P6.6 ANALYSIS AND EVOLUTION OF BALANCE IN UNSTABLE BAROTROPIC JETS

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1. INTRODUCTION

The nature of balance in the atmosphere is of central importance to the dynamics of both the troposphere and the stratosphere, and unbalanced motions such as inertia-gravity waves play a significant role in many aspects of atmospheric behavior. In light of the importance of upper-tropospheric jets for the generation of inertia-gravity waves (IGWs) in the atmosphere, this study examines the evolution of unstable barotropic jets to assess the nature and evolution of balance in these features. This issue is explored using the simplest non-trivial dynamical framework in which balanced and unbalanced flows can coexist, namely the one-layer shallow-water equations.

In this study, numerical simulations of initially balanced zonal jets on an f plane are investigated for evidence of the breakdown of balance and the generation of inertia-gravity waves during the life cycles of the instabilities to these jets. The presence of unbalanced flow is typically inferred via various quantities that provide indirect measures of imbalance, such as the existence of strong ageostrophy, large Rossby and/or Lagrangian Rossby numbers, and large values of horizontal divergence and its material derivative. Nevertheless, these quantities are based on specific balance constraints (i.e., quasigeostrophy, semigeostrophy, or nonlinear balance), and it should be noted that assessment of balance based on the inaccuracy of these constraints allows for the possibility that the unbalanced flow so identified includes higher-order balanced motions not accounted for in the system under consideration in addition to inertia-gravity waves. We choose to start with a balanced initial state and allow it to evolve, estimating possible unbalanced dynamical quantities using a second-order potential vorticity inversion. In strong jets, the Rossby and Froude numbers are not small compared to unity, therefore the applicability of traditional diagnosis is unclear.

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In the following section, the shallow-water model is described, along with the methodology used for this investigation. Section 3 provides results from these simulations for a single set of initial jet parameters, and in section 4, discussion and future work are presented.

2. MODEL AND DIAGNOSTIC CALCULATIONS

A one-layer shallow-water equation model in Cartesian f plane geometry is used to simulate the life cycles of the instabilities to the basic-state zonal profile

$$U(y) = U_o \operatorname{sech}^2\left(\frac{y}{y_o}\right), \tag{1}$$

where U_o is the maximum jet speed, and y_o is the initial width of the jet. Initially, random perturbations of infinitesimal amplitude are added to the basic-state jet, and the model is run to grow an unstable wave. The time t = 0 in the simulation shown here corresponds to the time at which the maximum meridional wind is 1.5 m s^{-1}. The boundary conditions in x are periodic and in y are solid wall with a damping layer to prevent reflection of waves back into the domain. A coordinate transformation is used in the y direction so that the boundaries are far from the jet region. The domain length is equal to one wavelength of the unstable mode. Associated with this jet profile, it is possible to define a Rossby and Froude number:

$$Ro_J = \frac{U_o}{f_o y_o}\ ,\ Fr_J = \frac{U_o}{\sqrt{gh_o}}, \tag{2}$$

where h_o is the depth of the layer, g is the acceleration due to gravity, and f_o is the Coriolis parameter. The latitude is set to 40^o N for these simulations. In addition, it may be useful to consider local values of Ro and Fr, defined by

$$Ro = \frac{\zeta}{f_o}\ ,\ Fr = \frac{|U|}{\sqrt{gh}}, \tag{3}$$

where ζ is the relative vorticity.

We also define a parameter, γ,

$$\gamma = \frac{\delta_{\max}}{\zeta_{\max}}, \quad (4)$$

where $\delta_{\max}$ is the maximum horizontal divergence in the domain, and $\zeta_{\max}$ is the maximum relative vorticity in the domain. This parameter provides a crude estimate of the validity of nonlinear balance following the ad hoc scale analysis of Haltiner and Williams (1980), and its relevance may be verified *a posteriori* by evaluation of terms in the nonlinear balance equation.

