P4.12 DECORRELATION IN RANGE OF OVERSAMPLED WEATHER RADAR SIGNALS USING FIR FILTER

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1. INTRODUCTION

Due to rapid advances in digital technology and the demands of the telephone industry, digital receivers suitable for use in weather radars have become readily available. These receivers have the capability to produce samples of weather signals at rates that are several times higher than the reciprocal of the transmitted pulse width τ. It is natural to assume that, given the sampling rate of L/τ (where L is a positive integer) one could improve the estimates' variance by averaging L sample-time autocorrelations (at lags 0 and 1) in range. Unfortunately, simple averaging does not yield the maximum variance reduction because samples are correlated in range. Torres and Zrnić (2003) have proposed an approach that uses the prior knowledge of autocorrelation along range to decorrelate samples. The scheme operates on blocks (i.e., vectors) of L samples in range whereby each vector is multiplied by an $L\times L$ matrix producing a new vector of uncorrelated samples. Thus, each spectral moment is estimated from an $L\times M$ block (M is the number of samples along sample-time) of samples, where samples along range-time are not correlated. Consequently, range-time averaging of L autocovariances results in an optimal variance reduction given by the oversampling factor. In this paper, an alternative realization of the same concept is presented. Namely, the use of FIR (Finite Impulse Response) filters to obtain decorrelated samples in range is investigated. The study has been motivated by the fact that many digital receiver processors feature built-in FIR filters with programmable coefficients (e.g., GC4016 digital receiver chip from Texas Instruments).

2. THEORY

Let us assume that the transmitted pulse has an arbitrary envelope shape $p(l)$ of length K, and the digital receiver outputs samples at a rate L/τ (for the sake of briefness we follow the notation of Torres and Zrnić 2003). Then, the induced weighting to each contiguous elemental shell (or "slab") in the complex sample $V(l,n)$ of the composite echo (located at position l in range, and n in sample-time) is given as (Torres and Zrnić 2003):

$$V(l,n)=\left[\sum_{i=0}^{K-1}s(l+i,n)p(K-1-i)\right]*h(l), \qquad (1)$$

where $s(l,n)$ is the contribution of the elemental shell, $*$ denotes convolution and $h(l)$ is the receiver filter impulse response. For simplicity, we consider the system where the bandwidth of the radar receiver up to the point of Analog-to-Digital (A/D) conversion is greater than or equal to L/τ and the magnitude of the frequency response is flat for frequencies $[-L/\tau, L/\tau]$. Then the impulse response of the receiver filter $h(l)$ is dependent only on the coefficients of the digital filter applied to the complex samples. Let us re-write equation (1) as:

$$V(l,n)=\sum_{i=0}^{K-1}s(K-1+l-i,n)\left[h(i)*p(i)\right]. \qquad (2)$$

It is obvious that if $h(i)*p(i) = \delta(i-K+1)$ (where $\delta(i) = 1$ for $i = 0$ and zero otherwise) we obtain $V(l,n) = s(l,n)$. The FIR filter coefficients calculation can now be formulated as the least squares inverse filtering problem (solution of which is readily available in the signal processing literature). Following Hayes (1996) the coefficients can be obtained as a solution of the *Wiener-Hopf equations* given below in matrix form as:

$$\left[\mathbf{C_V^{(R)}}\right]^*\mathbf{H}=\mathbf{P}, \qquad (3)$$

where $\mathbf{C_V^{(R)}}$ is the Toeplitz-Hermitian correlation matrix before the receiver filter. Elements of this matrix can be found as $R_V^{(R)}(l) = p(l)*p^*(-l)$ ((2) in Torres and Zrnić 2003). Vector $\mathbf{P} = [p^*(K-1)\ p^*(K-2)\ldots\ p^*(0)]^T$, and $\mathbf{H} = [h(0)\ h(1)\ldots h(K-1)]$ contains the filter coefficients. The approximation error is given by:

$$Err=1-\sum_{l=0}^{K-1}h(l)p(K-1-l). \qquad (4)$$

3. THE NOISE ENHANCEMENT FACTOR

Additive noise is present in every system so we can write $V(l,n) = V_S(l,n) + V_N(l,n)$ where the subscripts S and N stand for signal and noise, respectively. Then, if we denote the transformed signal with $X(l,n) = X_S(l,n) + X_N(l,n)$ we can calculate the resulting autocorrelation of $X_N(l,n)$ as:

$$R_{X_N}^{(R)}(m)=N\sum_{k=m}^{K-1}h(k)h^*(k-m), \qquad (5)$$

where N is the noise power. Consequently, the noise becomes colored, and the noise component of the total power of the transformed signal is:

$$\left|X_N(l,n)\right|^2=N\sum_{k=0}^{K-1}\left|h(k)\right|^2. \qquad (6)$$

Hence, the noise enhancement factor (NEF) of the deconvolving FIR filter is:

$$NEF=\sum_{k=0}^{K-1}\left|h(k)\right|^2. \qquad (7)$$

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4. THE RANGE-WEIGHTING FUNCTION

