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OPTIMAL SAMPLING STRATEGIES FOR HAZARDOUS WEATHER DETECTION USING NETWORKS OF DYNAMICALLY ADAPTIVE DOPPLER RADARS

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1. INTRODUCTION

The National Science Foundation Engineering Research Center for Collaborative Adaptive Sensing of the Atmosphere (CASA) is developing a revolutionary new paradigm for overcoming fundamental limitations in current radar technology such as the inability to sample the lower parts of the atmosphere (McLaughlin, 2005). CASA was created in fall 2003 and is led by the University of Massachusetts at Amherst with several partners including the University of Oklahoma, Colorado State University, and the University of Puerto Rico at Mayaguez. CASA is establishing a system of distributed, collaborative, and adaptive sensor (DCAS) networks that are unique because they can dynamically adjust their scanning strategies and other attributes collaboratively with other CASA radars to sense multiple atmospheric phenomena while at the same time meeting multiple end user needs.

In the first phase of its research program, CASA is placing test beds of small, inexpensive, low-power Doppler weather radars on existing infrastructures, such as cell phone towers, to test the DCAS concept. This network, called NetRad, will consist of four dual-polarization, mechanically-scanning Doppler radars that will be operating in central Oklahoma beginning in late winter of 2005. By 2008, the network is expected be expanded and include phased-array radars.

The DCAS networks are designed to overcome the fundamental limitations of current approaches to sensing and predicting atmospheric hazards. Distributed refers to the use of large numbers of solid-state radars that are spaced appropriately to overcome blockage due to the Earth's curvature, resolution degradation caused by beam spreading, and large temporal sampling intervals resulting from today's use of mechanically scanned antennas.

The radars can operate collaboratively by means of coordinated targeting of multiple radar beams based on atmospheric and hydrologic analysis tool such as detection, predicting, and tracking algorithms. By utilizing this collaboration, the system is able to determine needs and allocate resources such as radiated power, beam position, and polarization diversity towards regions of the atmosphere where a particular threat exists. The adaptive capabilities refer to the ability of the CASA radars and the associated computing and communications infrastructure to rapidly reconfigure in response to changing conditions in a manner that optimizes the response to competing end user demands. For example, this system could track tornadoes for public warning while simultaneously collecting information on the parent storm and providing quantitative precipitation estimates for input to hydrologic prediction models.

The objective of this paper is to develop a technique to apply, test, and analyze a sampling strategy to provide preliminary results of how to use the CASA radars to optimally adaptively sample the atmosphere. Although it may seem logical to simply scan a real tornado with as fine a temporal and spatial sampling as possible, that approach may be unnecessary and waste resources, especially when other phenomena are present simultaneously and competing for the same resource.

We approach this problem using an idealized, analytic vortex to serve as a proxy for a tornado. We then sample this flow field, as if it were being observed by one CASA radar, and use variational techniques to fit the pseudo-observations to the models of an idealized tornado vortex. We seek the minimum in the cost function, which defines the best (in a least squares sense) fit between model and pseudo-observations, across a variety of parameters including but not limited to azimuthal sampling interval, number of vertical levels sampled, distance of the radar from the vortex, and the number of radars available. This approach is similar to that used by Wood (1997) in the context of NEXRAD. In this paper, we evaluate the azimuthal sampling interval and the distance of the radar from the center of the vortex.

2. RANKINE COMBINED VORTEX APPROXIMATION

We use an approximation to the Rankine (1901) combined vortex (RCV) to prescribe the flow field for

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our simulated tornado. The RCV has a rotational velocity that increases linearly from zero at the center of the vortex to a maximum at the core radius. Beyond the core radius the rotational velocity decreases, with the velocity being inversely proportional to the distance from the rotation center. This traditional form of the RCV contains a cusp at the radius of maximum winds that represents a discontinuity. In order to obtain the Rankine combined vortex approximation (RCVA), we first consider the traditional RCV:

$$v = V_{\max} f(r) \tag{1}$$

where $V_{\max}$ is the maximum tangential wind and

$$f(r) = \begin{Bmatrix} \frac{r}{R}, & 0 \le r < R \\ \frac{R}{r}, & R \le r \end{Bmatrix} \tag{2}$$

where *r* is the radial distance to the vortex center and *R* is the core radius at which maximum tangential wind V_{max} occurs. A graph of this function (Figure 1) illustrates the discontinuity at the dimensionless radius of 4.

