QUANTITATIVE ANALYSIS OF WEATHER RADAR ATTENUATION CORRECTION ACCURACY

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1 INTRODUCTION
At short wavelengths (e.g. C- and X-band), the attenuation of the radar signal by the precipitation along its path is a critical issue for quantitative radar rain estimates that has been recognized for a long time (e.g. Hitschfeld and Bordan, 1954). A recently developed stochastic simulator of range profiles of raindrop size distributions (DSD) provides a controlled experiment framework to investigate the accuracy and robustness of attenuation correction algorithms (Berne and Uijlenhoet, 2005).

This paper focuses on the quantification of the influence of uncertainties concerning the radar calibration, the parameterization of a power-law relation between the radar reflectivity Z and the specific attenuation k, and total path integrated attenuation (PIA) estimations. The analysis concerns single frequency, incoherent and non-polarimetric radar systems. Two attenuation correction algorithms are studied: a forward algorithm based on the analytical solution proposed by Hitschfeld and Bordan (1954) and a backward algorithm based on the solution proposed by Marzoug and Amayenc (1994). From DSD range profiles, the corresponding profiles of bulk rain variables are derived. Using a Monte Carlo approach, the accuracy of the two algorithms is quantified for the different sources of error previously mentioned.

2 DSD SIMULATOR
The DSD simulator used in the following has been proposed by Berne and Uijlenhoet (2005). It enables to generate realistic DSD range profiles. It is based on the exponential DSD, which two parameters \( N_t \) and \( \alpha \) are considered to be random variables

\[
N(D|N_t, \alpha) = N_t \alpha e^{-\alpha D}, \tag{1}
\]

where \( N(D|N_t, \alpha) \) denotes the drop concentration in the diameter interval \([D, D + dD]\) given \( N_t \) and \( \alpha \). The latter are assumed to be jointly lognormally distributed and their logarithms are assumed to follow a bivariate first order vector auto-regressive process. Assuming Taylor’s hypothesis with a constant velocity of 12.5 m s\(^{-1}\) (see Berne and Uijlenhoet, 2005), the parameters have been derived from DSD measurements for an intense rain period of 45 minutes, collected during the HIRE’98 experiment in Marseille, France (see Table 1).

The generated DSD profiles have a total length of 30 km, with a spatial resolution of 25 m. From these DSD profiles, the corresponding profiles of bulk rain variables (e.g. \( R \), \( Z \) and \( k \)) are easily derived. This controlled experiment framework allows to apply a Monte Carlo technique to quantify the respective influence of the different sources of uncertainty in attenuation correction.

3 ATTENUATION CORRECTION ALGORITHMS
As mentioned in the introduction, we consider incoherent, single frequency and non-polarimetric radar systems. Two different types of algorithms will be studied in the following. The measured attenuated reflectivity \( Z_a(r) \) reads

\[
Z_a(r) = \delta_c A(r) Z(r), \tag{2}
\]

where \( \delta_c \) is the calibration error factor and \( A(r) \) is the two-way attenuation factor at the range \( r \) \((0 \leq A \leq 1)\). Assuming the \( Z-k \) relation reads

\[
Z = \delta_\alpha \alpha k^{\delta_\beta}, \tag{3}
\]

where \( \delta_\alpha \) (\( \delta_\beta \) respectively) is the error factor in \( \alpha \) (\( \beta \)). Therefore, \( A \) can be written as

\[
A(r) = \exp \left[ -0.2 \ln(10) \int_0^r \left( \frac{Z(s)}{\delta_\alpha} \right)^{1/(\delta_\beta)} ds \right]. \tag{4}
\]
Hitzschfeld and Bordan (1954) (HB hereafter) proposed an analytical solution to express $Z$ as a function of $Z_a$:

$$Z(r) = Z_a(r)/\left[\delta_e^{1/(\delta_e \beta)} - \frac{0.2 \ln(10)}{\delta_e \beta} \int_0^r (Z_a(s)/\delta_e \alpha)^{1/(\delta_e \beta)} ds \right]^{\delta_e \beta}. \tag{5}$$