As defined in Doviak and Zrnić (1993), the range-weighting function weights the reflectivity field in range. As previously shown, the deconvolving FIR filter attempts to transform samples $V(l,n)$ into $X(l,n)$ so that $X_S(l,n) \approx s(l,n)$. Henceforth, it makes sense to estimate the power using the formula:

$$\hat{S}_X = \sum_{l=0}^{L-1} w_F(l)\left(\frac{1}{M}\sum_{m=0}^{M-1}\left|X(l,n)\right|^2 - N\cdot NEF\right), \qquad (8)$$

where $w_F(l)$ is the weight assigned to each elemental shell echo $s(l,n)$. Following a similar derivation as in Doviak and Zrnić (1996) that leads to (4.22), but in the discrete domain, results in the expression for the range-weighting function:

$$\left|W(n)\right|^2 = \sum_{l=0}^{L-1} w_F(l)\left|\sum_{k=0}^{K-1} h(k)p(K-1-n+l-k)\right|^2, \qquad (9)$$

where n ranges from $-K+1$ to $K+L-2$. As an example, let us consider the pulse shape obtained as a return from a tower by the KOUN research WSR-88D radar, located in Norman, OK. The return signal was oversampled by a factor of 5 where the spacing between adjacent samples was 50 m. The pulse envelope and its phase profile are given in Fig. 1.

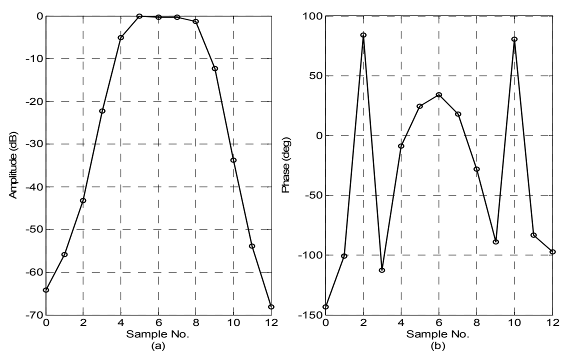

Figure 1. Amplitude and phase of the transmitted pulse.

The 3 dB pulse width is 1.57 μs (roughly corresponding to 250 m), but as can be seen from Fig. 1 (a) the pulse tails span more than just five samples. To take this into account as well as to obtain a filter that better approximates the desired impulse response, the FIR filter length was chosen to be K = 7. Then by setting $w_F(l) = 1/L$ and calculating $|W(n)|^2$ for an L of 1 and 5, we get the range-weighting functions shown in Fig. 2. Note that because the phase profile of the transmitted pulse (Fig. 1 (b)) is not flat the coefficients of the FIR filter are complex.

It is apparent from Fig. 2 that the support of the range-weighting functions in both cases is fairly wide (it spans $2K+L-2$ elemental shells). Particularly, given the range gate length of 5 samples (the case shown in Fig. 2 (b)), where the desired range resolution is 250 m, one can easily observe that the spectral moment estimate for the range gate encompassing elemental shells 0 to 4 will have contributions from two surrounding range gates. These contributions, however, will be attenuated by at least 10 dB. Nevertheless, this presents a serious problem in cases when strong signals are present in these range gates because they would contaminate the signal of interest.

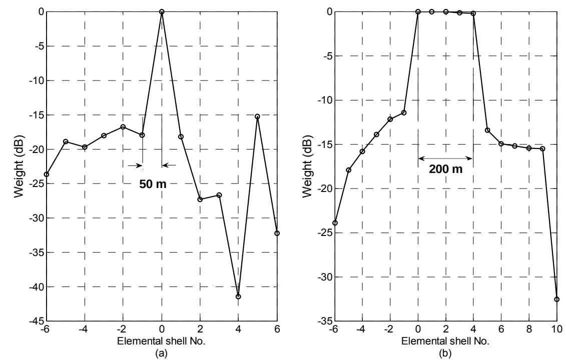

Figure 2. Range-weighting functions using an oversampling factor of 5, and a filter of length 7 with (a) no sample averaging, and (b) 5-sample averaging with uniform weights.

