P9R.2 MEASUREMENTS OF POLARIMETRIC PARAMETERS AT LOW SIGNAL-TO-NOISE RATIOS

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1. Introduction

The **S**imultaneous transmission and reception of electromagnetic waves with **H**orizontal and **V**ertical polarizations (**SHV**) has been implemented on the polarimetric research and development (WSR-88D) radar in Norman (Doviak et al. 2000). Six variables are measured with the radar in each radar resolution volume: reflectivity, Z, Doppler velocity, v, spectrum width, σ_v, differential reflectivity, Z_{DR}, differential phase, φ_{dp}, and modulus of the copolar correlation coefficient, ρ_{hv}. The first three are the base radar moments of the WSR-88D, the latter three are polarimetric variables. Another polarimetric variable, the specific differential phase, K_{dp}, is calculated from φ_{dp}. Definitions of the parameters can be found in Doviak and Zrnic (1993) and Bringi and Chandrasekar (2001).

On the WSR-88D, the spectral moments are displayed and stored if signal-to-noise ratios, SNR, exceed thresholds (2 for Z and 3.5 dB v and σ_v). Compatible thresholds will be applied to the polarimetric variables. Hence, there is need to considering polarimetric estimates at low SNR. The polarimetric estimates are prone to noise bias at SNR less than 15 dB. This vulnerability becomes more pronounced with range due to the drop of the scattered power.

In SHV polarimetric configuration, the transmit power is split into two channels that makes SNR in each channels 3-dB less comparing to thepower on current WSR-88Ds. In this paper, we consider two problems related to low SNR: 1) how to avoid noise impact on Z_{DR} and ρ_{hv} measurements (sections 2 and 3) and 2) how to make effective SNR larger with special signal processing (section 4).

2. Uncertainty of the noise level

In the SHV mode, differential reflectivity, the differential phase and modulus of the copolar correlation coefficient are calculated as (see e.g., Doviak and Zrnic 1993; Bringi and Chandrasekar 2001):

$$\hat{Z}_{DR} = 10\log\frac{\hat{P}_h - N_h}{\hat{P}_v - N_v}, \qquad (1)$$

$$\hat{\varphi}_{dp} = \arg(\hat{R}_{hv}), \qquad (2)$$

$$\hat{\rho}_{hv} = \frac{|\hat{R}_{hv}|}{[(\hat{P}_h - N_h)(\hat{P}_v - N_v)]^{1/2}}, \qquad (3)$$

where the circumflex denotes estimates, $\hat{P}_h$ and $\hat{P}_v$ are the estimates of the powers in the channels for horizontally (h) and vertically (v) polarized waves, N_h and N_v are the mean noise powers in the channels, and $\hat{R}_{hv}$ is the estimate of the copolar correlation which is calculated from complex voltages $e_m^{(h)}$ and $e_m^{(v)}$ in the H and V channels as

$$\hat{R}_{hv} = \frac{1}{M}\sum_{m=1}^{M} e_m^{(h)} e_m^{(v)*}; \qquad (4)$$

M is the number of samples used in the estimate, m numerates the samples, and the asterisk denotes complex conjugate. It follows from (1) and (3) that Z_{DR} and ρ_{hv} depend on the weather signal powers in the channels $\hat{S}_h$ and $\hat{S}_v$ which are obtained as:

$$\hat{S}_h = \hat{P}_h - N_h, \quad \hat{S}_v = \hat{P}_v - N_v. \qquad (5)$$

We will refer to relations (1) to (3) as conventional estimates.

System noise is measured on WSR-88D at elevation 22°. Then this noise is used in (5) at low elevations in the presence of precipitation during the whole volume coverage pattern. It is known that system noise is different for different elevations due to ground noise and thermal noise from precipitation. Internal system noise also varies over time. It is seen from (1) and (3) that if the noise powers differ from true noise, the estimates of Z_{DR} and ρ_{hv} are biased. Consider briefly the following three sources that change system noise: thermal radiation from precipitation and the ground, time

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variations of the internal system noise, and electromagnetic emission from thunderstorms.