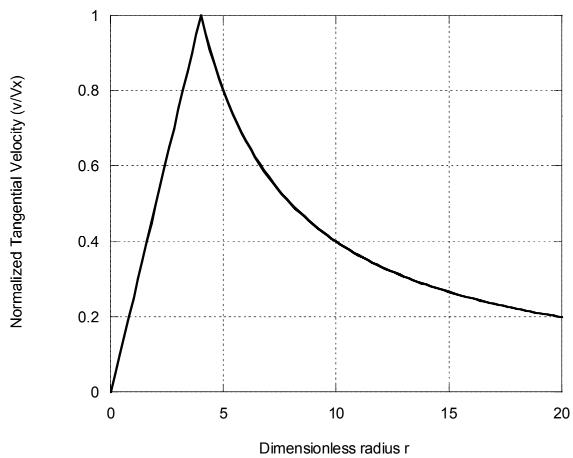

Figure 1: Tangential velocity in a Rankine combined vortex with cusp at dimensionless radius of 4.

Because of the discontinuous nature of the first derivative of the RCV, difficulties in solving the cost function occur in the retrieval technique. In order to overcome this difficulty, a new function with no discontinuity in the core radius yet retains the features of the RCV can be introduced (White, personal communication):

$$\phi_n(r,R) = \frac{2nR^{2n-1}r}{(2n-1)R^{2n} + r^{2n}} \tag{3}$$

where n =1,2…. After testing various combinations of (3) with different values of n, the approximating function that best fits (2) and that is used herein as the RCVA is:

$$\hat{f}(r) = \frac{1}{2}\left[\phi_1(r) + \phi_2(r)\right] = \frac{\phi_1 + \phi_2}{2} \tag{4}$$

Figure 2 shows a comparison of the approximation's fit (labeled as "(phi1+phi2)/2") to the traditional RCV (labeled as "RCV or f(r)"), as well as the case where n =1 and n =2 for Equation (3).

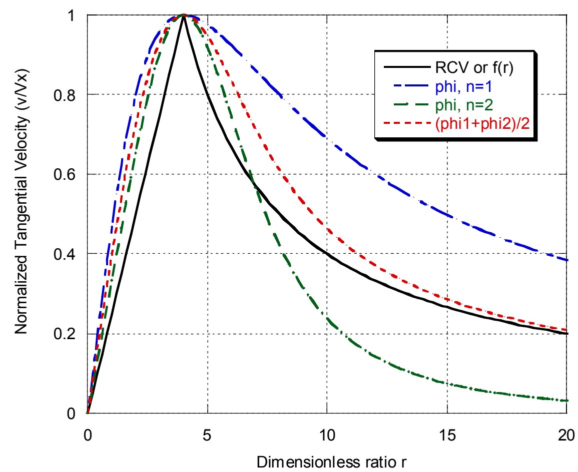

Figure 2: Comparison of RCV (Equation 2) to the RCVA (Equation 3) with varying values of n.

In this paper, the RCVA idealized flow will be used as the proxy of a tornado and as the model to which the flow is fit using variational techniques. A similar approach was used by Wood (1997).

3. VARIATIONAL RETRIEVAL TECHNIQUE

In order to determine how to best observe the tornado proxy with CASA radars, a one dimensional analysis technique is used that is based on the variational principle used by Wood (1997). This variational principle optimally estimates the maximum tangential wind speed, V_{max}, and core radius at which this wind speed occurs, R_{max}. Simulated Doppler velocities are generated by the combined NSSL Doppler radar simulation and retrieval technique; the velocities correspond to the RCVA idealized flow discussed in section 2 but are sampled in a manner that includes beam broadening, weighting along the beam center, etc. Simulation of the radar sampling process does not directly follow that of an actual radar because the mean Doppler velocity is calculated by averaging the Doppler velocity components within the effective radar beam. Normally, the radar pulses are averaged to produce a simulated mean Doppler velocity value. The program retrieves the maximum tangential velocity and the core radius of an axisymetric vortex from a single Doppler velocity signature of the vortex.

