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## 1. Introduction

The range resolution of a pulse radar is determined by pulse width and receiver bandwidth. For the operational Weather Surveillance Radar 1988-Doppler (WSR-88D) the range resolution is 250 m when a short pulse is used (Crum and Alberty 1993). However, high-resolution is often needed to resolve fine-scale atmospheric structure and dynamics. An intuitive approach of increasing the range resolution is to decrease the width of transmitted pulse. As a result, however, the sensitivity is degraded due to the decrease of transmitted energy contained in one pulse. Many efforts have been made to mitigate resolution limitation such as pulse compression (Mudukutore et al. 1998; Schmidt et al. 1979) and range imaging (RIM) (Palmer et al. 1999). In this work, an advanced technique of improving range resolution using range oversampling is developed and demonstrated.

Radar signals from adjacent range gates are independent when they are digitally sampled at a rate of the reciprocal of pulse width and an ideal receiver of infinite bandwidth is used (Doviak and Zrnić (1993), Section 4.6). If a higher sampling rate is employed, the range weighting function (defined as a convolution of the transmitted pulse shape and the receiver impulse response Bringi and Chandrasekar (2001)) is overlapped in range. Therefore, signals from adjacent gates are correlated due to radar returns from a common region. This is termed range oversampling. Range oversampled signals can be processed and averaged to reduce the variance of the three spectral moment estimates, reflectivity, mean radial velocity, and spectrum width (Ivić et al. 2003; Torres and Zrnić 2003). However, the range resolution is degraded by approximately a factor of two due to the average process. A novel resolution enhance-

ment technique using the same range oversampling data is developed by (Yu et al. 2005) and is termed RETRO (Resolution Enhancement Technique using Range Oversampling).

## 2. RETRO

For a pulse radar with infinite bandwidth, the range resolution is defined as  $\Delta R = c\tau/2$ , where  $\tau$  and  $c$  are the pulse width and the speed of light, respectively. In general, radar returns are digitally sampled at a rate of  $1/\tau$  such that the center of the resolution volumes are spaced by  $\Delta R$  in range, and therefore spectral moment estimates from each volume are independent (Doviak and Zrnić (1993), Section 4.6). For the case of oversampling with a factor of  $L$  (sampling rate is  $1/L\tau$ ), adjacent resolution volumes will be overlapped and shifted by  $\Delta R/L$ . Therefore signals from contiguous volumes are correlated in range due to the radar returns from the region shared by adjacent resolution volumes.

In RETRO  $L$  range oversampled signals are processed to estimate spectral moments at a fine resolution equivalent to the size of subvolume ( $\Delta R/L$ ). It is assumed that the estimated high-resolution signal  $Y_j(mT_s)$  from the  $j$ th subvolume is a linear combination of oversampled signals  $\mathbf{V}$  as shown in the following vector notation.

$$Y_j(t) = \mathbf{w}_j^H \mathbf{V} \quad (1)$$

where the complex weighting function  $\mathbf{w}_j$  is a column vector of size  $L$ . The Hermitian (transpose conjugate) is denoted by a superscript of  $H$ . Furthermore, the autocorrelation function of  $Y_j$  is defined as  $R_{Y_j}(\tau) = \langle Y_j(t + \tau)Y_j^*(t) \rangle$  and can be derived in the following form.

$$R_{Y_j}(\tau) = \mathbf{w}_j^H \mathbf{R}_V \mathbf{w}_j \quad (2)$$

where  $\mathbf{R}_V = \langle \mathbf{V}\mathbf{V}^H \rangle$  is the autocorrelation matrix of range oversampled signals with a size of  $L \times L$ . The ensemble average is denoted by  $\langle \cdot \rangle$ .

