P9R.1 THE ESTIMATION OF MODERATE RAIN RATES WITH OPERATIONAL POLARISATION RADAR.

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1. INTRODUCTION

Observations of differential reflectivity, Z_{Dr}, provide information on the mean raindrop shape and hence raindrop size, and when combined with the conventional reflectivity, Z, should yield more accurate rainfall rates, R. Ryzhkov et al (2005) report fractional errors of radar estimates of hourly gauge totals of about 50-70% using:

$$R = aZ^b Z_{DR}^c \quad (1).$$

rather than just over 80% from Z alone. Brandes et al (2003) found that using Z and Z_{DR} reduced the rms errors for rainfall accumulations from 7.8 to 6.7mm. In this paper we are concerned with much lower rain rates in the range 3-10 mm hr $^{-1}$ which are responsible for most of the flood producing rain in temperate regions. For such rain rates the differential phase shift is usually too small to be exploited, and the values of Z_{DR} are generally less than 0.8dB. In addition, for the operational C-band polarisation radars being installed in Europe the noise in Z_{DR} at a single range gate is about ±0.5dB. Ryzhkov et al (2005) average Z_{DR} over five adjacent gates and two radials thus reducing the Z_{DR} noise by about a factor of three for a resolution of about 1 x 2km. In this paper we suggest an alternative approach which is particularly appropriate for the very low values of Z_{DR} for rain rates nearer 3 mm hr^{-1}. We examine the behaviour of Z and Z_{DR} at each gate over a small region, and derive the normalised drop concentration N_w over that region. If we assume that the drop spectra over this region have the same characteristics, then from N_w we can derive the appropriate value of '*a*' to be used in $Z=aR^b$ over that region.

2. REPRESENTATION OF RAINDROP SIZE SPECTRA BY GAMMA FUNCTIONS.

The natural variability of rain drop spectra is well captured by the normalized gamma function:

$$N(D) = N_W\, f(\mu)\left(\frac{D}{D_0}\right)^{\mu} \exp\left(-\frac{(3.67+\mu)D}{D_0}\right) \quad (2)$$

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with three independent parameters, N_w, the normalised concentration, D_o, the median volumetric drop diameter, and μ a shape factor for the width of the spectrum. The normalization factor $f(\mu)$ is chosen so that for a given N_w the value of the liquid water is independent of μ and to ensure that when $\mu=0$ we have the conventional exponential form $N(D)=N_W \exp(-3.67(D/D_o))$.

Integration over suitably weighted values of equation (1) and assuming the terminal velocity is proportional to $D^{0.67}$, predicts Z varying as $N_W D_o^{7}$ and R as $N_W D_o^{4.67}$. Eliminating D_o gives $Z=aR^b$ with $b=1.5$ and a varying as $1/\sqrt{N_w}$ (Bringi and Chandrasekar, 2001). The oft quoted "factor of two" error in the value of R would then arise in natural rain if N_W varies by up to a factor of ten with 'a' changing by up to a factor of three. If we suppose N_w varies as $1/D_o$, as may occur in stratiform rain, then b ≈1.63 and 'a' depends on $N_w^{0.63}$. Alternatively, for tropical rain if N_W varies as D_o then we have b=1.4, and 'a' depending upon $N_w^{0.4}$.

For the Marshall-Palmer value of $N_W = 8000$ mm^{-1}m^{-3}, numerical integration of (2) with the correct terminal velocities yields $Z=218R^{1.52}$ and $Z=138R^{1.54}$ for a μ of 0 and 5, respectively. Similar in form to a 'conventional' relationship o $Z=200R^{1.5}$. For other values of N_W we have:

$$a = a_{MP}\sqrt{\frac{8000}{N_W}} \quad (3)$$

Where a_{MP} varies from 138 to 218 depending on the choice of μ. If drop spectra are well behaved and this range of μ encompasses the natural variability then we would expect the retrieved rainfall accuracy to have a 25% range. If such an accuracy is to be achieved then absolute calibration of Z is essential. This is difficult to achieve with rain gauge comparisons, but calibration of Z to within 10% is possible by exploiting the redundancy of the polarisation parameters, Z, Z_{Dr}, and K_{DP} in heavy rain (Goddard et al, 1994). It is also necessary to ensure that the absolute calibration of Z_{DR} is better than 0.1dB which can be achieved by observing light drizzle.