5. PERFORMANCE ANALYSIS

To assess the performance of the deconvolution FIR filter, the variance reduction of the autocovariance processor (Doviak and Zrnić 1996) for three estimators is compared. These are:

a. Oversampling-and-average-based (OAB). This estimator operates on oversampled data and each estimate is obtained though averaging L correlated autocovariances in range.
b. Whitening-transformation-based (WTB) (Torres and Zrnić 2003). This estimator operates on oversampled data to which the whitening transformation has been applied.
c. Deconvolving-filter-based (DFB). This estimator operates on oversampled data that has been transformed using a deconvolving FIR filter.

Power estimates are computed using (8) where, for the OAB and WTB, weights $w_F(l)$ are set to be $1/L$ (the estimator then reduces to the same one given in Torres and Zrnić 2003). To estimate the velocity and spectrum width, the autocorrelation in sample-time at lag 1 is calculated as:

$$\hat{R}_X^{(T)}(1) = \sum_{l=0}^{L-1}\frac{w_F(l)}{(M-1)}\sum_{m=0}^{M-2} X^*(l,m)X(l,m+1), \qquad (10)$$

where the weights $w_F(l)$ are the same as the ones used for the power estimator. The Doppler velocity is calculated as (Doviak and Zrnić 1993):

$$\hat{\upsilon} = -\frac{\upsilon_a}{\pi}\arg\left[\hat{R}_X^{(T)}(1)\right], \qquad (11)$$

where υ_a is the unambiguous velocity. The spectrum width is obtained as follows (Doviak and Zrnić 1993):

$$\hat{\sigma}_\upsilon = \frac{\upsilon_a\sqrt{2}}{\pi}\left|\ln\left[\frac{\hat{S}_X}{\hat{R}_X^{(T)}(1)}\right]\right|^{1/2} \operatorname{sgn}\left\{\ln\left[\frac{\hat{S}_X}{\hat{R}_X^{(T)}(1)}\right]\right\}. \qquad (12)$$

To find the weights for the DFB estimator, the range-weighting function in the case of OAB is calculated from (9) by setting $w_F(l)$ to $1/L$ and $h(k) = \delta(k)$ (shown in Fig. 3). These weights are then used as $w_F(l)$ values for the DFB. The resulting range-weighting functions are both shown in Fig. 3 for comparison. To achieve the shape of the range-weighting function in Fig. 3, the DFB estimator averages twice as many autocovariances in range as opposed to OAB (or WTB). Consequently, the support of the DFB range-weighting function is twice as wide. As previously stated, this makes the DFB estimator prone to contamination from strong signals located outside the region covered by the OAB range-weighting function.

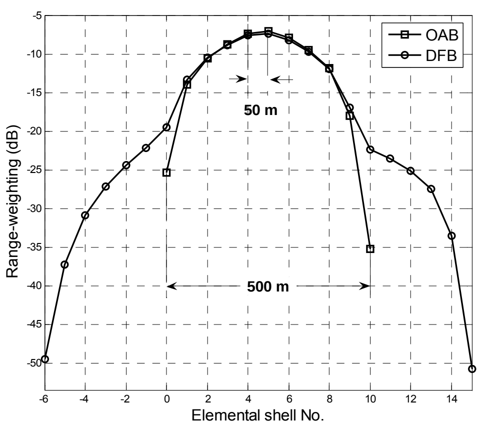

Figure 3. Range-weighting functions for OAB and DFB estimators.

In order to preserve the power, the FIR filter coefficients need to be scaled so that:

$$\sum_{l=0}^{L-1} w_F(l) \left| \sum_{k=0}^{K-1} \frac{h(k)}{scale} V(k-l,n) \right|^2 = \frac{1}{L} \sum_{l=0}^{L-1} \left| V(l,n) \right|^2. \qquad (13)$$

If the weights $w_F(l)$ are scaled so that their sum is one, one gets:

$$scale = \sqrt{\frac{\sum_{n=0}^{2K-2} \left| h(n) * p(n) \right|^2}{\sum_{k=0}^{N-1} \left| p(n) \right|^2}}. \qquad (14)$$