It is well known that thermal radiation coming to the radar antenna from precipitation increases the noise level hence this increase depends on the antenna pointing direction. On polarized KOUN (WSR-88D), we have observed noise increase of 0.8 dB. At S-band, attenuation can reach 8 dB (e.g., Ryzhkov and Zrnic 1995) and corresponding noise increase can exceed 1 dB. At X-band, Fabry 2003 observed 1-dB noise variations due to thermal noise from precipitation. Let N_a be the power of additional noise, then the noise increase of 0.8 dB (i.e., $10\log[(N_h + N_a)/N_h] = 0.8$) corresponds to $N_a/N_h = 0.2$ and 1.5-dB increase corresponds to $N_a/N_h = 0.4$. Here we consider noise increase due to thermal radiation from the ground and precipitation up to 1.5 dB.

The internal system noise varies over time due to hardware imperfections. These would influence formulas (1) and (3) unless the change is computed for each radial. Fig. 1 presents an example of noise records from the KOUN's H channel. With the antenna in the park position (azimuth = 0º; elevation = 22º), 400 consecutive range gates on a radial were split in four equal parts and the mean noise power was calculated for each part; 128 samples were averaged for each range gate so that four estimates of the mean noise power were obtained. This measurement was made over approximately 50 seconds and the result is presented in Fig. 1. It is seen that all four curves are highly synchronous exhibiting time variations of the gain. Time scale of these variations is of few seconds and variations are about 1 dB. Such variations we observe frequently but not all the time; most of the time they are within 0.5 dB.

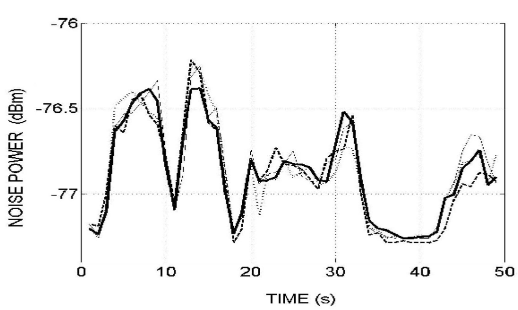

Fig. 1. Temporal variations of the noise level in the horizontal channel of the WSR-88D KOUN. 30 March 2004, 2343 UT.

Thunderstorms emit radiation in a broad frequency band including S-band. This radiation can be intercepted by the antenna and results in excessive noise. Fig. 2 presents two consecutive reflectivity profiles beyond 50 km recorded through a heavy thunderstorm; noise is seen beyond 68 km since no thresholds have been applied. Time interval between the records is 263 ms. The number of samples per estimate is 256. One can see a change of about 10 dB of the noise level. Such large changes are less frequent than smaller ones. This type of noise is not thermal but is white within the bandwidth of the radar receiver.

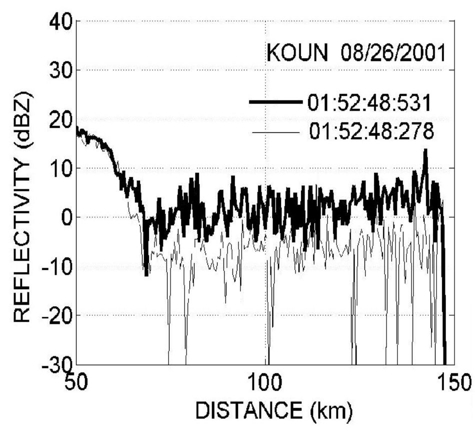

Fig. 2. Two reflectivity profiles. Azimuth is 35.2º and the elevation is 8.6º. UTC time is shown in a format of hour:minute:second:millisecond at the beginning of the records. Date is August 26, 2001.