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Now the key is to determine the complex weighting function  $w_j$ . A relatively simple and robust method developed by Capon (1969) was successfully applied to atmospheric radar to improve angular resolution using signals from spatially spaced receivers (Palmer et al. 1998; Yu et al. 2000), and to obtain high-resolution vertical profile of reflectivity and three-dimensional wind field using shifted frequencies (Palmer et al. 1999; Yu and Brown 2004). Using Capon's approach, the weighting vector in RETRO can be obtained by solving the following constrained optimization.

$$\min_{\mathbf{w}_j} R_{Y_j}(0), \quad \text{subject to } \mathbf{q}_j^H \mathbf{w}_j = 1 \quad (3)$$

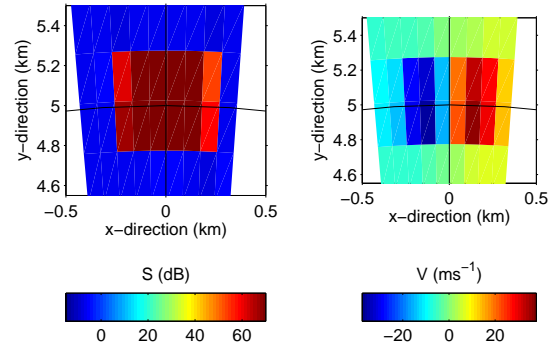
where  $\mathbf{q}_j$  is a column vector of size  $L$  specifying the weights for the true high resolution signals at the  $j$ th subvolume to produce oversampled signals in  $\mathbf{V}$ . The optimization ensures that the effect of interference is minimized while the response of the weighting function at the  $j$ th subvolume is unity. Interference refers to signals from subvolumes other than the subvolume in which the RETRO estimation is performed. The Capon weighting vector can be derived using a Lagrange method (Capon 1969; Palmer et al. 1998; Yu et al. 2005). As a result, time series of high-resolution signals at a subvolume  $Y_j$  can be generated by substituting the weighting function into (1). Reflectivity and radial velocity at subvolumes can be estimated by processing  $Y_j(mT_s), m = 0, 1, \dots, M - 1$ , using either spectral technique or autocovariance methods (Doviak and Zrnić (1993), Chapter 6), where  $T_s$  is the pulse repetition time and  $M$  is the number of samples.

Yu et al. (2005) have shown that the estimated high-resolution signal power ( $\hat{P}_j$ ) is a weighted sum of the true high-resolution power  $\sigma_l^2$  at  $2L - 1$  subvolumes.

$$\hat{P}_j = \sum_{l=1}^{2L-1} E_{jl} \sigma_l^2 \quad (4)$$

where the weight  $E_{jl}$  is the  $l$ th element of square of the RETRO response of the weighting function  $\mathbf{E}_j$ , and  $j = 1, \dots, 2L - 1$ . Thus, performance of high-resolution power estimate is determined by the weighting function which is adaptive to the atmospheric structure. Note that a deterministic bias in RETRO power estimate will be obtained for the case of uniform reflectivity. For example, the theoretical bias for  $L = 10$  is approximately 7.4 dB given that the correlation of oversampled signals in range has a triangular shape (Torres and Zrnić 2003). Nevertheless, for the case of uniform reflectivity resolution is not of primary interest and the whitening-based technique may be used to improve the statistical accuracy of the estimates. Yu

(a) Signal power and velocity from conventional sampling



(b) Signal power and velocity from RETRO

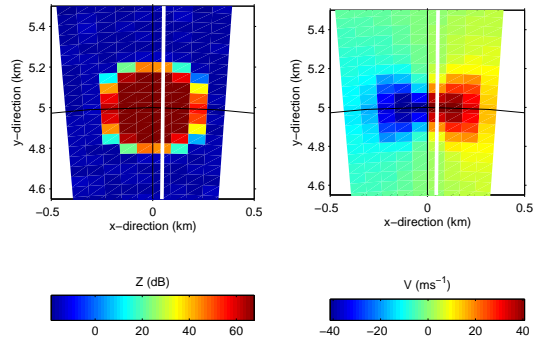


Figure 1: Reflectivity and velocity fields of a tornado vortex observed by a virtual WSR-88D using (a) conventional sampling ( $L = 1$ ) and (b) RETRO ( $L = 5$ ).

et al. (2005) have further shown that the high-resolution RETRO velocity is a biased estimator except for the case of uniform velocity field. However, the bias can be significantly small such that the RETRO high-resolution velocity estimates still reflects the true velocity field.

