# EXPERIMENTAL INVESTIGATION OF X-BAND POLARIMETRIC ALGORITHM FOR DROP SIZE DISTRIBUTION RETRIEVAL

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## 1. ABSTRACT

This paper investigates a technique for estimating Drop Size Distribution (*DSD*) model parameters from attenuation-corrected *X*-band radar data. The *DSD* model is assumed to be a three parameter "normalized" gamma distribution. Closely matched *X*-band (*XPOL*) and *S*-band (*S-POL*) dual-polarization radar measurements collected during the International $H_2O$ Experiment (*IHOP*) are used to assess the *X*-band based *DSD* retrieval. The goal of this comparison is to understand the relative accuracy of *X*-band *DSD* retrievals relative to those based on *S*- (2.8 GHz) band radar observation. The study further provides limited evaluation of the X-band *DSD* estimates using in-situ 2D-video disdrometer measurements available from the Ionian Sea Rainfall Experiment (*ISREX*).

## 2. INTRODUCTION

Dual-polarization weather radars have an advantage over single polarization systems as they introduce multi-parameter measurements by transmitting horizontal and vertical polarized electromagnetic waves. As the electromagnetic waves propagate through the atmospheric medium it undergoes scattering, differential attenuation, differential phase shifts, and depolarization. These multi- radar measurements are carrying information that can be used to estimate signal attenuation, raindrop particle size, and thermodynamic phase.

First Seliga and Bringi (1976, 1978) used polarimetric radar data, assuming an exponential *DSD*, to retrieve median volume diameter ($D_0$) by directly relating to the differential reflectivity ($Z_{DR}$) measurement (i.e., the ration of horizontal to vertical polarization reflectivity). In this study, dual-polarization measurements are used to estimate the governing parameters of a generalized gamma *DSD* model suggested by Ulbrich (1983) that characterize the natural variation of the *DSD* and associated rainfall rates. This paper is investigating a retrieval technique by comparing *X*-band based *DSD* parameter estimates to those derived from *S*-band dual-polarization measurements, as well as from *in situ* 2D-video disdrometer measurements.

## 3. BACKGROUND

### 3.1. ATTENUATION CORRECTION METHOD

The horizontal polarization reflectivity, $Z_{mH}$ ($mm^6m^{-3}$), and differential reflectivity, $Z_{mDR}$ (dB), measured by *XPOL* at range gate, *r*, are related to the corresponding equivalent (non-attenuated) radar parameters ($Z_H$ and $Z_{DR}$) as follows:

$$Z_{mH}(r) = Z_H(r)\times 10^{-0.2\int_0^r A_H(s)ds} \qquad (1)$$

$$Z_{mDR}(r) = Z_{DR}(r)\times 10^{-0.2\int_0^r A_{DP}(s)ds} \qquad (2)$$

$A_H$ and $A_{DP}$ (dB/km) are the specific and differential rain-path attenuation, respectively. Quantitative use of *X*-band radar data in estimation of precipitation parameters (rainfall rates, water content, raindrop size distribution) requires knowledge of the attenuation-corrected (i.e., $Z_H$ and $Z_{DR}$) radar parameters.

In dual-polarization systems $A_H$ and $A_{DP}$ profiles are derivable from $\Phi_{DP}$ measurements. Calculations based on raindrop spectra show that the gradient of $\Phi_{DP}$, i.e., specific differential phase shift, $K_{DP}$ (deg/km), relates almost linearly with $A_H$ and $A_{DP}$ (Bringi et al. 2001). Consequently, the two-way path-integrated specific and differential attenuation values at a radar range are equivalent to the $K_{DP}$ integration along that range (Matrosov et al. 2005; Anagnostou et al. 2005):

$$\int_0^r A_H(s)ds = \gamma\times\int_0^r K_{DP}(s)ds \qquad (3)$$

$$\int_0^r A_{DP}(s)ds = \varphi\times\int_0^r K_{DP}(s)ds \qquad (4)$$

The measured differential phase change, $\Phi_{DP}$, at range gate r is related to $K_{DP}$ as:

$$\Phi_{DP}(r) = \delta(r) + 2\int_0^r K_{DP}(s)ds \qquad (5)$$

where $\delta$ is the backscatter phase shift. Assuming negligible $\delta$ effect, one can relate directly the phase change to path-integrated attenuation. The iterative filter of Hubbert and Bringi (1995) is devised in this

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study to remove measurement noise and eliminate potential $\delta$ contamination in $\Phi_{DP}$ data.