3. NOISE IN DIFFERENTIAL REFLECTIVITY AND ITS EFFECTS ON RAINFALL RATES.

Using ‘hybrid’ or simultaneous transmission of horizontally and vertically polarised radiation, the accuracy of Z_{DR} is limited by the co-polar correlation of the signals. The accuracy improves with spectral width because there are more independent samples. For the C-band systems being installed in the UK and France correlations in rain reach 0.98 and tests show that the single gate noise in Z_{DR} is about 0.5dB for a dwell of about 100msec needed for operational scanning and for a spectral width of 1m s^{-1} which is typical for light rain. In heavy rain there is more turbulence and the Z_{DR} noise is reduced. These figures are in agreement with theory (Bringi and Chandrasekar, 2001).

In this paper we use data from the Chilbolton S-band radar. The 25m antenna with its well matched sidelobes yields values of correlation in the rain of *0.99* and coupled with the slow scanning this leads to a noise in the Z_{DR} at each gate in light rain of about *0.15*dB. To model the performance of the C-band radars extra noise can be introduced into the values of Z_{DR} .

This large error in measurement means that rainfall rates calculated on a gate-by-gate basis will be noisy and may be biased. There also is the problem of negative Z_{DR}, for example if the true Z_{dr} is *0.5* dB but a standard deviation of *0.5* dB means that *16* % of points will be recorded negative. In many early forms of (1) Z_{DR} was expressed in dB, and so negative values would result in negative rainfall. More recently (e.g Ryzhkov et al 2005) this has been avoided by using differential reflectivity expressed in linear terms, Z_{dr}.

4. THE AREA INTEGRATED Z/ZDR TECHNIQUE.

To overcome the high level of noise in individual values of Z_{DR} at low rainfall rates we suggest that the behaviour of Z and Z_{DR} data over a regions be considered, and calculate the implied value of N_W for the region, which will then be used to calculate a from $Z=aR^b$. Using a normalised gamma distribution (2)) with $\mu=5$ and Andsager et al. (1999) drop shapes (an essentially identical curve is found using the Goddard et al, 1982 shapes) we can calculate the position of a line of constant N_W in Z / Z_{dr} space as displayed in Figure 1. Because Z_{DR} is independent of N_w but Z scales with N_W , lines of constant N_w are displaced in the vertical as shown in Figure 1.

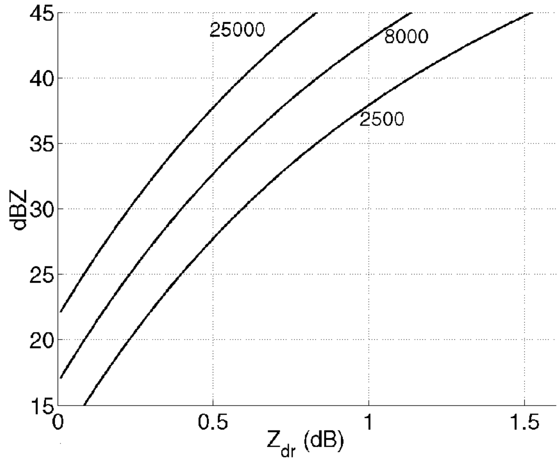

Figure 1. Lines of constant N_W in Z/Z_{dr} space. Shown are the curves for N_W = *2500, 8000 and 25000* $mm^{-1}m^3$.

The procedure for finding the value of N_w for the values of Z and Z_{DR} over a region is straightforward. The error in Z_{DR} is much larger than the error in Z, so the value of N_w is chosen which minimises the square of the residuals of the observed values of Z_{DR} about this line as demonstrated in Figure 2.

We have assumed that the targets are all in the rain. Any spurious targets with spurious values of Z and Z_{DR} must be removed before the fit is performed. An efficient way or removing ground clutter, anaprop and melting snow in the bright band is to accept as rain only data points with a linear depolarisation ratio, L_{DR}, < -20 dB.

Once the value of N_W has been found, we calculate the value of a to be used using (3) and find R from $Z = a R^{1.5}$.

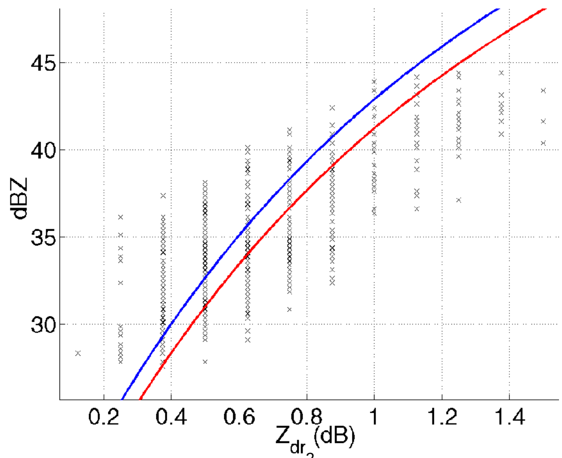

Figure 2. Example of *25* km^2 of data, taken with the Chilbolton S-Band radar, showing in blue the line for N_W = *8000* $mm^{-1}m^3$ and the best fit red line for N_W = *2200* $mm^{-1}m^3$ which corresponds to a=*247* if μ =5.