The second-order potential vorticity (PV) inversion for the f plane shallow-water equations is based upon the methods described by McIntyre and Norton (2000) and Mohebalhojeh and Dritchel (2002). The invertibility principle of potential vorticity (Hoskins et al. 1985) stems from the fact that the PV (Q) is a balanced and materially conserved property,

$$\frac{DQ}{Dt} = 0, \quad (5)$$

where

$$Q = \frac{\zeta + f_o}{H + h}, \quad (6)$$

is the shallow-water PV. The relative vorticity, ζ, is defined as $(\partial v/\partial x - \partial u/\partial y)$, and h is the free surface deviation from a constant mean depth, H. Through a combination of asymptotic and heuristic arguments, a second-order potential vorticity inversion is formulized to solve for balanced dynamical quantities. The PV inversion equation set is

$$g\nabla^2 h = f_o\zeta + 2J(u,v) - \mathbf{V}\cdot\nabla\delta - \delta^2 \quad (7)$$

$$\begin{aligned}(gH\nabla^2 - f_o^2)\delta &= -g\nabla^2(\mathbf{V}\cdot\nabla h + h\delta) + \\ &f_o(\mathbf{V}\cdot\nabla\zeta + \zeta\delta) - [2J(u,v) - \mathbf{V}\cdot\nabla\delta - \delta^2]_t\end{aligned} \quad (8)$$

$$\zeta = (H + h)Q - f_o \quad (9)$$

$$\zeta_t = -\mathbf{V}\cdot\nabla\zeta - (f_o + \zeta)\delta \quad (10)$$

$$\mathbf{V} = \mathbf{k}\times\nabla\psi + \nabla\chi \quad (11)$$

$$\mathbf{V}_t = \mathbf{k}\times\nabla\psi_t, \quad (12)$$

where $\mathbf{V} = (u,v)$ is the horizontal wind vector, ∇ is the horizontal gradient operator, $\mathbf{k}$ is the vertical unit vector, $\delta = (\partial u/\partial x + \partial v/\partial y)$ is the horizontal divergence, ψ is the streamfunction, χ is the velocity potential, and subscript t denotes terms and variables that are evaluated using diagnostic estimates of the corresponding time derivatives $\partial V/\partial t$ and $\partial\zeta/\partial t$. Together, with suitable boundary conditions and the relations

$$\zeta = \nabla^2\psi \quad \text{and} \quad \delta = \nabla^2\chi, \quad (13)$$

the system (Eqs. (7)-(13)) can be solved iteratively for the balanced height, vorticity, divergence, and winds from the knowledge of the potential vorticity field, Q.

3. RESULTS

The simulation shown here is for values of U_o = 60 m s^{-1}, h_o = 750 m, and y_o = 450 km. The initial Ro_J and Fr_J for this simulation are 1.33 and 0.70, respectively. Examining the evolution of the wave, it is important to note that the evolution of the maximum local Rossby and Froude numbers varies only slightly and remains O(1) throughout the entire simulation because the jet speed does not vary greatly throughout the simulation (Fig.1). From a formal scaling perspective (e.g., Spall and McWilliams, 1992), it is required that either Ro or Fr be small for nonlinear balance to be valid. None of the conditions are met here for this strong jet.

Next, the evolution of the diagnostic parameter, γ, is shown in Figure 2. It is apparent that γ remains less than O(10^{-1}) for the duration of the simulation, so that the maximum horizontal divergence is not very large compared to the maximum relative vorticity. This implies to a good approximation that nonlinear balance should remain valid throughout the entire simulation. This immediately differs with regard to formal scale analyses previously noted, and further study in terms of higher-order balance relations is needed to confirm this implication.

In order to gain further insight into the evolution of balance given large Rossby and Froude numbers, a second-order potential vorticity inversion is performed to calculate the balanced and unbalanced divergence for this wave to determine if there is substantial evidence of unbalanced flow and possible IGW generation. For simulation t = 80 h, the wind and potential vorticity fields are shown in Fig. 3 showing the instability of the jet and wave structure. At finite amplitude, a sinuous mode resembling a Von Karman vortex street develops with alternating positive and negative vortices indicating the cyclonic and anticyclonic vorticity in the field as the barotropic wave grows in amplitude. The growth rate of this wave is 1×10^{-5} s^{-1}, which corresponds to an e-folding time of approximately 34 hours.

