5M.4

Mountain Waves and Boundary Layers

Ronald B. Smith
Department of Geology and Geophysics
Yale University, Connecticut, USA

1. Abstract

Linear hydrostatic 3-D mountain wave theory is extended to include a thin frictional boundary layer (BL), parameterized using two characteristic relaxation times for wind adjustment. The friction amplifies the BL wind response and shifts it upstream so that the wind maxima occur in regions of favorable pressure gradient; not at points of minimum pressure. We prove that variations in BL thickness always reduce the mountain wave amplitude. The wave momentum flux reduction by the BL is greater than the pressure drag reduction, indicating that part of the pressure drag is taken from BL momentum. The BL has its biggest effect on narrower hills (say 1 to 10km in width) where frictional equilibration does not occur. The BL effect is sensitive to the ratio of the friction coefficients at the bottom and top of the BL, causing a diurnal variation in wave amplitude. The boundary layer improves the linear theory description of windy peaks. Low level flow splitting is enhanced and wave breaking aloft is reduced. The BL also decreases the amount of upslope orographic precipitation.

2. Formulation

Our goal is to construct the simplest possible model that captures the physics of the interaction between BLs and mountain waves. We begin by imagining a thin homogenous layer of fluid near the ground, with thickness $H+\eta$ and speed $U_B = \overline{U}_B + u'_B$ beneath a deep free atmosphere with wind speed $U = \overline{U} + u'$. The pressure (P) imposed by the free atmosphere penetrates the thin boundary layer so that each fluid layer within the BL feels the same horizontal pressure gradient force; a standard assumption in classical BL theory (e.g. Schlichting et al., 2000) The parameter C_B represents friction at the surface and C_T represents friction between the BL and the free atmosphere. Increased surface roughness should increase C_B. According to Monin-Obukhov scaling, upper BL turbulence will respond to surface heat flux. Positive heat flux will cause thermal convection that will increase C_T. Negative heat flux will stabilize the upper BL and reduce C_T. The steady momentum equation for the BL is

$$U_B \frac{dU_B}{dx} = -\frac{dP}{dx} - C_B U_B + C_T (U - U_B) \tag{1}$$

while the wind in the free atmosphere obeys

$$U \frac{dU}{dx} = -\frac{dP}{dx} \tag{2}$$

In the absence of any disturbance, $P, U = \overline{U}, U_B = \overline{U}_B$ are constant so from (1)

$$\overline{U}_B = \frac{C_T}{C_B + C_T}\overline{U} = \frac{r_C}{1+r_C}\overline{U} \tag{3}$$

where the friction ratio $r_C = C_T / C_B$. As r_C increases, the BL wind speed $\overline{U}_B \to \overline{U}$.

Assuming that the perturbations are periodic of the form $f(x) = \mathrm{Re}[\hat{f}e^{ikx}]$, (1, 2) give

$$ik\overline{U}_B \hat{u}_B = -ik\hat{P} - C_B \hat{u}_B + C_T(\hat{u} - \hat{u}_B) \tag{4}$$

and $$ik\overline{U}\hat{u} = -ik\hat{P} \tag{5}$$

According to (4, 5) the BL wind speed can respond more strongly to an imposed pressure gradient than the free atmosphere because of its smaller advective velocity (i.e. $\overline{U}_B < \overline{U}$); but it is also limited by the friction terms. The wind response ratio ($\hat{R}$) can be found from (4, 5)

$$\hat{R} = \frac{\hat{u}_B}{\hat{u}} = \frac{ik\overline{U} + C_T}{ik\overline{U}_B + C_B + C_T} \tag{6}$$

According to (6), the wind speed in the BL responds to free atmosphere wind speed perturbations with a different amplitude and phase. Note that when $\overline{U}, \overline{U}_B, C_T, C_B$ are all positive (in (6)), complex $\hat{R}$ lies in the 1st or 4th quadrant depending on whether k is positive or negative. In both cases, the phase shift represents an upstream shift to the BL wind perturbation.

To complete the analysis, the mass conservation equation is used to compute the displacement of the BL top, and this displacement is matched with the wave properties in the free atmosphere, that is,

$$\hat{w}(z) = \hat{A}e^{imz} + \hat{B}e^{-imz} \tag{7}$$

giving

$$\hat{A} = -\hat{q}\hat{B} + \frac{ikU\hat{h}}{1-\gamma} \tag{8}$$

where $\hat{h}(k)$ is the Fourier Transform of the terrain, the complex reflection coefficient is $\hat{q} = \frac{1+\gamma}{1-\gamma}$ and $\gamma = \hat{\beta}\overline{U}m/k = i(\overline{U}/\overline{U}_B)Hm\hat{R}$. Further details are given in Smith et al, (2005) and Smith (2005).