In order to use the variational technique, we must first calculate initial guesses of V_{max} and R_{max} that will be used to solve a set of nonlinear equations. By solving the set of nonlinear equations, we can then solve a set of linear equations using Gaussian elimination to get the retrieved values of V_{max} and R_{max}.

There are several steps in determining the initial guesses of V_{max} and R_{max} that serve as the input values to the cost function. These guesses, known as V_{rot} and R_a, must be close enough to the solution to give convergence and are given by:

$$V_{rot} = \frac{\Delta V_d}{2} \tag{5}$$

where V_{rot} is the rotational Doppler velocity of the vortex and V_d is the difference in the incoming (V_-) and outgoing Doppler velocity (V_+) peaks of the vortex and is defined as:

$$\Delta V_d = \frac{V_+ - V_-}{2} \tag{6}$$

The apparent radius, R_a, is given by:

$$R_a = \frac{D_a}{2} \tag{7}$$

where D_a is the apparent diameter and is the distance between the incoming and outgoing Doppler velocity peaks.

After specifying input values such as beamwidth and azimuthal interval, the range, elevation, and azimuth subpoints within the beamwidth volume and the two-way antenna pattern can be calculated. From this, the radial distance, *r*, from the circulation center to the subpoints of beamwidth volume center in Cartesian coordinates can be computed.

Using the radial distance *r*, the radial variation of the tangential component of the RCVA can be found by using $n = 1$ and $n = 2$:

$$\begin{aligned} v_{rcva} &= V_{\max} \frac{\phi_1 + \phi_2}{2} \\ &= V_{\max} \left(\frac{R_{\max} r}{R_{\max}{}^2 + r^2} + \frac{2 R_{\max}{}^3 r}{3 R_{\max}{}^4 + r^4} \right) \end{aligned} \tag{8}$$

With the tangential component, the mean Doppler velocity can be calculated within the beamwidth volume. Now the initial guesses for V_{max} and R_{max} are calculated.

These initial guesses will be used in the variational technique where Newton's Method is used to solve a set of nonlinear equations. Newton's Method essentially estimates the partial derivatives of the function by difference quotients, where a small change in the value of the variable is made called delta (Gerald and Wheatley, 1984). This small change in the value of the function is then divided by the change in the value of the variable; this process is done for each variable in each function. When using this technique, the initial guesses for the values of the variables must be close enough to the solution to give convergence. It is important to note also that delta should be small enough to give a realistic approximation to the partial derivative but not so small as to lead to an extreme amount of round-off.

The variational technique includes a cost function defined as:

$$J = \sum_{i=1} \left(V_i - \tilde{V}_i \right)^2 \tag{9}$$

where V_i is the modeled wind profile, $\tilde{V}_i$ is the observation, and i is the total number of observations. V_i is given by:

$$V_i = \frac{\sum_j G_j Z_j (v_{rcva} \cos\gamma \cos\phi)}{\sum_j G_j Z_j} \tag{10}$$

In (10), G is the Gaussian weighting function, Z is reflectivity (assumed to be uniform in this paper), γ is the angle between the radar viewing direction at a target point and the tangential velocity and φ is the elevation angle. To determine the optimal estimate, the cost function J, which is a function of V_{max} and R_{max}, is minimized. A necessary condition for this minimization is:

$$\frac{\partial J}{\partial V_{\max}} = 0 = 2 \sum_i \left(V_i - \tilde{V}_i \right) \frac{\partial V_i}{\partial V_{\max}} \tag{11}$$

$$\frac{\partial J}{\partial R_{\max}} = 0 = 2 \sum_i \left(V_i - \tilde{V}_i \right) \frac{\partial V_i}{\partial R_{\max}} \tag{12}$$

From (11) and (12), we can see that two more partial derivatives are needed:

$$\frac{\partial V_i}{\partial R_{\max}} = \frac{1}{2} \left(\frac{\partial \phi_1}{\partial R_{\max}} + \frac{\partial \phi_2}{\partial R_{\max}} \right) \cos\gamma \cos\varphi \tag{13}$$

$$\frac{\partial V_i}{\partial V_{\max}} = \frac{1}{2} (\phi_1 + \phi_2) \cos\gamma \cos\varphi \tag{14}$$