We conclude that estimating noise level variations to within 1 dB requires separate computations for each radial; this is still few years into the future for the operational network. Uncompensated noise biases the estimates of differential reflectivity and the copolar correlation coefficient. In Fig. 3, biases of Z_{DR} and ρ_{hv} are plotted for $N_a/N_h = 0.2$ and 0.4 that correspond to 0.8 and 1.5 dB of noise increase. It is seen from the figure that biases of differential reflectivity can exceed 0.1 dB in magnitude for SNR less than 13 dB. On KOUN, hardware accuracy of Z_{DR} measurements is 0.1 dB (Zrnic et al. 2005) so that noise variations between 1.1 dB and 1.5 dB can bias Z_{DR} by a larger value than hardware uncertainties. The copolar correlation coefficient is more sensitive to uncompensated noise: its bias can be 0.005 in magnitude for SNR as high as 18 dB. At SNR of about 3.5 dB, 1.5 dB of noise deviation can bias Z_{DR} and ρ_{hv} much more than the hardware inaccuracy. Thus, it is desirable to devise algorithms immune to the level of system noise. Such estimators are considered in the following section.

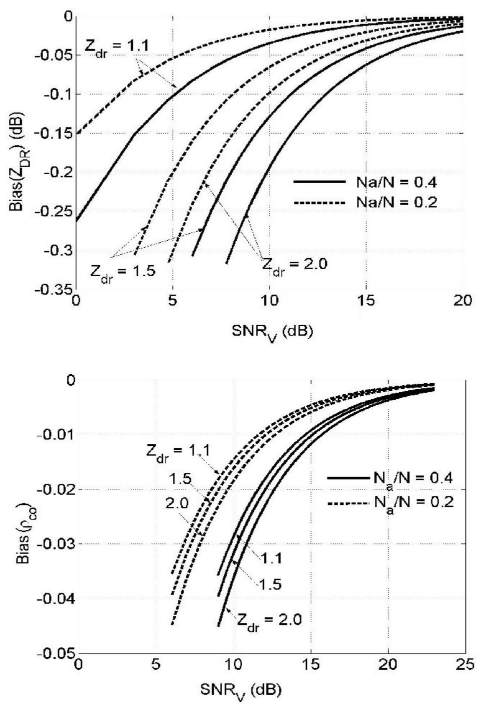

Fig. 3. Biases of the $\hat{Z}_{DR}$ (top) and $\hat{\rho}_{hv}$ (bottom) estimates due to additional white noise N_a for equal noise levels $N_h = N_v = N$ and two N_a/N ratios.

3. Z_{DR} and ρ_{hv} estimates free from noise bias

Following the notations by Doviak and Zrnic (1993) we write the correlation functions for the H and V channels at lag T as:

$$R_h(T) = S_h \rho_h(T) \exp(j\pi v_h / v_a), \qquad (6a)$$

$$R_v(T) = S_v \rho_v(T) \exp(j\pi v_v / v_a), \qquad (6b)$$

where the superscripts h and v denote the parameters that are calculated using the pulse trains in the H and V channels, T is the pulse repetition interval (T = 1/PRF), v_a is the unambiguous velocity ($v_a = \lambda/4T$, λ is the wavelength), $\rho_h(T)$, $\rho_v(T)$ are the temporal correlation coefficients, and j is imaginary one. Values without the circumflex are true means, for instance $< \hat{R}_h(mT) >= R_h(mT)$, where the brackets stand for ensemble average.

Copolar correlation $R_{hv}(nT)$ can be calculated for arbitrary lag n similarly to (4). Assuming that ρ_{hv} is not dependent on time and is determined by average shapes, the mean canting angles of the hydrometeors, and the drop size distribution we write

$$R_{hv}(T) = (S_h S_v)^{1/2} \rho_{hv}\ \rho_{(hv)}(T) \cdot \exp(j\pi v_{hv} / v_a + j\varphi_{dp}), \qquad (7a)$$

$$R_{hv}(0) = R_{hv} = (S_h S_v)^{1/2} \rho_{hv} \exp(j\varphi_{dp}), \qquad (7b)$$

where time variations of shapes and the canting angles affect the temporal correlation coefficient $\rho_{(hv)}(T)$, and the superscript hv mean that the parameters are calculated using the pulse trains in both H and V channels. To shorten the notations we write $R_{hv}(0) = R_{hv}$. The modules of functions (6) and (7) do not depend on the Doppler velocities:

