac{\sqrt{\sigma_x^2 y^2 + \sigma_y^2 x^2}}{x^2 + y^2}\right]. \tag{22}$$

Since the differential phase depends upon the measurement of the phase angle, the maximum resolution of the measurement will be twice the coherence values in angular units. If the coherence is 40 dB, then the data resolution of the differential phase error will be ± 1.2 deg. Therefore, the spacing between measurements should be at least 1.2 deg.

The boundaries on the differential phase measurement will depend intimately upon the system coherency. Namely, twice the system coherency reported in deg. For a coherency of 40 dB, the measurement bounds will be at most ± 1.2 deg. The selected bounds, ± 1.0 deg, was chosen for it is a nice round number.

The evaluation procedures are as follows.

1) Turn on Test Equipment.

2) Set the waveform generators as follows:

Waveform Generator	Frequency (MHz)	Waveform	Amplitude (dBm)	Phase Offset (deg)
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

3) Set the oscilloscope as summarized below.

Setting	Scale	Impedance
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4) In the control software, set the following signal processor parameters:

Parameter	Value
CCOR	35
Filter	0
Noise	1
SQI	10
Radar Mode	Dual Pol

5) Bring up a display to view data. (In EDGE, the A-Scope display is preferred)

6) Set the software to display as follows:

Moment	Differential Phase (PDP)
Mode	Sim ZDR
Cursor	80 km in range

7) Adjust the waveform generators as follows:

Waveform Generator	Description
Burst	Select Zero Phase Button
Horizontal	Adjust phase to match representation on the oscilloscope between the vertical and horizontal waveforms. Select Zero Phase Button
Vertical	Select Zero Phase Button

8) Using the signal processor A-Scope facility in the differential phase (PDP) mode, fine adjust the vertical phase to a representation of 0.0 degrees.

9) Select the Zero Phase Button on the vertical waveform generator.

10) Adjust the horizontal phase from -10 deg to +10 deg in 2 deg increments. Record the Differential Phase values from the display software.

11) Adjust the horizontal phase from -180 deg to +180 deg in 30 deg increments. Record the Differential Phase values from the display software.

12) Return the horizontal phase to 0.0 deg.

13) Using the signal processor A-Scope facility in the differential phase (PDP) mode, fine adjust the horizontal phase to a representation of 0.0 degrees.

14) Select the Zero Phase Button on the horizontal waveform generator.

15) Adjust the vertical phase from -10 deg to +10 deg in 2 deg increments. Record the Differential Phase values from the display software.

16) Adjust the vertical phase from -180 deg to +180 deg in 30 deg increments. Record the Differential Phase values from the display software.

17) Return the vertical phase to 0.0 deg.

3.6 Cross-Polar Correlation Coefficient (ρ_{HV})

Another polarimetric variable is the cross-polar correlation coefficient at zero lag. Its mathematical form is,

$$\rho_{hv}(0) = \frac{\left|\hat{R}_{hv}(0)\right|}{\sqrt{\hat{R}_{hh}(0)\hat{R}_{vv}(0)}}. \qquad (23)$$

This is a function of three measured quantities, the cross-polar correlation function normalized by the horizontal and vertical power respectively.

The error in the cross-polar correlation coefficient is a multivariate function. Defining,

$$\begin{aligned}\zeta_{\mathrm{Re}} &= \mathrm{Re}\left(\hat{R}_{hv}(0)\right)\\ \zeta_{\mathrm{Im}} &= \mathrm{Im}\left(\hat{R}_{hv}(0)\right),\end{aligned}$$

and

$$\begin{aligned}\sigma_{\zeta_{\mathrm{Re}}} &= \mathrm{Re}\left(\sigma_{hv}(0)\right)\\ \sigma_{\zeta_{\mathrm{Im}}} &= \mathrm{Im}\left(\sigma_{hv}(0)\right),\end{aligned}$$

we have,

$$\zeta = \left|\hat{R}_{hv}(0)\right| = \sqrt{\zeta_{\mathrm{Re}}^2 + \zeta_{\mathrm{Im}}^2} \qquad (24)$$

and

$$\begin{aligned}\sigma_\zeta = \frac{1}{\left|\hat{R}_{hv}(0)\right|}\Big\{&\Big[\mathrm{Re}\left(\sigma_{hv}(0)\right)\mathrm{Re}\left(\hat{R}_{hv}(0)\right)\Big]^2\\ &+\Big[\mathrm{Im}\left(\sigma_{hv}(0)\right)\mathrm{Im}\left(\hat{R}_{hv}(0)\right)\Big]^2\Big\}^{\frac{1}{2}}\end{aligned} \qquad (25)$$