Contained in (13) are two final partial derivatives that involve the RCVA discussed in section 2. Recall (2):

$$\phi_n = \frac{2nR^{2n-1}r}{(2n-1)R^{2n} + r^{2n}} \quad n = 1,2,...$$

Also recall that the best approximation is when $n = 1$ and $n = 2$, which results in:

$$\phi_1 = \frac{2Rr}{R^2 + r^2} \tag{15}$$

$$\phi_2 = \frac{4R^3 r}{3R^4 + r^4} \tag{16}$$

Therefore, with some simplifications:

$$\frac{\partial \phi_1}{\partial R} = \frac{2r^3 - 2R^2 r}{(R^2 + r^2)^2} \tag{17}$$

$$\frac{\partial \phi_2}{\partial R} = \frac{(12R^2 r^5 - 12R^6 r)}{(3R^4 + r^4)^2} \tag{18}$$

The nonlinear equations were solved for the values of the computed partial derivatives of each function for each variable. The next step in retrieving V_{max} and R_{max} is to solve a set of linear equations using Gaussian elimination with partial pivoting. Because the set of linear equations are all augmented to the coefficient matrix and all the solutions are acquired at once, the set of equations can be solved with multiple right hand sides. The solutions are returned in the space of the augmentation columns (Gerald and Wheatley, 1984). From this routine, retrieved values of V_{max} and R_{max} are obtained.

4. DESCRIPTION OF EXPERIMENT

In this paper, we present results from prescribed changes in two parameters, the azimuthal sampling interval and the distance of the radar from the center of the vortex. This is done for two tornadoes, a strong F1 tornado (Tornado A) and a F4 tornado (Tornado B). The output is the maximum tangential wind speed, V_{max}, and core radius at which this wind speed occurs, R_{max} as mentioned in section 3. The results presented in this paper are for a single CASA radar sampling a single vortex with assumed uniform reflectivity across it.

The input values for the retrieval program include a maximum tangential wind speed of 50 ms^{-1} for Tornado A and 100 ms^{-1} for Tornado B. A core radius of 200 m was used for Tornado A and 400 m for Tornado B. Because the CASA radars have a two degree beamwidth, this value was used as the input beamwidth. The effective beamwidth is another value to consider for the retrieval program. Effective beamwidth is the azimuthal broadening of a horizontally rotating beam at a given range, and depends on three radar parameters: antenna rotation rate, the time interval between pulses, and the number of pulses transmitted (Doviak and Zrnić, 1993, 193-197). For a two degree beamwidth, it was calculated to be 2.90 degrees (Brown et al., 2002). The elevation angle used in this paper was assumed to be 0°; we also assume that the simulated measurements are free of noise.

The results presented in this paper vary the azimuthal sampling interval and the distance of the vortex from the radar. The azimuthal sampling intervals used are 0.5 degrees, 1.0 degree (both oversampling), a "baseline" experiment of 2 degrees (no over or undersampling), and 2.5 degrees and 3.0 degrees (both undersampling). A corresponding effective beamwidth was used for each interval according to Brown et al, 2002. The distance of the vortex from the radar varies in increments of 2.5 km starting at 2.5 km and increasing to 30 km, which is the maximum range for the CASA radars.

5. RESULTS

The model was run for each combination of azimuthal sampling interval and distance discussed in section 4 for Tornado A and Tornado B. The results are presented in Tables 1-4, as well as the initial guesses for V_{max} and R_{max}, denoted by V* and R*.

The results for Tornado A show that oversampling produces the best results, while undersampling leads to several values for V_{max} and R_{max} that cannot be retrieved mostly beyond 10 km from the radar. This is because there are not a sufficient number of data points in the region being scanned; as range from the radar increases, the distance between data points of the Doppler velocity azimuthal profile of the vortex signature increases relative to the size of the vortex. Therefore, sampling a hook echo, especially in a weak tornado, can be difficult for a one dimensional case. Also from the tables you can see that the initial guesses of the program, V* and R*, both degrade with range from the radar.

The results for Tornado B show that oversampling is slightly better than undersampling, illustrated by values that are very close to the original values of 100 ms^{-1} and 400 m. Again, some values cannot be retrieved because there are not a sufficient number of data points in the region being scanned.

