1. INTRODUCTION

Differential phase has many unique properties that set it apart from other polarimetric variables. Among them is its immunity to radar miscalibration, attenuation in precipitation, and partial beam blockage. Specific differential phase $\Phi_{DP}$ is less sensitive to variations in drop size distribution at the higher end of the raindrop spectrum and is less contaminated by hail than radar reflectivity factor $Z$ and differential reflectivity $Z_{DR}$. $K_{DP}$ – based estimates of rainfall are particularly advantageous at shorter radar wavelengths for which $Z$ and $Z_{DR}$ are substantially biased due to attenuation and differential attenuation.

On the negative side are extreme noisiness of $K_{DP}$ for weak weather echoes, lack of adequate radial resolution, and its vulnerability to the gradients of $Z$ and total differential phase $\Phi_{DP}$ within the radar resolution volume. The latter manifests itself as oscillatory behavior of $\Phi_{DP}$ causing spurious values of $K_{DP}$ of both signs. As a result, accurate estimation of $K_{DP}$ in localized convective cells is a challenge. Because of these problems, some researchers avoid using $K_{DP}$ for quantitative rain measurements and hydrometeor classification altogether and rely primarily on $Z$ and $Z_{DR}$.

In this paper, a new procedure for differential phase processing is suggested that ensures the radial resolution of $K_{DP}$ similar to the one for $Z$ (without degrading the statistical accuracy of the $K_{DP}$ estimate) and substantially mitigates the effects of nonuniform beam filling.

2. IMPROVEMENT OF RADIAL RESOLUTION

One of the three drawbacks of $K_{DP}$, its noisiness at lower rain rates, can be easily addressed by not using $K_{DP}$ for rainfall estimation at rates lower than $5 – 10$ mm h$^{-1}$. A synthetic polarimetric algorithm (a leading candidate for operational implementation on the dual-polarization WSR-88D radar) stipulates the use of $Z$ and $Z_{DR}$ if rain rate estimated from $Z$ is less than $6$ mm h$^{-1}$ (Ryzhkov et al. 2005).

Addressing the second $K_{DP}$ deficiency, inadequate radial resolution, requires more significant changes in the processing of differential phase. In the previous studies (e.g., Gorgucci et al. 2000), it was noticed that relatively poor radial resolution of the $K_{DP}$ estimates leads to underestimation of rain in the middle of small-size convective cells and to overestimation at their periphery. As a result, the shape of such cells is often distorted in the $K_{DP}$ or $R(K_{DP})$ fields. This might pose a serious problem for accurate measurement of rain associated with intense, localized convection.

Gorgucci et al. (2000) modeled radial nonuniformity of rain as linear or step functions of range. In this study, we use measured fields of radar reflectivity in the presence of localized convection to quantify a “smearing effect” of $K_{DP}$ processing. For our analysis, we selected data collected by the KOUN WSR-88D polarimetric radar on June 5, 2003 between 11 and 12 UTC over the 50 x 40 km area containing ARS micronetwork of rain gages (Ryzhkov et al. 2005).

As a first step, we simulated $K_{DP}$ data with the radial resolution of 0.5 km which is compatible with the one for radar reflectivity after $Z$ data are averaged over 3 successive range gates. For computing simulated $K_{DP}$, we assume that high-resolution estimates of rain rate from $Z$ and $K_{DP}$ are perfectly matched (Ryzhkov et al. 2005):

$$R(Z) = 0.01710^{0.0714Z} = 44.0K_{DP}^{0.822}. \quad (1)$$

In other words, high-resolution $Z$ data were used to simulate “intrinsic” $K_{DP}$ ($K_{DP}^{(s)}$ hereafter).

As a second step, simulated $K_{DP}$ values were used to generate radial profiles of total differential phase

$$\Phi_{DP} = 2\int K_{DP}^{(s)} dr. \quad (2)$$

Simulated profiles of $\Phi_{DP}$ were then treated as “measured” and a standard procedure was applied to smooth them and to obtain coarser resolution estimates of simulated $K_{DP}^{(s)}$. Two estimates of specific differential phase were obtained from $\Phi_{DP}$ as a slope of a least squares fit for two range averaging intervals, corresponding to 9 and 25 successive gates (Ryzhkov et al. 2005). For any particular range gate, the lightly filtered (9 gates) estimate of $K_{DP}^{(s)}$ is selected if $Z > 40$ dBZ, otherwise the heavily filtered estimate (25 gates) is used. Thus, radial resolution of the $K_{DP}^{(s)}$ estimate is about 6 km for relatively light rain ($R < 12$ mm h$^{-1}$) and about 2 km for more intense rain. Finally, rain rate estimate from simulated values of low-resolution $K_{DP}$ was obtained using Eq (1).