$$|R_h(T)| = S_h \rho^{(h)}(T),\ \ |R_v(T)| = S_v \rho^{(v)}(T),$$

$$|R_{hv}(T)| = (S_h S_v)^{1/2} \rho_{hv} \rho_{(hv)}(T), \qquad (8)$$

Three temporal correlation coefficients in the latter equations can differ. They are functions of the spectral width and thus depend on the velocity spread, oscillations and wobbling of the hydrometeors. Primary contribution to the spectral width is the spread of velocities; contributions from wobbling and oscillations are small (Zrnic and Doviak 1998, Melnikov and Zrnic 2003). That is why signals in the two SHV channels are highly correlated and we can write $\rho_h(T) = \rho_v(T) = \rho_{(hv)}(T) = \rho(T)$. Then Z_{DR} and ρ_{hv} can be obtained from (8) as:

$$Z_{DR} = 10\log\frac{S_h}{S_v} = 10\log\frac{|R_h(T)|}{|R_v(T)|}. \qquad (9)$$

$$\rho_{hv} = \frac{|R_{hv}(T)|}{(|R_h(T)||R_v(T)|)^{1/2}}. \qquad (10)$$

The modules in (9) and (10) do not depend on noise so that these two estimates are not biased by white noise.

Independence of estimators (9) and (10) of noise bias is demonstrated in Figs. 4 and 5. In the figures, vertical cross-sections of a non-precipitating cloud are depicted. The cloud was weak and radar returns had low SNR. Radar data are time series sequences of in phase and quadrature phase components (I and Q) from both H and V channels. A digital ground clutter filter with the notch width of ±1 m s^{-1} was applied to the data before processing. SNR threshold for data presentation in Figs 4 and 5 is 0 dB. To make difference in estimator's performance visually clear, no noise correction was applied to the conventional estimators.

In Fig. 4, ZDR fields are presented calculated with the conventional and 1-lag estimators. It is seen that the figure for conventional estimator (left image) has much more reds than the 1-lag one (right image). Increase of ZDR with the conventional estimator is attributed to noise impact. It is worth noting that the radar echo below 2.5 km is the return from insects with high intrinsic differential reflectivity.

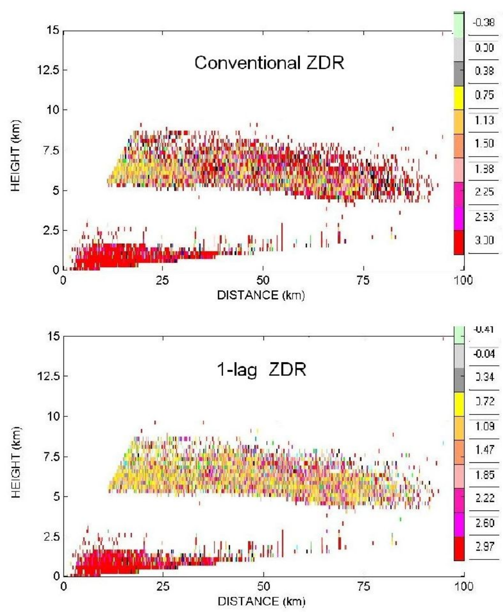

Fig. 4. Differential reflectivities of a non-precipitating cloud measured with the conventional (top) and 1-lag (bottom) estimators. WSR-88D KOUN, April 4, 2005, 0236 UT, azimuth is 89.6°.

In Fig. 5, fields of ρ_{hv} are presented calculated with the conventional and 1-lag estimators. It is seen that the figure for conventional estimator (left image) has much lower values than the 1-lag one (right image). Most of the 1-lag values are close to one.