6. FUTURE WORK

The next phase of this research will use a two dimensional program that can use a two dimensional array of Doppler velocity data in range and azimuth, thus alleviating the insufficient data point problem that occurs in the one dimensional case. This program will use the RCVA. Several more parameters will be changed also, including the number of radars adaptively sampling a vortex, adding translation to the tornado, the number of vortices, and possibly using non uniform reflectivity.

Other idealized flows will be used as well in the two dimensional retrieval program, including the Burgers-Rott vortex (BRV). These flows will be used to create the flow as well as serve as the model to fit.

The same parameters that are being changed for the RCVA case will be used for these flows as well. Metric for optimization such as a cost function, probability density function, and information content will also be used.

7. CONCLUSIONS

CASA is developing a revolutionary new paradigm for overcoming fundamental limitations in current radar technology such as the inability to sample the lower parts of the atmosphere. To facilitate this, CASA is establishing a system of distributed, collaborative, and adaptive sensor (DCAS) networks that are unique because they can dynamically adjust their scanning strategies and other attributes collaboratively with other CASA radars to sense multiple atmospheric phenomena while at the same time meeting multiple end user needs.

The purpose of this paper is to show preliminary results of how to best use CASA radars and the DCAS idea in order to find the optimal sampling strategy for the radars. An optimal sampling strategy would maximize the amount of information that can be extracted from the atmosphere while using the minimum amount of resources to meet end user needs. Results of changing the azimuthal sampling interval as well as the distance a radar is from the center of a vortex for a tornado of F1 intensity indicates that oversampling produces better results than undersampling. For a stronger tornado of F4 intensity, both oversampling and undersampling produce results that are in good agreement with the original input values. Both experiments in this one dimensional case have data that cannot be retrieved due to lack of data points; the use of a two dimensional model will help alleviate this problem.

8. REFERENCES

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Experiment	1		2		3		4		5		6.00	
∆Azimuth (degrees)	0.5		1		1.5		2		2.5		3.00	
EBW (degrees)	2.07	V*	2.24	V*	2.53	V*	2.9	V*	3.35	V*	3.82	V*
Range (km)												
2.5	50.0	49.0	50.0	48.7	50.0	48.5	50.0	47.2	50.0	47.7	50.0	47.1
5	50.0	46.8	50.0	44.8	50.0	43.8	50.0	42.4	50.0	42.8	50.0	38.5
7.5	50.0	43.0	50.0	42.2	50.0	41.1	50.0	39.6	50.0	35.9	50.0	35.2
10	50.0	40.2	50.0	35.6	50.0	33.1	50.0	33.7			50.0	30.1
12.5	50.0	35.8	50.0	34.2	50.0	33.3	50.0	28.5				
15	50.0	33.6	50.0	32.9	50.0	29.4					50.0	22.1
17.5	50.0	31.1	50.0	29.4	50.0	26.1					50.0	19.4
20	50.0	28.6	50.0	27.1	50.0	23.4						
22.5	50.0	26.4	50.0	25.0	50.0	21.1						
25	50.0	24.4	50.0	23.1	50.0	19.1						
27.5			50.0	21.4	50.0	17.5						
30	50.0	21.1	50.0	19.9	50.0	16.1						

Table 1: Retrieved maximum tangential velocity, V_{max}, for Tornado A in m/s. Blank cells represent data that cannot be retrieved. V is the initial guess.*

Experiment	1		2		3		4		5		6.00	
∆Azimuth (degrees)	0.5		1		1.5		2		2.5		3.00	
EBW (degrees)	2.07	R*	2.24	R*	2.53	R*	2.9	R*	3.35	R*	3.82	R*
Range (km)												
2.5	0.20	0.20	0.20	0.22	0.20	0.22	0.20	0.17	0.20	0.22	0.20	0.22
5	0.20	0.22	0.20	0.18	0.20	0.17	0.20	0.17	0.20	0.22	0.20	0.17
7.5	0.20	0.20	0.20	0.26	0.20	0.26	0.20	0.26	0.20	0.33	0.20	0.26
10	0.20	0.20	0.20	0.18	0.20	0.17	0.20	0.35			0.20	0.35
12.5	0.20	0.22	0.20	0.22	0.20	0.17	0.20	0.44				
15	0.20	0.26	0.20	0.26	0.20	0.33					0.20	0.52
17.5	0.20	0.31	0.20	0.31	0.20	0.39					0.20	0.61
20	0.20	0.03	0.20	0.35	0.20	0.46						
22.5	0.20	0.39	0.20	0.39	0.20	0.52						
25	0.20	0.44	0.20	0.44	0.20	0.60						
27.5			0.20	0.48	0.20	0.65						
30	0.20	0.52	0.20	0.52	0.20	0.72						