5. EVALUATION WITH TIPPING BUCKET RAINGAUGES.

To evaluate the technique, 0.2mm resolution tipping bucket rain gauge data have been compared with instantaneous rainfall rates from PPI radar scans performed every 45 seconds. The radar range gate is 300m and the azimuthal resolution better than 200m. The beam height is between 200 and 400m above the gauges so a correction of up to one minute is made for the time to the drops to fall to the ground. Rainfall rates calculated using just the pixel above the gauge were found to be virtually identical to those derived the eight radar pixels which were nearest neighbours.

Figures 3-6 compares four gauges on 21 April 2004 with the conventional $Z = 200\ R^{1.5}$ and the Z/Z_{DR} integrated technique, where the upper μ=5 trace has rain rates about 25% higher than the lower μ=0 line. Also plotted is the form of (1) suggested by Chandrasekar and Bringi (2001) and also tested by Ryzhkov et al (2005) for much higher rain rates:

$$R = 0.0067 Z^{0.927} Z_{dr}^{-3.43} \qquad (4)$$

where Z_{dr} is in linear units. No trace is included for the form of (4) with differential reflectivity in dB as the negative values caused the algorithm to go unstable.

In Fig 3 the integrated Z/Z_{DR} technique correctly identifies $a \approx 100$ during the heavier rainfall (12.0 to 13.0hrs) increasing to ≈ 200 during the subsequent lighter rainfall. For the other three examples the performance of the three algorithms is much more similar.

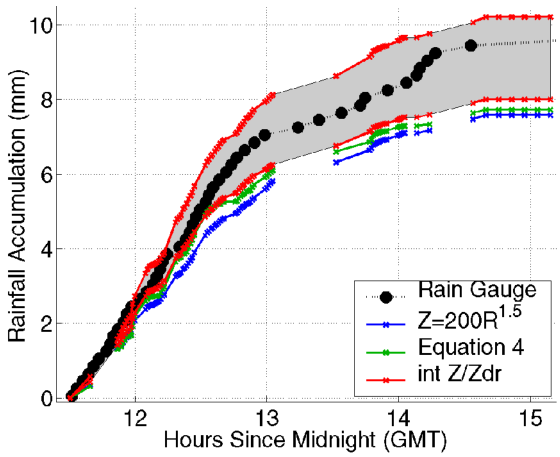

Figure 3. Observed rainfall and rainfall from the three radar algorithms for Tisbury gauge range 43.7km.

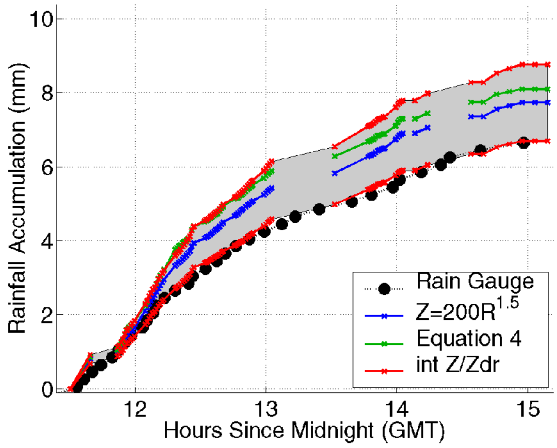

Fig 4: As for Fig 3 but for the Easterton gauge at 40.1km range.