The HB algorithm is a forward algorithm because the integral is between 0 and $r$. However, the difference in its denominator can be close to 0 and this makes the algorithm highly unstable. To avoid instability problems, another family of attenuation correction algorithms has been developed. It is based on the knowledge of an estimate $A_0$ of the PIA at a given range $r_0$. The estimate $A_0$ can be uncertain, that is

$$A(r_0) = \delta_A A_0, \tag{6}$$

where $\delta_A$ is the error factor in $A_0$. The reformulation of Eq.(5) starting from $r_0$ and going backward to the radar guarantees the stability of the algorithms. As an example, we use the solution proposed by Marzoug and Amayenc (1994) (MA hereafter):

$$Z(r) = Z_a(r)/\left[(\delta_e \delta_A A_0)^{1/(\delta_e \beta)} + \frac{0.2 \ln(10)}{\delta_e \beta} \int_r^{r_0} (Z_a(s)/\delta_e \alpha)^{1/(\delta_e \beta)} ds \right]^{\delta_e \beta}. \tag{7}$$

The main drawback of such a backward algorithm is that it requires a reliable estimation of the PIA at a given range.

To study the accuracy of the algorithms, we use a Monte Carlo technique. The analysis focuses on attenuation correction using Eqs. (5) and (7). One thousand profiles of $N_t$ and $\lambda$ (hence of $Z$, $k$ and $Z_a$) are generated. To be consistent with operational radar sampling resolutions, the high spatial resolution (25 m) profiles are averaged at a lower spatial resolution of 250 m. On each profile a $Z$-$k$ power-law relation is fitted by means of a non-linear regression technique. It must be noted that they constitute the best possible power-law relations. The exact PIA value is calculated as the difference between the non-attenuated and the attenuated $Z$ profiles. Then the two algorithms are applied using the fitted relations on the 1000 profiles.

Figure 1 shows the median, as well as the 10% and 90% quantiles, of the distribution of the root mean square error (RMSE) calculated between the exact $Z$ profiles and the $Z_c$ profiles obtained by applying the two attenuation correction algorithms without any uncertainty (i.e. $\delta_c = \delta_a = \delta_b = \delta_A = 1$). The significant dispersion of the distribution (Fig. 1 is in log scale) is explained by the fact that the use of a deterministic power law between $Z$ and $k$ is not fully consistent with the stochastic nature of these variables.

Obviously, the MA algorithm ($0.05 < \text{median} < 0.3 \text{ dBZ}$) is more stable and accurate than the HB algorithm ($0.05 < \text{median} < 10 \text{ dBZ}$), which diverges in about 1 in 5 cases. These RMSE values will constitute the reference values for the quantification of the influence of the different sources of uncertainty, as detailed in the following sections.

4 INFLUENCE OF THE UNCERTAINTY IN CALIBRATION

Radar systems can be affected by calibration errors. In this section, the influence of the uncertainty in calibration on the accuracy of the attenuation correction algorithms is quantified. For better visual inspection, the calibration error is expressed in dBZ as $\epsilon_c = 10 \log(\delta_c)$ and varies in the interval $[-5,+5]$. The additional error due to uncertain calibration is calculated as the ratio between the RMSE values for a given calibration error and the reference RMSE values. Figure 2 presents the median, as well as the 10% and 90% quantiles, of the distribution of RMSE ratio as a function of the calibration error.
The other error factors ($\delta_\alpha$, $\delta_\beta$ and $\delta_A$) are fixed to 1. When $\epsilon_c > 0$, the median values are similar for

the two algorithms but the dispersion is larger for the HB algorithm. When $\epsilon_c > 0$, the distribution remains similar for the MA algorithm. For the HB algorithm, Eq. 5 shows that $\delta_c < 1$ (or $\epsilon_c < 0$) results in more diverging profiles because the denominator can be close to zero for smaller $Z_a$ values, as illustrated in Fig. 2 by the sharp increase of the median when $-1 < \epsilon_c < 0$ and the divergence of the algorithm when $\epsilon_c < -1$ (absence of points).

As expected, the RMSE ratio rapidly increases when $\epsilon_c = 0$, that is the calibration error decreases significantly the accuracy of the two algorithms. For instance, the median RMSE ratio value is about 3 when $\epsilon_c = 1$ for the MA algorithm. It is about 4 (8 respectively) when the error is about $20\%$ for the HB algorithm.