The total divergence and unbalanced divergence fields at t = 80 h are also indicated in Fig. 4. The unbalanced divergence field resulting from the potential vorticity inversion is $O(10^{-1})$ compared to the total divergence at this time. Furthermore, integration of the model at this point does not reveal any imbalances greater than $O(10^{-1})$ when compared to the total divergence. The structure of the unbalanced divergence field clearly indicates the absolute maximum unbalanced divergence occurs downstream from the jet streak in the wind field in the region of highly curved flow and strongest parcel deceleartions which are consistent with previous numerical and observational studies (e.g., Uccellini and Koch (1987) and Koch and O'Handley (1997)). Hence, the magnitude of the imbalance is very small, and IGW formation is unlikely to occur, even though clear structures of unbalanced divergence are readily apparent. The growth rate ($O(10^{-5})$) indicates that the possible slow evolution of the wave may prevent large imbalances. This is consistent with previous analyses and shows that the evolution of a barotropic wave utilizing the shallow-water equations remains balanced to a high degree, even though Rossby and Froude numbers are O(1) in strong jets.

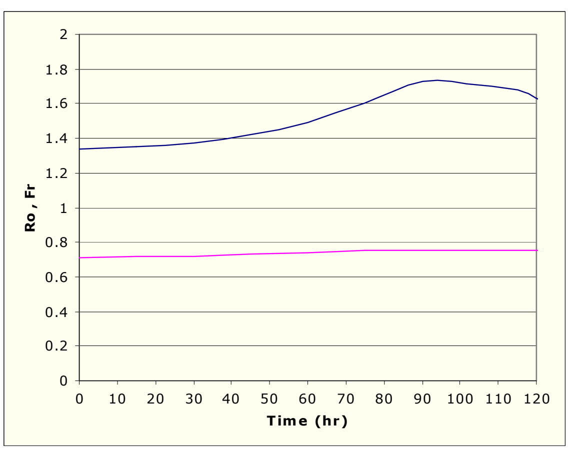

Figure 1. Time series of local Rossby number (blue) and local Froude number (pink).

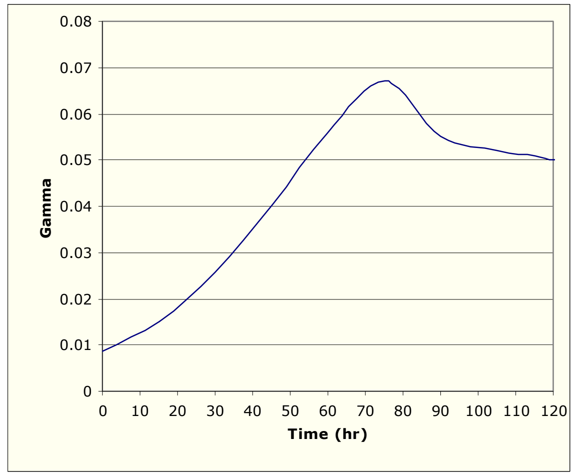

Figure 2. Time series of the ratio of maximum horizontal divergence to relative vorticity during the barotropic wave growth.

4. DISCUSSION AND FUTURE RESEARCH

The results of the diagnostic parameters and potential vorticity inversion throughout the simulation reveal that nonlinear balance should remain valid for strong jets, even though the Rossby and Froude numbers are O(1) throughout the simulation. Even though the presence of unbalanced flow, either in numerical simulations or in atmospheric data, is typically inferred via various diagnostic quantities, it is possible that these quantities are based on specific balance constraints (i.e., nonlinear balance, quasigeostrophy, semigeostrophy). It should be noted that an assessment of balance based on these constraints will not be an exact representation of the unbalanced flow.

Also, there are a few limiting factors to the generation of IGWs in the shallow-water model. In addition to the slow growth rate, there is a possibility, in the shallow-water system, that there is a constraint on the growth of the mean-square divergence that may limit the potential for gravity wave generation and/or amplification. Also, possible effects of the time Asselin filter and damping coefficient used in the model may damp possible IGWs to quite an extent. Implementation of an initially unbalanced simulation initiated continuous IGW propagation from the jet region, and in this model, the IGWs generally were not affected to a great extent as they radiated from the jet region.

