P9R.10 Effect of the spatial variability of precipitation on the differential propagation phase shift

Julie Deshaies*, Chérif Hamzaoui and Enrico Torlaschi
Université du Québec à Montréal

1. Introduction*

The interaction between the electromagnetic waves emitted by a polarimetric radar and precipitation is in general complex because of propagation through a non-homogeneous medium. This complexity makes the problem very complicated. To make progress in this field, one has to make assumptions in order to simplify the problem. This raises the question about the validity of these new assumptions. One of the most important assumptions done at this moment is to assume uniformity for cross range variability of precipitation. With new resolution methods, we can probe the problem with less assumptions.

The polarization allows us to identify hydrometeors (dry/wet snow, freezing rain, rain, hail) and biological scatterers (birds, insects). It also allows us to detect electrically active storm and to improve quantitative estimations of precipitation (Zrnic and Ryzhkov, 1999). This is defined as the variation of the electric field in time and in space. Polarimetric radar gives more information as the differential propagation phase shift ϕ_{DP} that represents the cumulative phase shift introduced by wave propagation through precipitation. Its range gradient is the specific differential propagation phase K_{DP} which is related to precipitation rate by

$$R = 5{,}1\left(\lambda K_{DP}\right)^{0{,}866} \qquad (1)$$

For the same precipitation rate, we see that K_{DP} is inversely proportional to the wavelength λ. The expected value of KDP is 0.5 °/km. For this value of K_{DP}, the precipitation rate has to be greater than 20 mm/h for S-band radar, 12 for C-band and 7 for X-band. This parameter has several advantages : it's independent of radar calibration, it's immune to propagation effects, to precipitation attenuation and to partial radar beam blockage. It's also less sensible to drop size distribution variations and to the presence of hail (Ryzhkov and Zrnic, 1996). Far of the radar, negative values of K_{DP} sometimes appear at the rear side of isolated convective cells or at the leading edge of squall lines when strong reflectivity azimuthal gradient is observed. We will determine and characterize the effects of cross and down range variability of precipitation on ϕ_{DP} in order to explain this kind of observations.

We examine first the propagation of a plane wave through a non-homogeneous slab of dielectric spheres and obtain the equation for the transmitted wave. Secondly, we consider the propagation of a plane wave through a non-homogeneous dielectric slab and use the perturbation method to derive an analytical solution for the transmitted wave. Finally, we compare the two solutions, define an effective propagation constant and establish the variability of the refractive index on the precipitation rate. This in turn helps us to assess the effect of spatial variability of precipitation on ϕ_{DP}.

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2. Non-homogeneous slab of dielectric spheres

We consider the propagation of a plane wave through a non-homogeneous slab of dielectric spheres as shown in fig.1. We use the same theory exposed in Bringi and Chadrasekar (2001) for the case with a constant number of spheres. We want to find the transmitted wave equation at an arbitrary point P far off the slab.

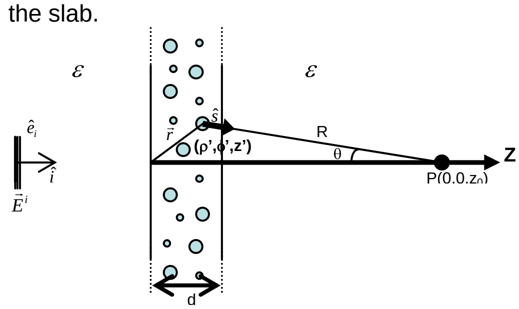

Fig. 1. Slab consisting of dielectric spheres.

We suppose that the wave interacting with a particular sphere is the unperturbed incident field. This approximation is valid for a sparse distribution of spheres. The scattered wave at P by a sphere at $\vec{r}$ is

$$\vec{f}(\hat{s},\hat{i})e^{-jk_0 z}\frac{e^{-jk_0 R}}{R} \qquad (2)$$

where $\vec{f}(\hat{s},\hat{i})$ is the vector scattering amplitude.

We define $n(\rho',\phi',z')$ as the number of spheres per unit volume. A volume element dV contains $n(\rho',\phi',z')dV$ spheres. We will remove the primes to simplify the notation but we have to keep in mind that $\rho'=\rho-\rho_0$, $\phi'=\phi-\phi_0$ and $z'=z$ for future use. This elementary volume is centered at $\vec{r}\equiv(\rho,\phi,z)$. Using the continuum approximation, the electric field $d\vec{E}^s$ due to $n(\rho,\phi,z)dV$ spheres in dV is

$$d\vec{E}^s(P)=\vec{f}(\hat{s},\hat{i})\frac{e^{-jk_0 R}}{R}e^{-jk_0 z}n(\rho,\phi,z)dV \qquad (3)$$

* *Corresponding author address:* Julie Deshaies, Centre ESCER/UQAM, Case postale 8888, succursale Centre-Ville, Montréal, QC Canada, H3C 3P8; e-mail: deshaies@sca.uqam.ca