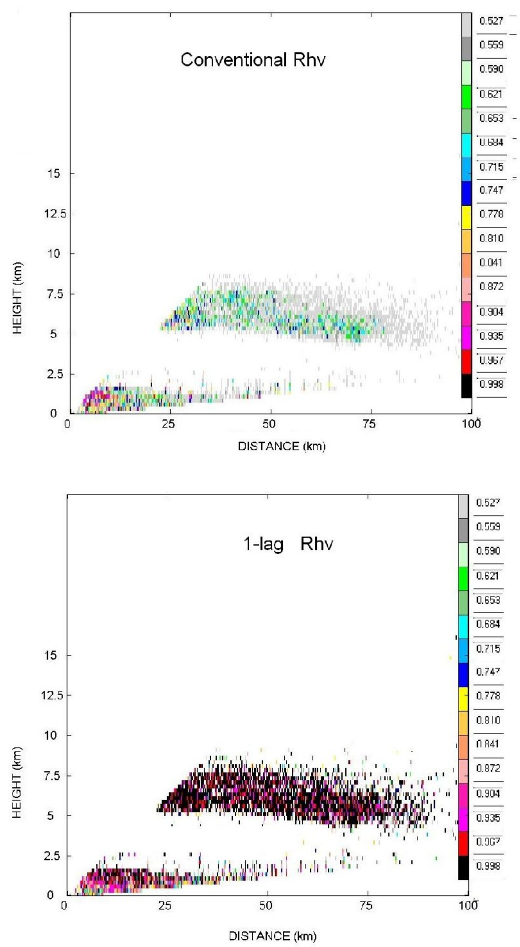

Fig. 5. Fields of the copolar correlation coefficients measured with the conventional (top) and 1-lag (bottom) estimators. Other parameters are as in Fig. 4.

We have conducted computer simulations of the conventional and 1-lag estimators for various SNR that showed that the 1-lag algorithms are free from noise bias. The radar data confirms this conclusion. The simulations also show that the 1-lag estimators have lower standard deviations than the conventional ones for SNR lower than 15 dB and spectrum widths lower than 6 m s^{-1} (S-band radar).

4. Possibilities to regain radar sensitivity

On the current WSR-88D radars, the received power is in one polarization (same as transmitted). In the SHV mode of the WSR-88D, the received power is partitioned into two channels hence the SNR per channel is two times smaller than what it is presently. Note, that in alternate transmission via a ferrite switch the loss in power could be even larger! So the SHV scheme leads to the loss of sensitivity of 3 dB in each polarimetric channel and the question is: can the loss be restored? We show here that this can be done to some extend.

We can capitalize on strong correlation between weather signals in the H and V channels. Summing voltages in the channels we can form this signal:

$$e_{sum} = e^{(h)} + e^{(v)} = s^{(h)} + n^{(h)} + s^{(v)} + n^{(v)}, \qquad (11)$$

where s and n are weather signals and noise voltages corresponding to the channels marked with the superscripts. The mean power of the sum signal is

$$P_{sum} = < [e^{(h)} + e^{(v)}][e^{(h)*} + e^{(v)*}] > = S_h + S_v + < s^{(h)} s^{(v)*} > + < s^{(h)} s^{(v)*} > + < n^{(h)} n^{(h)*} > + < n^{(v)} n^{(v)*} > .$$

We used the fact that the mean voltages in the channels are zero and the noise voltages in the channels are uncorrelated so the latter equation can be written as:

$$P_{sum} = S_h + S_v + 2(S_h S_v)^{1/2} \rho_{hv} \cos\varphi_{dp} + N_h + N_v .$$

Signal-to-noise ratio for the sum signal is

$$SNR_{sum} = \frac{S_h + S_v + 2(S_h S_v)^{1/2} \rho_{hv} \cos\varphi_{dp}}{N_h + N_v} . \qquad (12)$$

It follows from the latter that if there is no differential phase between the signals in the channels, SNR_{sum} is

$$SNR_{sum} = 2 SNR_h \frac{Z_{dr} + 2 Z_{dr}^{1/2} \rho_{hv} + 1}{4 Z_{dr}} . \qquad (13)$$

where Z_{dr} is differential reflectivity in linear units. Thus to achieve (13), we can measure the differential phase and then shift the voltages in the vertical channel by this phase. In cases with low signal power where it is difficult to measure φ_{dp}, two alternatives can be applied. If low reflecting echoes are not behind other echoes, the differential phase is the system differential phase which is known with sufficient accuracy. If low reflecting echoes are behind other echoes, the differential phase at low reflecting echoes is the phase φ_{dp} at the edge of a obscuring echo which is known also (i.e., can be measured).