The three BL parameters in our model can be combined with other terrain and atmospheric parameters to give three non-dimensional BL parameters;

$$p_1 = NH/U \quad p_2 = U/aC_B \quad p_3 = C_T/C_B \tag{9}$$

where "a" is a terrain length scale. The first "p_1" compares the BL thickness to the vertical wavelength. The second "p_2" compares the terrain advection time (a/U) to the BL relaxation time. The third "p_3" compares the two friction coefficients.

3. Results of the BL Model: Surface Wind Field

To summarize the effect of the BL on wave generation, we consider hydrostatic flow over a Gaussian Ridge. The thinning and thickening of the BL gives an effective topography that is lower and shifted upstream compared to the actual terrain (Figure 1a). Because of this shift, and the additional upstream shift that arises from friction in the BL, the BL wind pattern is shifted even further upstream (Figure 1b). This change is important with respect to the wind at the ridge crest. With no BL, the perturbation wind is zero at the crest. With a BL, the perturbation is strongly positive at the crest; agreeing with common experience of windy hill tops.

This comparison is repeated for a circular Gaussian hill in Figure 2. With no BL, the windward and lee ward slopes have negative and positive wind speed perturbations, with zero at the hill top. With a BL, the pattern is shifted upstream so that the peak is windy and slower winds are found upstream and downstream.

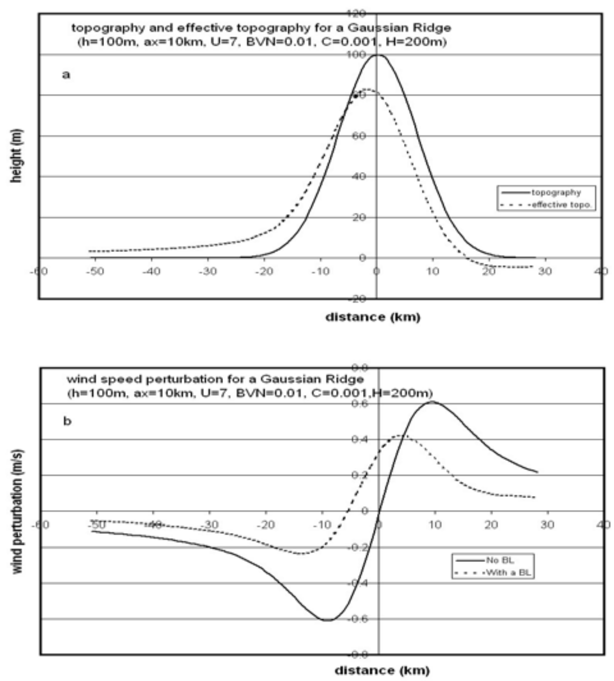

Figure 1: The influence of the BL on hydrostatic flow over an infinite Gaussian ridge. Mean conditions are U = 7m/s, N = $0.01s^{-1}$. The ridge is 100m high and 10 km wide. The BL has a depth of H = 200m and friction coefficients of $C_T = C_B = 0.001s^{-1}$. The non-dimensional parameters (9) are $p_1 = 0.29$, $p_2 = 0.7$, and $p_3 = 1$. Part a) is the terrain and the effective terrain. Part b) is the wind speed perturbation with and without the BL.