Defining additionally,

$$x = \hat{R}_{hh}(0),$$

$$\sigma_x = \sigma_{hh}(0),$$

and

$$y = \hat{R}_{vv}(0),$$

$$\sigma_y = \sigma_{vv}(0).$$

Then, the correlation coefficient can be written,

$$\rho_{hv} = \frac{\zeta}{\sqrt{xy}}. \qquad (26)$$

The standard deviation of the correllation coefficient is,

$$\sigma_{\rho_{hv}} = \sqrt{\left[\sigma_\zeta \frac{\partial\rho_{hv}}{\partial\zeta}\right]^2 + \left[\sigma_x \frac{\partial\rho_{hv}}{\partial x}\right]^2 + \left[\sigma_y \frac{\partial\rho_{hv}}{\partial y}\right]^2}.$$

Inserting the identities,

$$\frac{d}{d\zeta}\left(\frac{\zeta}{\sqrt{xy}}\right) = \frac{1}{\sqrt{xy}},$$

$$\frac{d}{dx}\left(\frac{\zeta}{\sqrt{xy}}\right) = \frac{-\zeta}{2\sqrt{x^3 y}},$$

and

$$\frac{d}{dy}\left(\frac{\zeta}{\sqrt{xy}}\right) = \frac{-\zeta}{2\sqrt{xy^3}},$$

we get,

$$\sigma_{\rho_{hv}} = \rho_{hv}\left\{\frac{\left[\mathrm{Re}\left(\sigma_{hv}(0)\right)\mathrm{Re}\left(\hat{R}_{hv}(0)\right)\right]^2}{\left|\hat{R}_{hv}(0)\right|^4} + \frac{\left[\mathrm{Im}\left(\sigma_{hv}(0)\right)\mathrm{Im}\left(\hat{R}_{hv}(0)\right)\right]^2}{\left|\hat{R}_{hv}(0)\right|^4} + \frac{\sigma_{hh}^2(0)}{\left|\hat{R}_{hh}(0)\right|} + \frac{\sigma_{vv}^2(0)}{\left|\hat{R}_{vv}(0)\right|}\right\}^{1/2} . \quad (27)$$

The expected values for the correlation coefficient follow a sin x/x shape with respect to frequency. As noted in the section on correlation coefficient estimation, the actual value will depend upon the number of samples and the PRF. However, one value will always be the case, when both channels have the same frequency, they will have a correlation of 1. The correlation coefficient will decrease in a (sin x/x) pattern from there.

The evaluation procedures are as follows.

1) Turn on Test Equipment.

2) Set the waveform generators as follows:

Waveform Generator	**Frequency (MHz)**	**Waveform**	**Amplitude (dBm)**	**Phase Offset (deg)**
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	-20.0	0.0

3) Set the oscilloscope as summarized below.

Setting	**Scale**	**Impedance**
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4) In the control software, set the following signal processor parameters:

Parameter	**Value**
CCOR	35
Filter	0
Noise	1
SQI	10
Number Samples	64
PRF	1180
Radar Mode	Sim ZDR

5) Bring up a display to view data. (In EDGE, the A-Scope display is preferred)

6) Set the software to display as follows:

Moment	Correlation Coefficient
Mode	Dual Pol
Cursor	80 km in range

7) The correlation coefficient on the display should be 1.0.

8) Adjust the frequency of the horizontal waveform generator to 59.9999850 MHz and increase in 0.0000050 MHz increments. Record the resultant correlation coefficients.

9) Reset the frequency of the horizontal waveform generator to 60.0.

10) Adjust the frequency of the vertical waveform generator to 59.9999850 MHz and increase in 0.0000050 MHz increments. Record the resultant correlation coefficients.

11) Reset the frequency of the vertical waveform generator to 60.0.

3.7 Linear Depolarization Ratio (L_{DR})

The linear depolarization ratio is obtained by transmitting in one dimension and receiving in both. The resulting cross-polar power to co-polar power in the linear depolarization ratio, L_{DR}.