Applying the superposition principle, the total scattered field at P is given by

$$\vec{E}^s(P) = \int_{\text{slab volume}} \frac{e^{-jk_0R}}{R} \vec{f}(\hat{s},\hat{i}) e^{-jk_0z} n(\rho,\phi,z) dV \tag{4}$$

where $dV = \rho d\rho \, d\phi \, dz$, $R^2 = \rho^2 + (z_0 - z)^2$ and $RdR = \rho \, d\rho$ for a given z. Then, $\vec{E}^s(P)$ can be written as

$$\vec{E}^s(P) = \int_{\phi=-\phi_0}^{2\pi-\phi_0} d\phi \int_{z=0}^{d} e^{-jk_0z} dz \int_{\sqrt{\rho_0^2+(z_0-z)^2}}^{\infty} \frac{e^{-jk_0R}}{R} (RdR) \vec{f}(\hat{s},\hat{i}) n(R,\phi,z) \tag{5}$$

In fig. 1, we see that $\mu \equiv \cos\theta = \hat{s}\cdot\hat{i} = (z_0 - z)/R$ and

$$\frac{d}{dR} \equiv \frac{d}{d\mu}\frac{d\mu}{dR} = \frac{d}{d\mu}\left[\frac{-(z_0-z)}{R^2}\right] \tag{6}$$

Integrating by parts,

$$\begin{aligned} \int_{\sqrt{\rho_0^2+(z_0-z)^2}}^{\infty} e^{-jk_0R} n(R,\phi,z) \vec{f}(\hat{s},\hat{i}) dR &= n(R,\phi,z) \vec{f}(\hat{s},\hat{i}) \frac{e^{-jk_0R}}{(-jk_0)}\Bigg|_{R=\sqrt{\rho_0^2+(z_0-z)^2}}^{R=\infty} \\ &\quad - \frac{1}{jk_0} \int_{\sqrt{\rho_0^2+(z_0-z)^2}}^{\infty} \frac{(z_0-z)}{R^2} \frac{d}{d\mu}\left[n(R,\phi,z) \vec{f}(\hat{s},\hat{i})\right] e^{-jk_0R} dR \end{aligned} \tag{7}$$

We suppose that $d\left(n(R,\phi,z) \vec{f}(\hat{s},\hat{i})\right)/d\mu$ is a well-behaved function and that the second integral is negligible because P is several wavelengths form the slab. Then,

$$\int_{\sqrt{\rho_0^2+(z_0-z)^2}}^{\infty} e^{-jk_0R} n(R,\phi,z) \vec{f}(\hat{s},\hat{i}) dR \approx n(R = \sqrt{\rho_0^2 + (z_0-z)^2}, \phi, z) \vec{f}(\hat{i},\hat{i}) \frac{e^{-jk_0(z_0-z)}}{(jk_0)} \tag{8}$$

where we neglected the contribution from the upper limit $(R \to \infty)$ (oscillating contribution). This is acceptable if $n(\rho,\phi,z)$ is null when $\rho \to \infty$. With the far-field approximation, we can define $R \approx (z_0 - z)$ since $\rho_0 << (z_0 - z)$. In this case we obtain

$$\vec{E}^s(P) = \frac{\vec{f}(\hat{i},\hat{i}) e^{-jk_0z_0}}{jk_0} \int_{\phi=-\phi_0}^{2\pi-\phi_0} d\phi \int_{z=0}^{d} n(\phi,z) dz \tag{9}$$

The total electric field at P is given by:

$$\vec{E}(P) = \hat{e}_i E_0 e^{-jk_0z_0} + \frac{\vec{f}(\hat{i},\hat{i}) e^{-jk_0z_0}}{jk_0} \int_{\phi=-\phi_0}^{2\pi-\phi_0} d\phi \int_{z=0}^{d} n(\phi,z) dz \tag{10}$$

where $\vec{f}(\hat{i},\hat{i})$ is the forward scattering amplitude in the direction $\hat{s} = \hat{i}$. The component of $\vec{E}(P)$ parallel to the incident linear polarization state $(\hat{e}_i)$ with $E_0 = 1V/m$ is

$$\hat{e}_i \cdot \vec{E}(P) = e^{-jk_0z_0}\left[1 + \left(\frac{1}{jk_0} \int_{\phi=-\phi_0}^{2\pi-\phi_0} d\phi \int_{z=0}^{d} n(\phi,z) dz\right) \hat{e}_i \cdot \vec{f}(\hat{i},\hat{i})\right] \tag{11}$$

This equation is valid to first-order in the slab thickness (d).

3. Non-homogeneous dielectric slab

To study the propagation of a plane wave through a non-homogeneous dielectric slab, we use a method based on the theory of perturbation developed in quantum mechanics. First, we have to know how the refractive index of precipitation varies in space. In other words, what are the extremum values that this index can take. To do so, we have to use a numerical recipe developed by Yves Gingras (1997) which calculates the propagation constants. With these constants, we can retrieve the values that can take the refractive index. We believe that the value of the refractive index varies around 1. In that case, we will use the Born expansion (Nayfeh, 2000) to find the electric field at a particular point.

With this result, we will be able to define a propagation constant that take into account the spatial variability of precipitation.

4. Summary

We showed in this paper a method how to avoid the approximation of uniformity in the cross range variablility of precipitation. The problem has been divided in two cases. First, we considered the plane wave propagation through a non-homogeneous slab of dielectric spheres and secondly, through a non-homogeneous dielectric slab. With the theory exposed in Bringi and Chandrasekar for a slab with a constant number of spheres, we found the electric field at a point P far off a slab with a number of spheres varying with space.

The next step will be to consider the propagation through a non-homogeneous dielectric slab. We will compare the two results and we will define a propagation constant accordingly. Finally, we will determine the effect of spatial variability of precipitation on ϕ_{DP}.

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