Future studies of the evolution of unbalanced flow from initially balanced jets may include

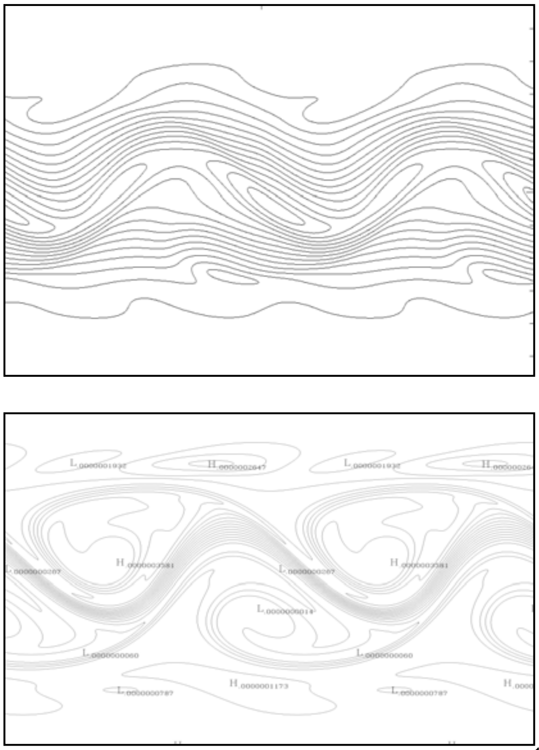

Figure 3. Wind speed (contour interval. 5 m s^{-1}) (top) and potential vorticity (contour interval 2×10^{-8} s^{-1}) (bottom) for t = 80 h.

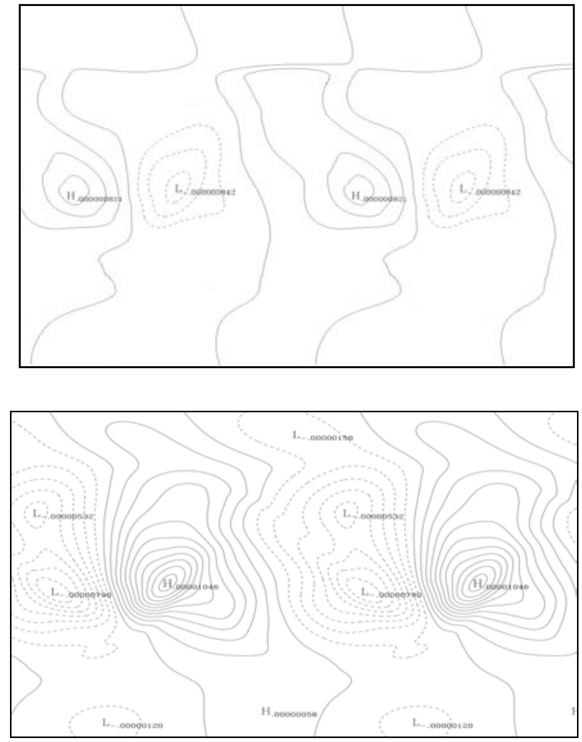

Figure 4. Horizontal divergence (contour interval 1×10^{-6} s^{-1}) (top) and unbalanced divergence contour interval 2.5×10^{-7} s^{-1}) (bottom) for t =80 h.

identification of inertia-gravity waves, both in the vicinity of the jet and in the far field following Ford (1994). Researching the potential impacts on the present study due to the addition of such factors as topography, beta-effect, and baroclinicity also should provide insight on the generation of unbalanced flow and IGWs. Nevertheless, the barotropic shallow-water model is the simplest non-trivial dynamical framework in which balanced and unbalanced flow can coexist; therefore, it is sensible to begin with this model and expand the study to more complicated regimes. In other words, a hierarchical approach should be taken by building upon simpler dynamical frameworks.

5. ACKNOWLEDGMENT

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