For argument, assume that the horizontal polarization is transmitted. Then, the linear depolarization ratio is given by

$$L_{DR} = 20\log\left(\frac{\left|\hat{R}_{hv}(0)\right|}{\left|\hat{R}_{hh}(0)\right|}\right). \quad (28)$$

As expected, the error estimate is that of a multivariate function. Defining,

$$\zeta_{\mathrm{Re}} = \mathrm{Re}\left(\hat{R}_{hv}(0)\right)$$
$$\zeta_{\mathrm{Im}} = \mathrm{Im}\left(\hat{R}_{hv}(0)\right),$$

and

$$\sigma_{\zeta_{\mathrm{Re}}} = \mathrm{Re}\left(\sigma_{hv}(0)\right)$$
$$\sigma_{\zeta_{\mathrm{Im}}} = \mathrm{Im}\left(\sigma_{hv}(0)\right),$$

we have,

$$\zeta = \left|\hat{R}_{hv}(0)\right| = \sqrt{\zeta_{\mathrm{Re}}^2 + \zeta_{\mathrm{Im}}^2} \qquad (29)$$

and

$$\sigma_\zeta = \frac{1}{\left|\hat{R}_{hv}(0)\right|}\left\{\left[\mathrm{Re}\left(\sigma_{hv}(0)\right)\mathrm{Re}\left(\hat{R}_{hv}(0)\right)\right]^2 + \left[\mathrm{Im}\left(\sigma_{hv}(0)\right)\mathrm{Im}\left(\hat{R}_{hv}(0)\right)\right]^2\right\}^{1/2} . \qquad (30)$$

Defining further,

$$x = \hat{R}_{hh}(0),$$

and

$$\sigma_x = \sigma_{hh}(0),$$

we obtain,

$$L_{DR} = 20\log\left(\frac{\zeta}{x}\right) = 20\left[\log\zeta - \log x\right] \qquad (31)$$

and

$$\sigma_{L_{DR}} = \sqrt{\left[\sigma_x \frac{\partial L_{DR}}{\partial x}\right]^2 + \left[\sigma_\zeta \frac{\partial L_{DR}}{\partial \zeta}\right]^2} . \qquad (32)$$

Using the identity,

$$\frac{d}{du}\log u = \frac{1}{u\ln 10},$$

we see that,

$$\sigma_{L_{DR}} = \frac{20}{\ln 10}\sqrt{\left[\frac{\sigma_x}{x}\right]^2 + \left[\frac{\sigma_\zeta}{\zeta}\right]^2} .$$

Changing back to the original variables,

$$\sigma_{L_{DR}} = \frac{20}{\ln 10}\left\{\frac{\sigma_{hh}^2(0)}{\hat{R}_{hh}^2(0)} + \frac{\left[\mathrm{Re}\left(\sigma_{hv}(0)\right)\mathrm{Re}\left(\hat{R}_{hv}(0)\right)\right]^2}{\left|\hat{R}_{hv}(0)\right|^4} + \frac{\left[\mathrm{Im}\left(\sigma_{hv}(0)\right)\mathrm{Im}\left(\hat{R}_{hv}(0)\right)\right]^2}{\left|\hat{R}_{hv}(0)\right|^4}\right\}^{1/2} \qquad (33)$$

The expected values for the differential reflectivity are simply the difference in the reflectivity values between the horizontal and vertical channels.

Since the functions that use L_{DR} have a coarse error variance with respect to L_{DR}, the variance bounds for the linear depolarization ratio was set to be about that of the reflectivity.

The evaluation procedures are as follows.

1. Turn on Test Equipment.

2. Set the waveform generators as follows:

Waveform Generator	**Frequency (MHz)**	**Waveform**	**Amplitude (dBm)**	**Phase Offset (deg)**
Burst	60.0	Sine	0.0	0.0
Horizontal	60.0	Sine	-20.0	0.0
Vertical	60.0	Sine	0.0	0.0

3. Set the oscilloscope as summarized below.

Setting	**Scale**	**Impedance**
Channel 1	1.0 V/div	50 Ω
Channel 2	1.0 V/div	50 Ω
Channel 3	1.0 V/div	50 Ω
Time Base	10 ns/div	
Trigger	Channel 1	

4. In the control software, set the following signal processor parameters:

Parameter	**Value**
CCOR	35
Filter	0
Noise	1
SQI	10
ZDR Offset	0.0
Radar Mode	Sim ZDR

5. Bring up a display to view data. (In EDGE, the A-Scope display is preferred)

6. Set the software to display as follows:

Moment	LDR
Mode	Dual Pol
Cursor	80 km in range

7. Adjust the amplitude of the vertical waveform generator from -40 dBm to +0.0 dBm in 5.0 dBm steps. Record the value of LDR.

8. Reset the amplitude of the vertical waveform generator to 0.0 dBm.

9. Adjust the amplitude of the horizontal waveform generator from -40 dBm to +0.0 dBm in 5.0 dBm steps. Record the value of LDR.