Three estimators are compared on simulated data for values of signal-to-noise ratio (SNR) ranging from 0 to 25 dB. Oversampled data was simulated using the pulse shape shown in Fig. 1. The results are given in Fig. 4. Observing the plots, it is apparent that both WTB and DFB estimators, as predicted, yield much smaller variances than OAB for all three Doppler spectral moments. Unfortunately, due to the noise enhancement factor present in both WTB and DFB estimator, this improvement disappears as SNR decreases. Furthermore, the curves show that the DFB estimator achieves somewhat larger variance reduction than WTB. This can be easily explained by the fact that the DFB estimator uses a block of $(2L+K-1)\times M$ samples to calculate each estimate as opposed $L\times M$ samples in the case of WTB estimator. Consequently, it becomes obvious that the DFB estimator does not achieve the maximum variance reduction given the number of samples it uses for estimation (i.e., it does not achieve the Cramer-Rao lower bound of Bamler (1991); thus, it is not optimal). This, however, is expected because the DFB estimator does not maximize the use of all the information from elemental shells (because it attempts to weight them differently) encompassed by the support of its range-weighting function, while the whitening transformation does.

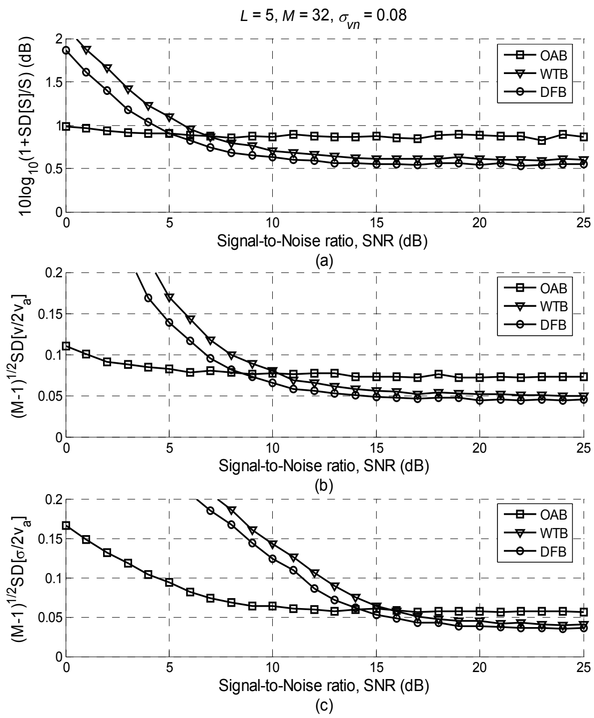

Figure 4. Normalized standard error of (a) signal power, (b) mean Doppler velocity, and (c) Doppler spectrum width vs the SNR for the pulse profile shown in Fig. 1. Curves were calculated from simulated data based on 2000 realizations for each SNR value.

6. TIME SERIES IMPLEMENTATION

For validation of the results obtained through simulations, all three estimators were applied to time series. Data used in the experiment was collected with the Echotek ECDR-GC814 digital receiver that had been incorporated into the RRDA (Research Radar Data Acquisition) subsystem (Ivić, Zahrai, and Zrnić 2003). The subsystem is capable of actively controlling the WSR-88D radar, producing Doppler spectral moments, and recording them as well as time series data on an array of hard disks (RAID). The system is capable of collecting data oversampled by a factor of 5 or 10. For this experiment, the weather signal was sampled at a rate five times greater than the inverse of the pulse width while the antenna was kept in a stationary position. To verify that all three estimators are unbiased with respect to each other, autocovariances averaged over 100 radials were used to estimate three Doppler spectral moments which are shown in Figs. 5

(a), 6 (a) and 7 (a). These were also used to estimate statistics on each of the three moments. The results are shown in Figs. 5 (b), 6 (b), and 7 (b). The total pool of data used for statistics estimation was 800 radials.