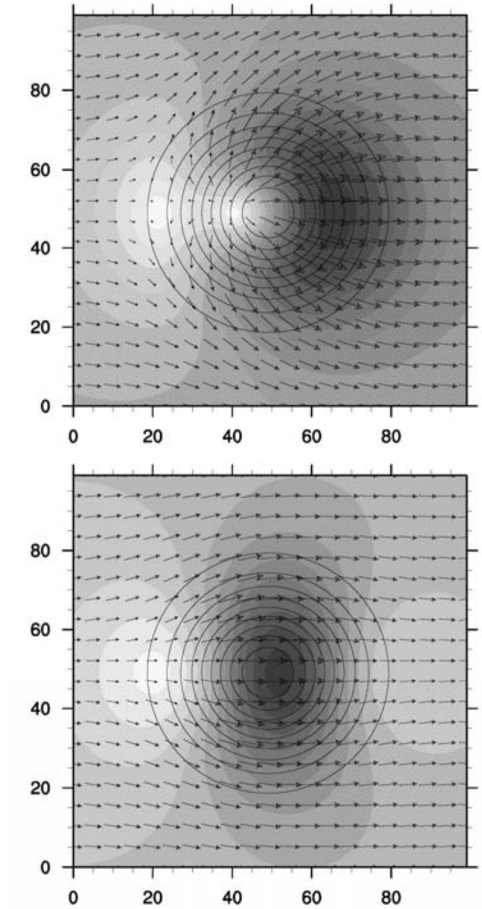

Figure 2: The wind field near a circular Gaussian hill. a) no BL, b) with a BL. The hill is 1 kilometer high with 100m contours. Surface winds are shown as vectors. The wind speed is shaded. The dimensional parameters are $U = 10ms^{-1}; N = 0.001s^{-1}; H = 100m; C_B = C_T = 0.001s^{-1}$; $a = 10km; h_m = 100m$. The non-dimensional parameters (9) are $P_1 = 0.1$, $P_2 = 1$, $P_3 = 1$.

4. Results of the Model: Drag and Momentum Flux

The BL also has an effect on the mountain pressure drag and the wave momentum flux (Figure 3). The BL impact is scale-dependent. For a wide hill or a fast-responding BL, the BL stays in frictional equilibrium with the free atmosphere. The influence of the BL is small. For narrower hills, the BL is disturbed more quickly than friction can act and the response is larger. The control parameter for this variation is p2 (see 9). As p2 increases, the hill drag drops below the inviscid value. Note that the wave momentum flux drops more than the wave drag, indicating that some of the mountain drag is captured in the BL.

A nocturnal BL, with its reduced turbulence level, response more strongly to wave-induced pressure gradients, so the BL effect is greater. Closed form expressions can be derived using "k-theory" (Smith 2005).

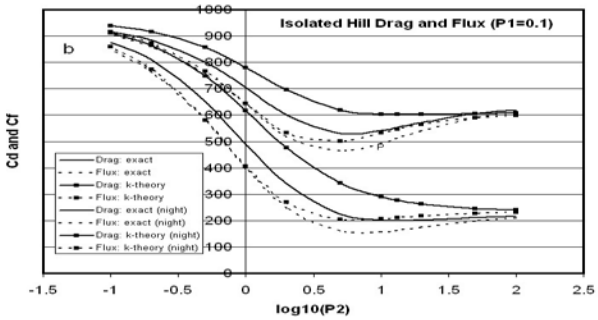

Figure 3: The hydrostatic Drag and Wave Momentum Flux coefficient for a Gaussian isolated hill are plotted against the log of the scale parameter $p_2 = U / C_B a$. Drag curves are solid. Flux curves are dashed. The k-theory value is also shown. Parameters used to make the plot are: $U = 10ms^{-1}$; $N = 0.01s^{-1}$; $H = 100m; C_B = C_T = 0.001s^{-1}$; $a = 10km; h_m = 100m$ so that $P_1 = 0.1$ and $P_3 = 1$. Curves marked "night" were computed with a reduced $C_T = 0.0005s^{-1}$ so $P_3 = 0.5$. All drag and flux values are multiplied by 1000. With no BL, the unscaled values would be $C_D = C_F = 0.9843$ for the circular Gaussian hill.

5. Conclusions

- The horizontal flow in the frictional BL responds to free stream pressure gradients strongly and with an upstream phase shift. Strong winds are predicted on hill tops.
- Horizontal divergence in the BL causes the BL thickness to vary, reducing wave generation, mountain drag and wave momentum flux. Some of the mountain drag is absorbed in the BL.
- The role of the BL in gravity wave generation is sensitive to the turbulent structure of the BL. Nocturnal BLs with weaker winds cause a larger reduction in wave generation.
- The linear theory of BLs introduces a new length scale to the mountain wave problem related to the downstream distance needed for a disturbed BL to return to frictional equilibrium. We estimate this distance to be $L_B = U / C_B \sim 10km$; midway between the conventional scales: $L_N = U / N \sim 1km$ and $L_f = U / f \sim 100km$.

6. Acknowledgements:

Qingfang Jiang , Idar Barstad and Jason Evans contributed useful ideas. This research was partly supported by the National Science Foundation, Division of Atmospheric Sciences (ATM-0112354).

REFERENCES

Jiang, Q., J.D. Doyle, R.B. Smith , 2005, Interaction between trapped waves and boundary layers, In Press JAS

Schlicting, H., K. Gersten, C. Mayes, 2000, *Boundary-Layer Theory*, Springer-Verlag, New York, 822p

Smith, R.B., Q. Jiang, J.D. Doyle, 2005, A theory of gravity wave absorption by boundary layers, In Press JAS.

Smith, R.B., 2005, Mountain Waves and Boundary Layers, Submitted to JAS