10. Reset the amplitude of the horizontal waveform generator to 0.0 dBm.

4 RESULTS

The test procedures described in the last section were applied to an EEC SImultaneous Dual POLarization (SIDPOL) weather radar. The system consists of a 250 kW magnetron transmitter and an EDRP-9 digital receiver and signal processor mounted above the elevation plane of the pedestal. This section details the results of the validation assessment.

The first step in the evaluation process and hence the validation procedures is the coherency measurements and calibration of the test configuration. For the test configuration described previously, both channels had a 40 dB coherency (40.0 dB horizontal, 40.1 dB vertical). Similarly for an IF amplifier output of 0.0 dBm, the horizontal and vertical waveform generator outputs were +1.02 dBm and +1.02 dBm respectively. For an –40.0 dBm IF amplifier output, the horizontal and vertical waveform generator outputs were –39.3 dBm and –39.4 dBm respectively.

The next step in the validation process is the reflectivity validation. Settng the waveform generators so that the output from the IF amplifiers are 0.0 dBm, we find the reflectivity levels at 80 km to be 76.9 dBZ and 76.8 dBZ for the horizontal and vertical channels respectively. The amplitudes of each channel were varied in 1 dB steps and the reflectivity tracked with a linearity better than 0.1 dB.

4.1 Reflectivity Factor (Z)

The next step in the validation process is the reflectivity validation. Settng the waveform generators so that the output from the IF amplifiers are 0.0 dBm, we find the reflectivity levels at 80 km to be 76.9 dBZ and 76.8 dBZ for the horizontal and vertical channels respectively. The amplitudes of each channel were varied in 1 dB steps and the reflectivity tracked with a linearity better than 0.1 dB.

4.2 Radial Velocity (V)

Validating the velocity product involves shifting the frequency from the burst frequency. The velocity associated with the shift is estimated from the relativistic Doppler shift formula developed previously and depends on the radar wavelength. The radar under test had a wavelength of 5.38 cm. Thus a frequency shift of 37.5 Hz results from a Doppler velocity shift of 1.0 m/s. For evaluation purposes, the expected velocities ranged from –10 m/s to +10 m/s in 1 m/s intervals. Ideally, the velocity error should be less than 1 m/s. The measured error, in both channels, was less than or equal to 0.03 m/s.

4.3 Spectrum Width (W)

The current test confiuration provides CW signals with constant phase. As such, we cannot evaluate the spectrum width at this time. However, the successful completion of the test configuration was the proof of concept for a new test signal generator.

4.4 Differential Reflectivity (Z_{DR})

First the Z_{DR} offset is measured and entered into the signal processor. The measured offset for this system was –0.03 dB. Afterwards, a series of measurements are undertaken to with expected differences between channels. It was expected that the error in Z_{DR} measurement would be no greater than 0.1 dB. This held true with the greatest error being 0.06 dB away from expected.

4.5 Differential Propagation Phase (Φ_{DP})

The differential phase measurement is the measurement of a very small value. As such, the validation procedure looks at the difference at 30 degree intervals throughout the range and in 2 degree intervals from –10 to 10 degrees.

For the entire range, the error was less than 1 degree, throughout the range and less than 0.3 degree for most of the range. At the edge of the range (±180 deg), there was some issue of folding.

For the smaller range, the error was less than 0.3 degree throughout.

4.6 Cross-Polar Correlation Coefficient (ρ_{HV})

Finally, the evaluation of the cross-polar correlation coefficient involves shifting the frequency of one channel with respect to the other channel. First the horizontal channel and then the vertical channel. The maximum difference from the expected value was 0.04.

4.7 Linear Depolarization Ratio (L_{DR})

The Linear Depolarization Ratio measures the how much the signal is changed in orietation. For meteorogical targets it varies grossly over a 30 dB interval. In the validation of this moment, the maximum error was 0.1 dB.

5 CONCLUSION

This paper discusses a test configuration and methodology for polarimetric base moments. For each moment, the signal processing formulae and error estimates were obtained. Criteria for pass or fail were described and justified. Finally, results from an EEC SImultaneous Dual POLarization (SIDPOL) weather radar were presented.

The system test configuration utilized three commercially available waveform generators with phase locking capabilities. One waveform generator produced the burst signal while the other two generators were the horizontal and vertical channel signals respectively. This configuration provided a proof of concept for a new radar test signal generator subsystem currently under development. The new TSG will allow the evaluation of the system using pulsed signals rather than CW signals and will allow the evaluation of the spectrum width.

6 REFERENCES

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