Weather signals have the modulus of the correlation coefficient ρ_{hv} very close to 1 and we want to restore weak echoes, i.e., powers reflected from small particles for which differential reflectivity (the ratio) is close to 1. It follows from (13) that for such regions, we have $SNR_{sum} \approx 2\ SNR_h$. If full energy goes to the H channel, the signal-to noise ratio is 2 SNR_h . It means that the summing of the voltages allows for almost full restoration of radar sensitivity. Because the differential phase is measured independently of powers, all we need to do is to shift the vertical voltages by the differential phase and calculate the power of summed voltages.

In Fig. 6, reflectivities and Doppler velocities are depicted. The left images were calculated for the single H channel with SNR threshold of 3.5 dB. The same threshold was applied for the right images that were calculated by coherent summation of signals in the H and V channels. It is seen that fields (b) and (d) have more points than images (a) and (c), thus more estimates have SNR larger than the noise threshold.

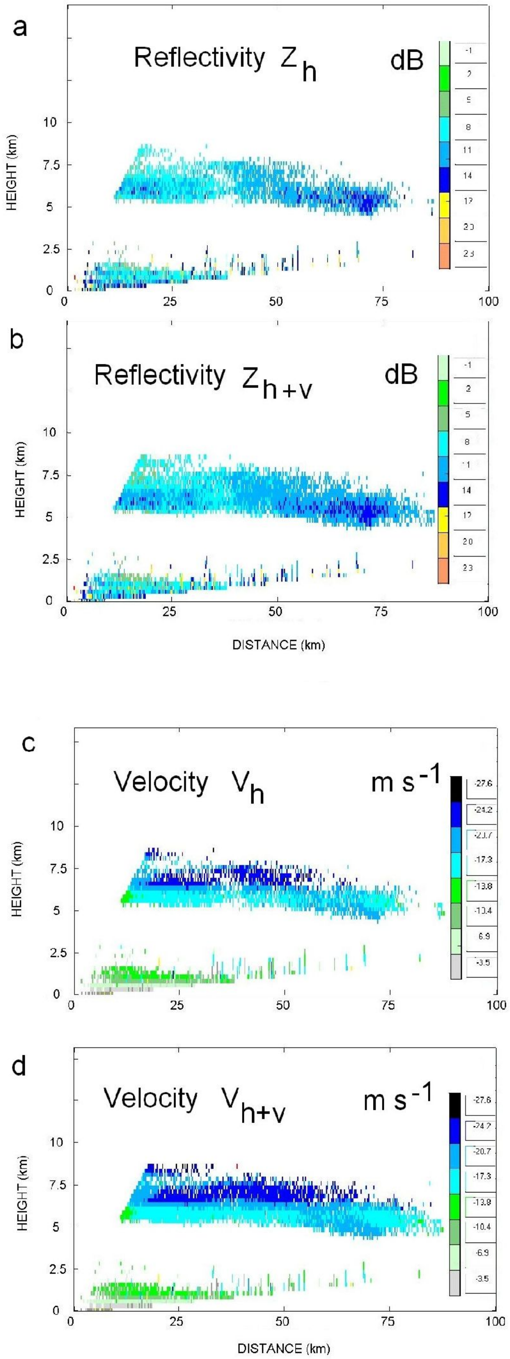

Fig. 6. Vertical cross-section of a non-precipitating cloud obtained with WSR-88D KOUN. The reflectivity fields have been calculated in the H-channel (a) and using coherent summation of signals in the H and V channels (b). The velocity field in the H channel (c) and one obtained with coherent summation (d). The date and time are the same as in Fig. 4.