### 3. Results

RETRO is tested and verified using numerical simulation of a tornado vortex. Oversampled data were generated based on a simulation scheme similar to the one used in Torres and Zrnić (2003). Ideal time series were generated independently at each subvolume according to designed Doppler

spectrum as described in Zrnić (1975). In this work, tornado spectra are simulated using a combined Rankine model (Zrnić and Doviak 1975; Yu et al. 2003). Subsequently, range oversampled signals were obtained through a weighted sum of the  $L$  ideal time series. The weights are determined by the range weighting function. In this work, a rectangular pulse of 250 m is simulated to demonstrate the resolution improvement on WSR-88D. Moreover, it is assumed that receiver has a bandwidth much larger than  $1/\tau$ , the same condition used in Torres and Zrnić (2003) and Ivić et al. (2003). A sequence of white Gaussian noise is added to each set of oversampled data according to a designed signal-to-noise ratio (SNR).

In simulation, a tornado is located at 5 km north of the radar in which the diameter of the maximum wind is 100 m and the maximum tangential wind is  $50 \text{ m s}^{-1}$ . A doughnut-shape reflectivity with a width of 40 m is simulated and the maximum reflectivity is located at the radius of 110 m from the center of the tornado. Level I time series data collected by a virtual WSR-88D with  $1^\circ$  beamwidth and 250 m pulse length scanning through the tornado were simulated with and without range oversampling. The later sampling scheme is termed conventional sampling (CS) (i.e.,  $L = 1$ ). Both signals were generated at a constant SNR of 40 dB. The signal power and mean radial velocity from CS and RETRO with  $L = 5$  are shown in Figure 1(a) and (b), respectively. It is clear that the reflectivity structure and velocity distribution in range are better resolved when RETRO is used. Furthermore, the range profile of signal power and radial velocity at the azimuth angle of  $1^\circ$ , as indicated by the white lines in Figure 1(b), is shown in Figure 2 for both CS and RETRO. The modeled signal power and radial velocity with 50 m range resolution are denoted by red lines. It is evident that RETRO can reconstruct the tornado reflectivity and velocity profile at 50 m resolution using a 250 m pulse and oversampling factor of 5. Furthermore, the performance of RETRO as a function of SNR, oversampling factor ( $L$ ), and the number of samples is investigated by Yu et al. (2005).

Doppler spectra reconstructed using RETRO at the subvolume of 4.8, 5.05, and 5.25 km are shown in Figure 3. The RETRO-produced and model spectra are denoted by black and red lines, respectively. It has been shown that tornado spectrum is typically broad closed to the center of tornado vortex and can be used to improve the detection (Yu et al. 2003, 2004). It is clear that not only the three spectral moments but the Doppler spectra at these subvolumes can be reconstructed us-

ing RETRO. Another application of RETRO is the mitigation of clutter contamination and is demonstrated in Yu et al. (2005).

## 4. Conclusions

In this work, the application of range oversampled signals is further exploited. A technique was developed to improve range resolution using range oversampling and is termed RETRO. Improving resolution using overlapped range weighting function can be postulated as an inversion problem and the Capon method was used to solve the problem because of its simplicity and robustness. In RETRO it is assumed that high-resolution signals is a linear combination of oversampled signals. The weighting function is obtained by solving a constrained optimization. As a result, RETRO power and velocity estimates are adaptive to the oversampled data and optimal estimates can be obtained. RETRO is demonstrated using numerical simulations for a tornado case. The variation of tornado reflectivity and velocity in range was observed at resolution of subvolume using Capon RETRO, while the conventional processing techniques cannot resolve them. The result suggests that fine resolution can be achieved by transmitting a longer pulse and oversampling returns in range. However, note that the performance of RETRO is limited when the reflectivity is uniform or slowly varying in range.