### *3.2. RAINDROP SIZE DISTRIBUTION*

The raindrop size distribution describes the probability density-distribution function of raindrop sizes. In this study, we will use a gamma distribution model (Ulbrich, 1983):

$$N(D) = N_0 D^{\mu} \exp(-\Lambda D) \tag{6}$$

The three governing parameters $N_0$ ($\mathrm{mm}^{-1-\mu}\mathrm{m}^{-3}$),$\mu$ and $\Lambda$ ($\mathrm{mm}^{-1}$) are adequately describes some of the natural variations in the shape of the raindrop size distribution and has been widely accepted from the radar meteorology community. The relation between $D_0$, $\Lambda$ and $\mu$ is given by:

$$\Lambda D_0 \cong 3.67 + \mu \tag{7}$$

The gamma raindrop size distribution governing parameters can be normalized in order to be considering more physically (Willis, 1984; Testud et al. 2001):

$$N(D) = N_w f(\mu) \left( \frac{D}{D_0} \right)^{\mu} e^{-(3.67+\mu)\left(\frac{D}{D_0}\right)} \tag{8}$$

where $N_w$ is the scaled version of $N_0$ defined in (6),

$$N_w = \frac{N_0}{f(\mu)} D_0^{\mu} \tag{9}$$

and

$$f(\mu) = \frac{6}{(3.67)^4} \frac{(3.67+\mu)^{\mu+4}}{\Gamma(\mu+4)} \tag{10}$$

with $f(0)$ and $f(\mu)$ is a unit less function of $\mu$. The $N_w$ ($\mathrm{mm}^{-1}\mathrm{m}^{-3}$) is an intercept parameter of an equivalent exponential distribution with the same water content *W* ($\mathrm{gr\ m}^{-3}$) and $D_0$ (mm) as the gamma *DSD* (Bringi and Chandrasekar, 2002).

## 4. PARAMETER ESTIMATION OF THE GAMMA DSD

Once *X*-band data has been corrected for attenuation (Anagnostou et al. 2004; 2005-in press, Matrosov et al. 2002; 2005, Park et al. 2005-in press) it is possible to proceed with the estimation of the three governing parameters ($N_w$, $D_0$ and $\mu$) that control the gamma drop size distribution.

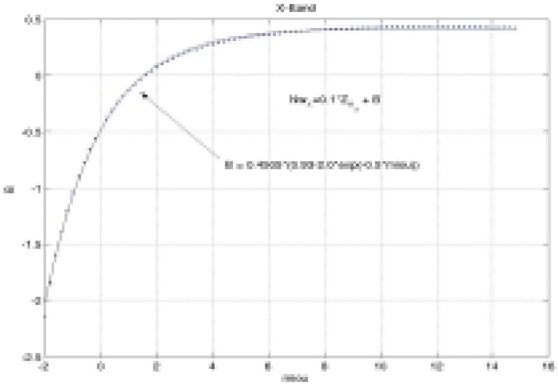
This method is based on the assumption that there are correlations among the *DSD* parameters therefore it is possible to reduce the unknown parameters and it is possible by deriving simple relations, based on the constrained-gamma model, to retrieve the three governing *DSD* parameters. Brandes et al. (2003, 2004) have shown the following form for the constrain relationship derived from measured raindrop spectra and based on the truncated moment method:

$$\Lambda = 1.935 + 0.735\mu + 0.0365\mu^2 \tag{11}$$

On the basis of the above constrained relation the following algorithm is proposed for estimating *DSD* parameters.