Another approach to increase the accuracy of the Doppler velocity and spectrum width measurements is averaging corresponding estimates from the two channels (Doviak and Zrnic 1998). A combined correlation function $R(T)$ can be constructed from the correlation functions for the H and V channels $R_h(T)$ and $R_v(T)$ as

$$R(T) = \frac{1}{2}\left[R_h(T) + R_v(T)\right]. \qquad (14)$$

Then the Doppler velocity is obtained as the argument of the correlation function: $v = \arg(R(T))$. This estimator is unbiased and its variance is

$$<\delta v^2> = \frac{v_a^2}{2\pi^2\rho^2(T)}\left\{\left(\frac{Z_{dr}}{Z_{dr}+1}\right)^2 \frac{2SNR_h[1-\rho^2(T)]+1}{(M-1)SNR_h^2} + \left(\frac{1}{Z_{dr}+1}\right)^2 \frac{2SNR_v[1-\rho^2(T)]+1}{(M-1)SNR_v^2} + \frac{Z_{dr}^2 + 2Z_{dr}\rho_{co}^2 + 1}{(Z_{dr}+1)^2}\frac{[1-\rho^2(T)]}{M_{I1}}\right\}. \qquad (15)$$

An improvement to the velocity estimate can be calculated from a ratio of standard deviations obtained from the variances (15) and variances in Doppler velocity estimate for one channel. This ratio is plotted in Fig. 7. For small particles, Z_{dr} is close to 1 and one can see that there is a substantial improvement in the measurements for SNR less than 10 dB. At SNR about 3.5 dB, the curves do not drop below 0.83, that is sufficiently close to 1 (no degradation).

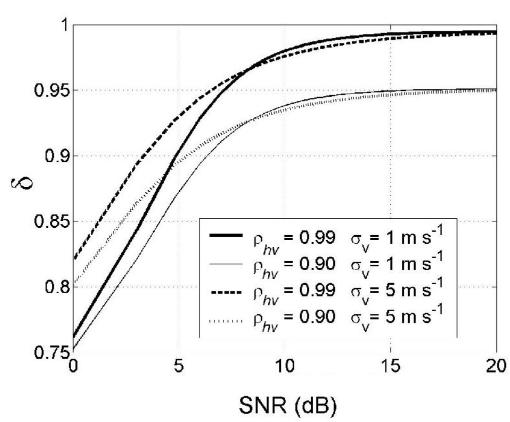

Fig. 7. The ratio of standard deviations of velocity estimates (15) and the one for the H-channel as a function of signal-to-noise ratio for the spectrum widths of 1 and 5 m s^{-1}. Z_{DR} = 0 dB, M = 64.

5. Conclusions

Conventional Z_{DR} and ρ_{hv} estimates are prone to bias due to system noise variations. The noise levels in the radar channels are hard to control with the accuracy better than 1.5 dB. Additional thermal noise from ground and precipitation, wideband electrical noise from thunderstorms, and variations of system noise make this control very difficult. Noise uncertainty of 1.5 dB can bias estimates of differential reflectivity and the modulus of the copolar correlation coefficient by much larger values than inaccuracy associated with the polarimetric WSR-88D hardware (KOUN).

Estimators of Z_{DR} and ρ_{hv} free from noise bias were devised. The method is based on calculation of the 1-lag cross correlation coefficients that are immune to bias from white noise. Our simulations and processed radar data confirm this conclusion. In comparison with the conventional estimators, the 1-lag estimators have lower standard deviations for spectral width lower than 6 m s^{-1} (S-band radars).

WSR-88D KOUN has two almost identical channels, i.e., H and V channels. Coherent summation of signals in the channels allows for increase of effective SNR in calculations of reflectivity, the Doppler velocity and spectrum width. Coherent summation is summation of voltages in the H and V channels with compensated differential shift between these. The sum of the correlation coefficients calculated for both channels can also be used to increase the accuracy of the Doppler velocity and spectrum width measurements at low SNR.

Acknowledgments

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