Table 2: Retrieved core radius, R_{max}, that the maximum tangential velocity occurs at in km for Tornado A. Blank cells represent data that cannot be retrieved. R is the initial guess.*

Experiment	1		2		3		4		5		6	
ΔAzimuth (degrees)	0.5		1		1.5		2		2.5		3	
EBW (degrees)	2.07	V*	2.24	V*	2.53	V*	2.9	V*	3.35	V*	3.82	V*
Range (km)												
2.5	100.0	99.2	100.0	99.1	100.0	98.9	100.0	98.1	100.0	97.9	100.0	96.3
5	100.0	98.1	100.0	97.4	100.0	96.9	100.0	94.4	100.0	95.3	100.0	94.3
7.5	100.0	96.0	100.0	95.3	100.0	93.8	100.0	89.1	100.0	85.6	100.0	86.7
10	100.0	93.5	100.0	89.7	100.0	87.6	100.0	84.7	100.0	85.6	100.0	77.0
12.5	100.0	90.6	100.0	89.3	100.0	87.0	100.0	83.7	100.0	79.6	100.0	74.9
15	100.0	86.1	100.0	84.5	100.0	82.3	100.0	79.1	100.0	71.9	100.0	70.4
17.5	100.0	84.0	100.0	77.8	100.0	76.7	100.0	73.3	100.0	64.4	100.0	65.2
20	100.0	80.4	100.0	71.2	100.0	74.8	100.0	67.4	100.0	57.7	100.0	60.2
22.5	100.0	76.0	100.0	70.1	100.0	70.7	100.0	61.9	99.5	52.0	100.0	55.5
25	100.0	71.7	100.0	68.3	100.0	66.6	100.0	56.9				
27.5	100.0	69.6	100.0	66.2	100.0	62.6	100.0	52.5			100.0	47.5
30	100.0	67.2	100.0	63.8	100.0	58.9					100.0	44.2

Table 3: Retrieved maximum tangential velocity, V_{max}, for Tornado B in m/s. Blank cells represent data that cannot be retrieved. V is the initial guess.*

Experiment	1		2		3		4		5		6	
ΔAzimuth (degrees)	0.5		1		1.5		2		2.5		3	
EBW (degrees)	2.07	R*	2.24	R*	2.53	R*	2.9	R*	3.35	R*	3.82	R*
Range (km)												
2.5	0.40	0.39	0.40	0.39	0.40	0.41	0.40	0.44	0.40	0.44	0.40	0.35
5	0.40	0.39	0.40	0.44	0.40	0.44	0.40	0.35	0.40	0.44	0.40	0.44
7.5	0.40	0.39	0.40	0.39	0.40	0.46	0.41	0.52	0.40	0.33	0.40	0.52
10	0.40	0.44	0.40	0.35	0.40	0.35	0.40	0.35	0.40	0.44	0.40	0.35
12.5	0.40	0.44	0.40	0.44	0.40	0.44	0.40	0.44	0.40	0.55	0.40	0.44
15	0.40	0.39	0.40	0.52	0.40	0.52	0.40	0.52	0.40	0.65	0.40	0.52
17.5	0.40	0.46	0.40	0.61	0.40	0.61	0.40	0.61	0.40	0.76	0.40	0.61
20	0.40	0.52	0.40	0.35	0.40	0.52	0.40	0.70	0.40	0.87	0.40	0.70
22.5	0.40	0.59	0.40	0.39	0.40	0.59	0.40	0.79	0.40	0.98	0.40	0.79
25	0.40	0.44	0.40	0.44	0.40	0.65	0.40	0.87				
27.5	0.40	0.48	0.40	0.48	0.40	0.72	0.40	0.96			0.40	0.96
30	0.40	0.52	0.40	0.52	0.40	0.79					0.40	1.05

Table 4: Retrieved core radius, R_{max}, that the maximum tangential velocity occurs at in km for Tornado B. Blank cells represent data that cannot be retrieved. R is the initial guess.*