A scatterplot of $R(Z)$ versus $R(K_{DP}^{(s)})$ for a one-hour dataset in the ARS area is shown in Fig. 1. As expected, lowering radial resolution leads to underestimation of rain for higher rain rates (often associated with localized convection) and its overestimation for lighter rain.
The ratio \( R(Z)/R(K_{DP}^{(g)}) \) can be considered as a correction factor for the rain estimate obtained from actually measured \( K_{DP} \):
\[
R_1(K_{DP}) = R(K_{DP}) \left( \frac{R(Z)}{R(K_{DP}^{(g)})} \right). \tag{3}
\]
In (3), \( R(K_{DP}) \) is a low-resolution (2 or 6 km) estimate of rain rate from the measured \( K_{DP} \), and \( R_1(K_{DP}) \) is a high-resolution estimate (0.5 km).

Note that the ratio \( R(Z)/R(K_{DP}^{(g)}) \) depends only on the shape of the high-resolution radial profile of reflectivity and is not sensitive to the type of \( R(Z) \) relation. Hence, estimator \( R_1(K_{DP}) \) combines advantages of \( Z \) - its high resolution and capability to precisely map rain cells, and numerous benefits of \( K_{DP} \) mentioned earlier.

### 3. MITIGATION OF NONUNIFORM BEAM FILLING

The impact of nonuniform beam filling on the quality of \( K_{DP} \) estimates was investigated by Ryzhkov and Zrnic (1998) and Gosset (2004). It was shown that variations of differential phase and radar reflectivity within the radar resolution volume may cause significant perturbations of the radial profile of \( \Phi_{DP} \). These perturbations can be quantified using relatively simple considerations.

Differential phase is determined from the argument of the complex cross-covariance \( R_{hv} \) (Ryzhkov and Zrnic 1998):
\[
R_{hv} = \int Z_{hv}(\theta, \varphi) \exp[i\Phi_{DP}(\theta, \varphi)] I(\theta, \varphi) \, d\theta \, d\varphi, \tag{4}
\]
where “cross-reflectivity” \( Z_{hv} \) is equal to \( \rho_{hv} (Z_hZ_v)^{1/2} \), \( \rho_{hv} \) is a cross-correlation coefficient, \( Z_h, Z_v \) are radar reflectivity factors at horizontal and vertical polarizations expressed in linear scale, and \( I(\theta, \varphi) \) is a two-way antenna pattern.

If linear dependencies of \( \log(Z_{hv}) \) and \( \Phi_{DP} \) on angles \( \theta \) and \( \varphi \) are assumed, then a simple analytical formula for perturbations of the measured \( \Phi_{DP} \) can be easily obtained for a two-dimensional Gaussian shape of the antenna pattern:
\[
\Delta \Phi_{DP} \approx 0.02 \Omega^2 \left( \frac{dZ_{hv}}{d\theta} \frac{d\Phi_{DP}}{d\theta} + \frac{dZ_{hv}}{d\varphi} \frac{d\Phi_{DP}}{d\varphi} \right), \tag{5}
\]
where \( \Omega \) is a one-way 3 dB antenna pattern width,
\[
Z_{hv} = 10 \log(Z_{hv}) = Z_{hv}(0) + \frac{dZ_{hv}}{d\theta} \theta + \frac{dZ_{hv}}{d\varphi} \varphi, \tag{6}
\]
and
\[
\Phi_{DP} = \Phi_{DP}(0) + \frac{d\Phi_{DP}}{d\theta} \theta + \frac{d\Phi_{DP}}{d\varphi} \varphi. \tag{7}
\]

Eq (5) shows that the perturbation of \( \Phi_{DP} \) occurs if both \( \Phi_{DP} \) and \( Z \) vary within the radar resolution volume. Transverse gradients of \( \Phi_{DP} \) always increase with range (if the contribution of backscatter differential phase is negligible), hence \( \Phi_{DP} \) perturbations are more pronounced at longer ranges from the radar. In situ fons of localized convection the magnitude of \( \Delta \Phi_{DP} \) can increase dramatically causing large positive and negative biases in the \( K_{DP} \) estimates obtained from the perturbed radial profile of \( \Phi_{DP} \). The corresponding errors in rain rate estimates can be much larger than those due to inadequate radial resolution of \( K_{DP} \).

Most common manifestation of this artifact is the appearance of negative values of \( K_{DP} \) observed at the rear side of convective rain cells. It should be emphasized that such negative \( K_{DP} \) are always coupled with positively biased \( K_{DP} \) that are not easily recognizable but equally detrimental for rainfall measurements.