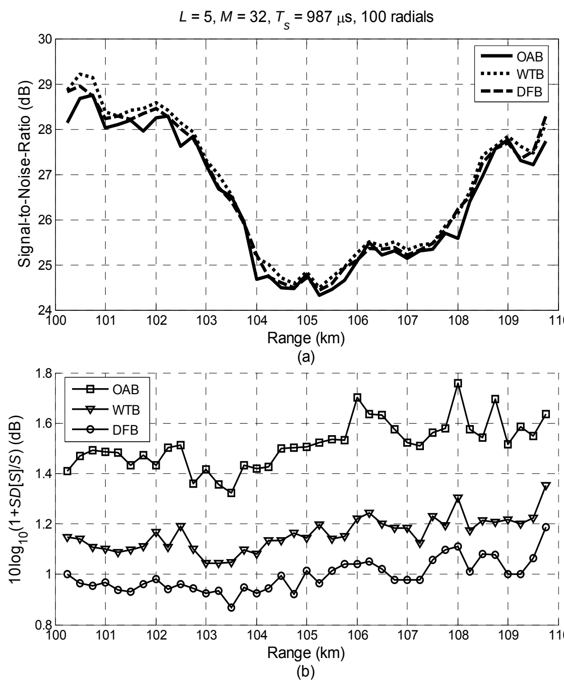

Figure 5. (a) Estimated SNR and (b) experimental normalized standard deviation of power estimates for L = 5.

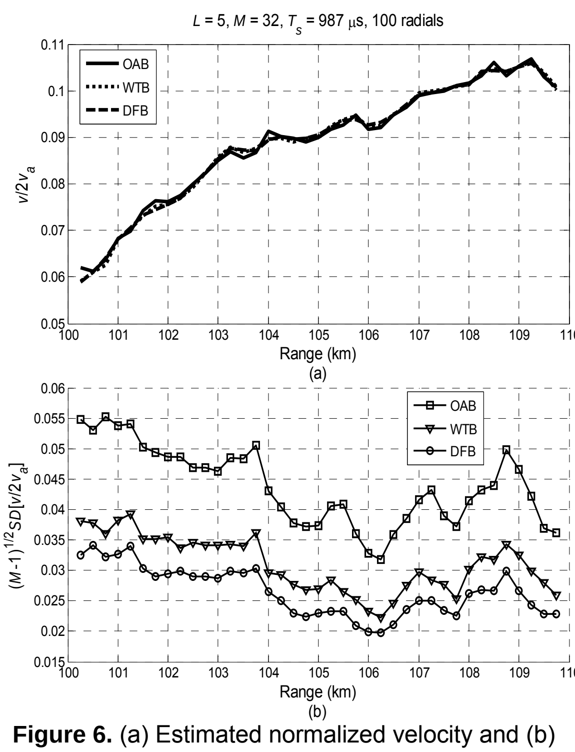

Figure 6. (a) Estimated normalized velocity and (b) experimental normalized standard deviation of velocity estimates for L = 5.

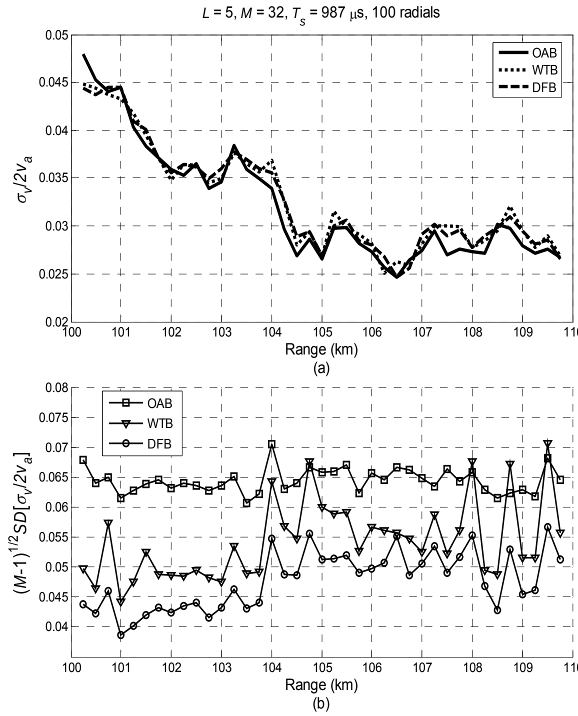

Figure 7. (a) Estimated normalized spectrum width and (b) experimental normalized standard deviation of spectrum width estimates for L = 5.

7. SUMMARY

Due to their wide use, many digital receiver circuits provide hardware based FIR filters with programmable coefficients. This makes FIR filters suitable for efficient hardware implementation. It is shown that by summing appropriately weighted sample-time autocovariance averages, it is possible to influence the shape of the estimators' range-weighting function. Furthermore, the possibility of using deconvolving FIR filters as an alternative to matrix-based whitening transformation was investigated. It was shown that an estimator using a deconvolving FIR filter that mimics the range-weighting function of the OAB estimator can be devised. However, the support of the range-weighting function of such an estimator is twice as wide as that of the OAB (or WTB) estimator. This has the potential of severely degrading the resolution in the case of strong reflectivity gradients.

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