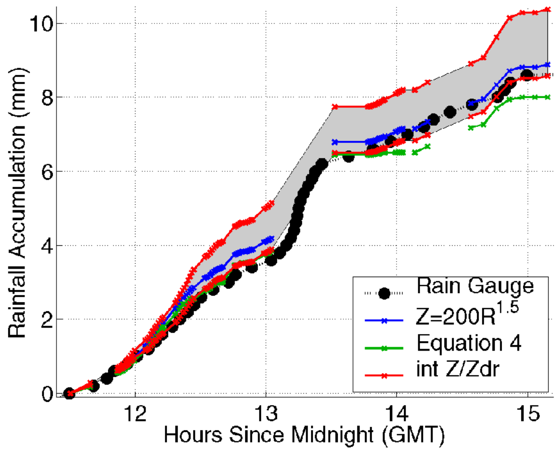

Figure 5. As for Fig 3 but for the Tidworth gauge at 18.2km

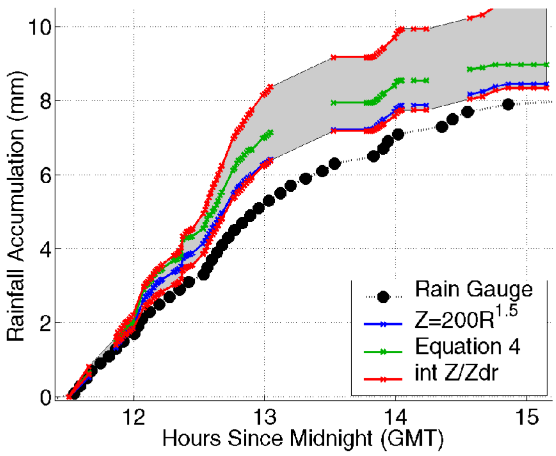

Fig. 6: As for Fig 3 but for the Winterbourne Stoke gauge range 32km.

6. ANALYSIS OF PPI.

Figures 7 and 8 display the values of Z and derived values of ‘a’ with 5km resolution for a single PPI taken with the operational Met Office C-band radar. These data were taken on a test rig which had some obscured azimuths where the data are missing. Note the values of ‘a’ below 100 in regions of lighter rain. The scatter plot of Z and Z_{DR} in Figure 9 confirm that this is associated with drizzle, with a larger concentration of small drops as shown by the lower than expected values of Z_{DR} leading to high drop concentrations.

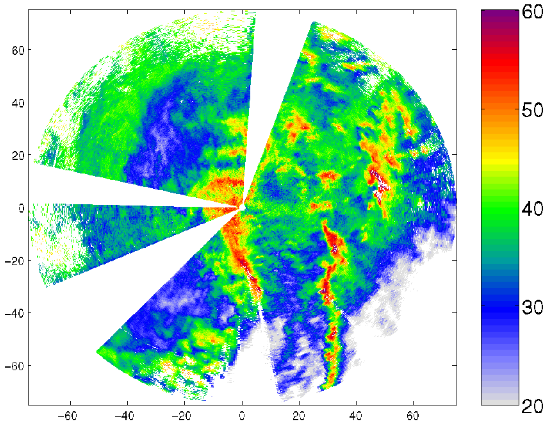

Figure 7: C-band PPI reflectivity data taken on 7 April 2005..

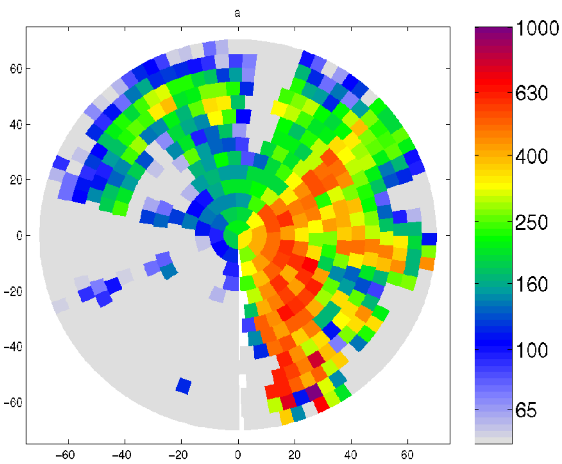

Figure 8: Values of ‘a’ derived from the Z and Z_{DR} data in figure 7.

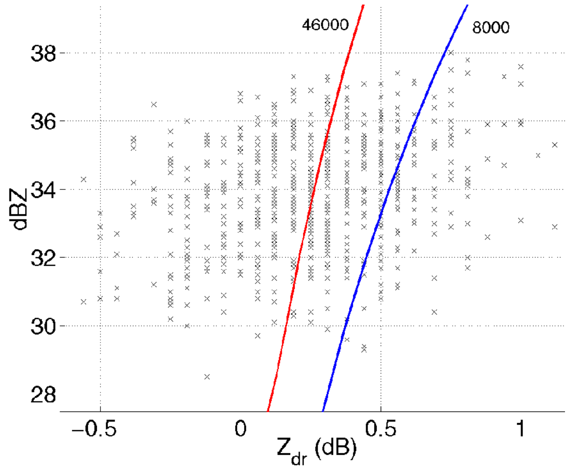

Figure 9. Scatter of the individual values of Z and Z_{DR} for one of the pixels in figure 8 with a=54, showing the lower than expected values of Z_{DR} indicating that light ‘drizzle’ rainfall with a high concentration of small drops.

7 CONCLUSIONS.

These first results indicate that the technique has potential for deriving improved rainfall rates even in light rainfall near to 3mm hr^{-1}.

8. ACKNOWLEDGEMENTS.

We thank the staff at Chilbolton and the RCRU at RAL for providing the Chilbolton radar data. RJT acknowledges the support of a NERC studentship, the Met Office and the Environment Agency.

9. REFERENCES.

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