Nonuniform beam filling also reduces the magnitude of the cross-correlation coefficient \( \rho_{hv} \). Such a reduction is described by the following formula:
\[
\rho_{hv}^{(g)} = \rho_{hv} \exp(-0.045 \Omega^2 \left( \left( \frac{d\Phi_{DP}}{d\theta} \right)^2 + \left( \frac{d\Phi_{DP}}{d\varphi} \right)^2 \right)). \tag{8}
\]

The drop in \( \rho_{hv} \) causes increase of statistical errors in estimates of \( K_{DP} \) that makes \( K_{DP} \) less reliable.

Since differential phase and its gradients are directly proportional to the radar frequency, the impact of nonuniform beam filling is much more pronounced at C and X bands (Ryzhkov and Zrnic 2005). At shorter wavelengths, perturbations of the radial profiles of \( \Phi_{DP} \) are caused by both nonuniform beam filling and backscatter differential phase (Gosset 2004).

The areas of potentially contaminated \( K_{DP} \) can be identified by examining transverse gradients of \( Z \) and \( \Phi_{DP} \) from the data at adjacent rays. Another way to avoid apparently wrong estimates of rain rate from \( K_{DP} \) is to check consistency between \( R(K_{DP}) \) and \( R(Z) \) or \( R(Z, Z_{DR}) \). The latter rainfall estimates are much less affected by nonuniform beam filling. Unfortunately, both \( R(Z) \) and \( R(Z, Z_{DR}) \) are not reliable estimators of rain in the presence of hail or “large drops”. In the former case, both relations dramatically overestimate rain, whereas in the areas of “large drops” (i.e., in updrafts or proximity of
hail cores) the R(Z) tends to overestimate and R(Z, Z_{DR}) - to underestimate rain due to very high Z_{DR}.

As one of the possible solutions, we suggest to compare R(K_{DP}) with R_{m}(Z, Z_{DR}) - the mean of the two estimates: R(Z) and R(Z, Z_{DR}) after they are capped at the level of 100 mm h^{-1}. Such comparison should be preformed only for moderate and heavy rain (R(Z) > 6 mm h^{-1}). If

\[ 0.2 \times R_{m}(Z, Z_{DR}) < R(K_{DP}) < 2.0 \times R_{m}(Z, Z_{DR}), \tag{9} \]

then the estimate of R(K_{DP}) is accepted, otherwise R(K_{DP}) has to be replaced with R_{m}(Z, Z_{DR}). After R(K_{DP}) is corrected for “smearing” effect using Eq (3) and checked for consistency with R_{m}(Z, Z_{DR}) using Eq (9), it can be used in the synthetic algorithm as described by Ryzhkov et al. (2005).

4. EXAMPLE

Here we demonstrate how the R(K_{DP}) estimate is improved after a two-step correction procedure is applied. We select the case on May 20, 2003 when a cluster of relatively small and intense convective cells was observed in the ARS area. Comparison of hourly accumulations from the KOUN radar and ARS gages indicates significant underestimation of rain total for one of the gages with the synthetic algorithm (Ryzhkov et al. 2003, page 87). Nonreliable estimate of K_{DP} is a primary cause of the problem.

Figs. 2 and 3 illustrate radial profiles of R(Z) and uncorrected and corrected profiles of R(K_{DP}) along the radial that is closest to the gage (Az = 237°). The two figures are from two successive radar scans at the time of most intense rain over the gage located at the distance 53.4 km from the radar.

Fig. 2. Radial profiles of R(Z) (thin lines) and uncorrected and corrected R(K_{DP}) (thick lines). May 20, 2003; 0332 UTC, Az = 237°.

Fig. 3. Same as in Fig. 2 but for 0338 UTC.

In Fig. 2a, the shapes of uncorrected profile of R(K_{DP}) and R(Z) are quite different, their maxima and minima
do not coincide, and large negative rain rates from $K_{DP}$ are seen at about 58 km from the radar. The corresponding profiles in Fig. 3a are better matched but large negative $R(K_{DP})$ is measured right above the gage (53 – 54 km).

Applying Eq (3) (step 1) restores radial resolution of the $R(K_{DP})$ estimate (Fig. 2b, 3b) and consistency check (9) (step 2) eliminates negative excursions of $R(K_{DP})$ whereby making it more realistic and in accord with the $R(Z)$ profile. As a result, the hourly rainfall estimate from the synthetic algorithm was considerably improved for this particular gage.

The performance of the newly suggested scheme for $K_{DP}$ processing was further validated by Kang et al. (2005) for several rain events.

5. REFERENCES


Kang, M., S.E. Giangrande, A.V. Ryzhkov, and D. Lee, 2005: Polarimetric rainfall measurements in the areas of localized strong convection. *This volume*.