5 INFLUENCE OF THE UNCERTAINTY IN THE PARAMETERIZATION OF THE Z-K RELATION

The two studied algorithms are based on the assumption of a power-law relation between $Z$ and $k$. To analyze the influence of the uncertainty in the parameters of the $Z$-$k$ power law on the accuracy of the two attenuation correction algorithms, an error factor between 0.7 and 1.3 is applied to the prefactor (the exponent respectively). The additional error due to uncertain parameterization of the $Z$-$k$ relation is calculated as the ratio between the RMSE values for a given prefactor (exponent) error and the reference RMSE values. Figure 3 presents the median, as well as the 10% and 90% quantiles, of the distribution of the RMSE ratio as a function of the relative deviation of the prefactor and exponent, with respect to the reference $Z$-$k$ relation. The other error factors ($\delta_\alpha$ and $\delta_A$) are fixed to 1. For the prefactor (top panel of Fig. 3), the distribution of the RMSE ratio is very similar for the two algorithms when $\delta_\alpha > 1$. However, the HB algorithm is more sensitive to the error in $\alpha$ when $\delta_\alpha < 1$, which is consistent with Eq. (5). For instance, the median RMSE ratio is about 3 when the error is about $20\%$ in the prefactor for the MA algorithm. It is about 3 (7 respectively) when the error is about $20\%$ for the HB algorithm.

The influence of the exponent is stronger for both algorithms, as indicated by the higher values of the RMSE ratio. It must be noted that the MA algorithm is more sensitive to the error in $\beta$. For instance, the median RMSE ratio is about 9 when the error in the exponent is about $20\%$ for the MA algorithm. It is about 3 when the error is about $20\%$ for the HB algorithm.

6 INFLUENCE OF THE UNCERTAINTY IN THE PIA ESTIMATE

The MA algorithm is more accurate and more robust than the HB algorithm, but it requires an additional parameter which is the estimate of the PIA at a given range. This section is devoted to the quantification of the influence of the uncertainty in this PIA estimate on the accuracy of the MA algorithm.
Similarly to $\epsilon_c$, we define $\epsilon_A = 10 \log(\delta_A)$. The error in the PIA estimate $\epsilon_A$ is generated as a Gaussian white noise with a standard deviation of 2.5 dB (Delrieu et al., 1999). The additional error due to uncertain PIA estimate $\epsilon_A$ is calculated as the ratio between the RMSE values for the uncertain PIA estimate and the reference RMSE values. Figure 4 presents the median, as well as the 10% and 90% quantiles, of the distribution of the RSME ratio as a function of $\epsilon_A$ for the MA algorithm. The other error factors ($\delta_c, \delta_\alpha$ and $\delta_\beta$) are fixed to 1. According to Eq. (7), Fig. 4 and Fig. 2 should be identical because $\delta_A$ and $\delta_c$ can be interchanged. In fact, for a given reference $Z_a$ profile, the error in the calibration and in the PIA estimate are generally different and therefore the deduced distribution of the RMSE ratio is slightly different. Nevertheless, the influence of $\delta_A$ is the same as $\delta_c$.

7 CONCLUSIONS

Attenuation correction is an important step for quantitative rain estimation using C- or X-band weather radar. In this paper, we focus on X-band incoherent, single frequency and non-polarimetric radar systems. We investigate the influence of uncertainties in the radar calibration, in the parameterization of a power-law relation between the radar reflectivity $Z$ and the specific attenuation $k$, and in the total path integrated attenuation (PIA) estimates on the accuracy of two attenuation correction algorithms. The first (HB algorithm) is based on a forward implementation and is known for its instability. The second (MA algorithm) is based on a backward implementation and is stable, but requires an additional piece of information which is the PIA at a given range from the radar. A stochastic model of DSD range profiles provides a controlled experiment framework, with fully consistent $Z$ and $k$ profiles, to quantify the influence of the different sources of uncertainty. An uncertainty of 1 dBZ in the measured $Z$ (or of 1 dB in the PIA estimate) leads to a multiplication of the RMSE by at least a factor 3. An uncertainty of about 20% in the prefactor (exponent respectively) leads to a multiplication of the RMSE by at least a factor 5 (2). The influence of the use of alternative DSD models (e.g. gamma, lognormal) is the subject of ongoing research.

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REFERENCES