## References

- Bringi, V. N. and V. Chandrasekar, 2001: *Polarimetric Doppler Weather Radar Principles and Applications*. Cambridge University Press, Cambridge, UK.
- Capon, J., 1969: High-resolution frequency-wavenumber spectrum analysis. *Proc. IEEE*, **57**, 1408–1419.
- Crum, T. D. and R. L. Alberty, 1993: The wsr-88d and the wsr-88d operational support facility. *Bull. Amer. Meteor. Soc.*, **74**, 1669–1687.
- Doviak, R. J. and D. S. Zrnić, 1993: *Doppler Radar and Weather Observations*. Academic, San Diego, Calif.
- Ivić, I. R., D. Zrnić, and S. M. Torres, 2003: Whiteness in range to improve weather radar spectral moment estimates part ii: experimental evaluation. *J. Atmos. Oceanic Technol.*, **20**, 1449–1459.

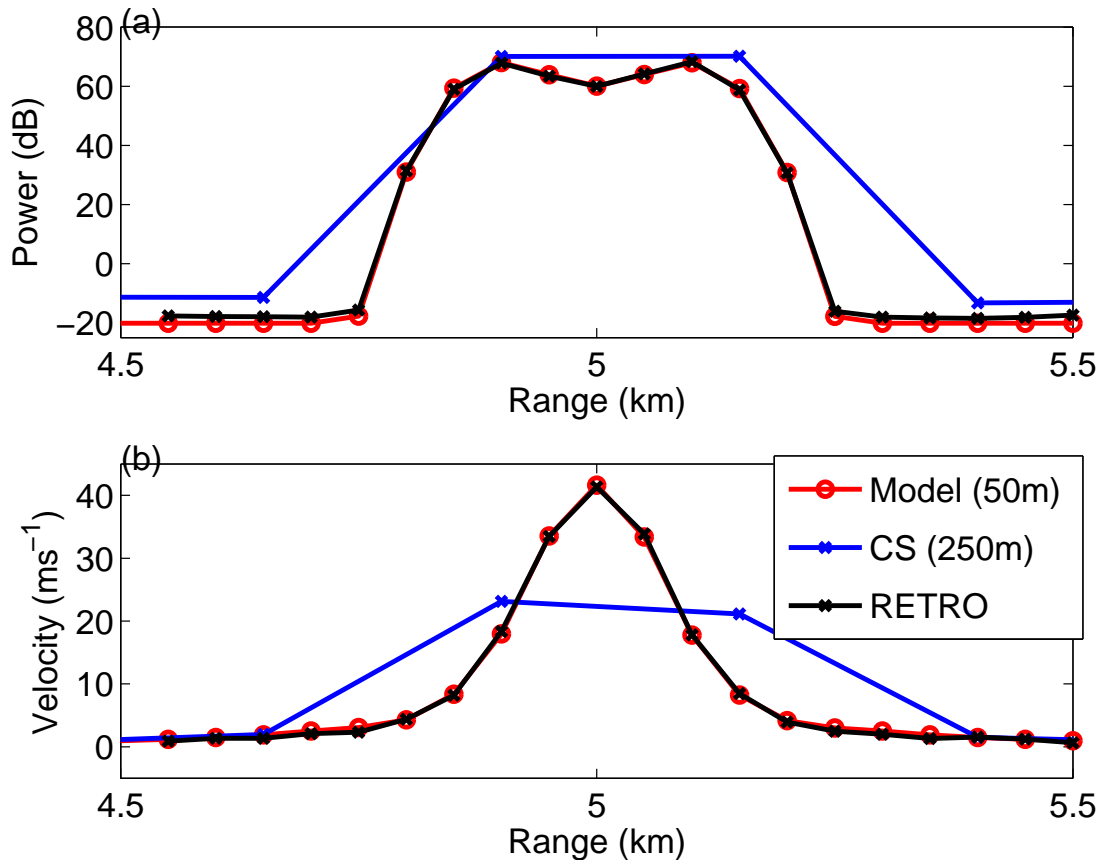


Figure 2: The range profile of (a) signal power and (b) radial velocity at  $1^\circ$  azimuth in Figure 1(b). It is evident the model reflectivity and velocity can be reconstructed using RETRO.

Mudukutore, A. S., V. Chandrasekar, and R. J. Keeler, 1998: Pulse compression for weather radars. *IEEE Trans. Geosci. Remote Sens.*, **36**, 125–142.

Palmer, R. D., S. Gopalam, T.-Y. Yu, and S. Fukao, 1998: Coherent radar imaging using capon's method. *Radio Sci.*, **33**, 1585–1598.

Palmer, R. D., T.-Y. Yu, and P. B. Chilson, 1999: Range imaging using frequency diversity. *Radio Sci.*, **34**, 1485–1496.

Schmidt, G., R. Ruster, and P. Czechowsky, 1979: Complementary code and digital filtering for detection of weak vhf radar signals from the mesosphere. *IEEE Trans. Geosci. Electro.*, **17**, 154–161.

Torres, S. M. and D. Zrnić, 2003: Whitening in range to improve weather radar spectral moment estimates. part i: formulation and simulation. *J. Atmos. Oceanic Technol.*, **20**, 1433–1448.

Yu, T.-Y. and W. O. J. Brown, 2004: High-resolution atmospheric profiling using combined spaced antenna and range imaging techniques. *Radio Sci.*, **39**, RS1101, doi:10.1029/2003RS002907.

Yu, T.-Y., R. D. Palmer, and D. L. Hysell, 2000: A simulation study of coherent radar imaging. *Radio Sci.*, **35**, 1129–1141.

Yu, T.-Y., A. Shapiro, D. Zrnić, M. Foster, D. Andra, R. Doviak, and M. Yeary, 2004: Tornado spectral signature observed by wsr-88d. *22<sup>nd</sup> Conference on Severe Local Storms*.

Yu, T.-Y., G. Zhang, A. Chalamalasetti, R. J. Doviak, and D. Zrnić, 2005: Resolution enhancement technique using range oversampling. *J. Atmos. Oceanic Technol.*, in press.

Yu, T.-Y., D. Zrnić, A. Shapiro, and M. B. Yeary, 2003: Feasibility of earlier tornado detection using doppler spectra. *31<sup>st</sup> Conference on Radar Meteorology*, 333–336.

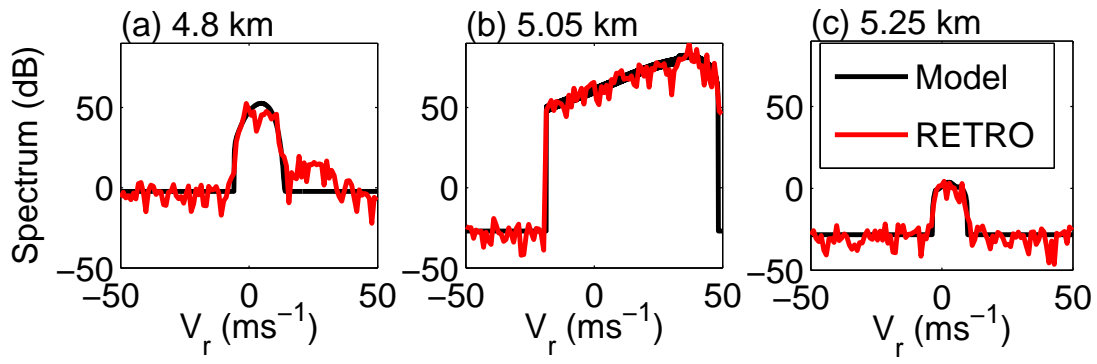


Figure 3: An example of selected RETRO spectra at range of 4.8, 5.05, and 5.25 km.

Zrnić, D. S., 1975: Simulation of weatherlike doppler spectra and signals. *J. Appl. Meteorol.*, **14**, 619–620.

Zrnić, D. S. and R. J. Doviak, 1975: Velocity spectra of vortices scanned with a pulsed-doppler radar. *J. Appl. Meteorol.*, **14**, 1531–1539.