The algorithm consists of two parameterizations in the sequence order presented below. First it retrieves the $\mu$ parameter from $Z_{DR}$ based on the exponential type relationship shown in Figure 1 (upper panel). The $N_w$ is then estimated as function of $Z_H$ on the basis of the linear model (Figure 1—lower panel), with intercept parameter varying non-linearly as function of the estimated $\mu$ value. The third parameter, $D_0$, is then estimated using relation (7). The relationships are in the form of look up tables derived from *T*-matrix scattering simulations on the basis of measured *DSD* spectra assuming a polynomial axis ratio relation as in Brandes et al. (2002):

$$\begin{aligned} \mathrm{r} = 0.9951 + 0.02510\mathrm{D} - 0.03644\mathrm{D}^2 + \\ 0.00530330\mathrm{D}^3 - 0.0002492\mathrm{D}^4 \end{aligned} \tag{12}$$



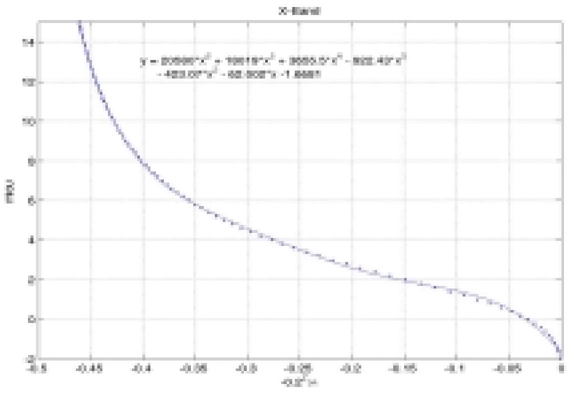


Fig. 1. Experimental relations of the $\mu$-$Z_{DR}$ and $N_w$-($Z_H$,$\mu$).

## 5. EVALUATION OF THE *XPOL* DSD RETRIEVAL

The algorithms presented in this study are assed based on two datasets. First, we use coincident *X*-/*S*-band data collected during the IHOP experiment. We selected two major mesoscale convective storm events (May 17th and June 15th and 16th, 2002) measured coincidentally by the National Observatory of Athens *X*-band dual-polarization Doppler radar on wheels (*XPOL*) and National Center of Atmospheric Research *S*-band dual-polarization Doppler radar (*S-POL*). In order to evaluate *X*-band *DSD* algorithm, *S-POL* observations are used as reference to retrieve the three governing *DSD* parameters using the well-established Brandes et al. (2004) algorithm. Figure 2 illustrates sample ray plots of $N_w$, $D_0$, and $\mu$ from matched *XPOL* and *S-POL* measurements taken from the June 16th case.

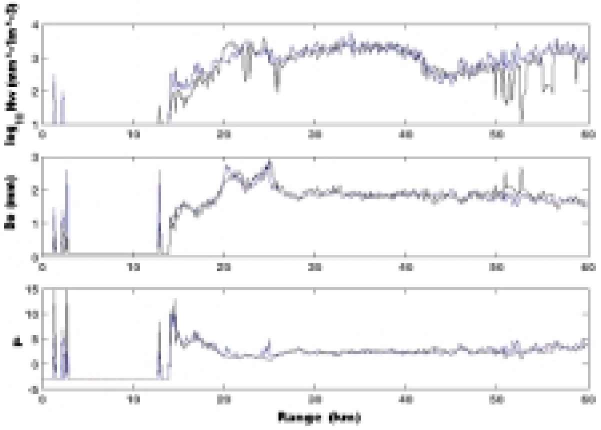


Fig. 2. Sample ray plots taken during June 16 showing good agreement of *DSD* parameters retrieved from *S-POL* observations (blue line) compared to retrievals from *XPOL* observations (black line) using the *XPOL DSD* algorithm.

Figure 3 shows histograms of DSD parameters derived from *XPOL and S-POL* measurements for all matched rays. We note close agreement between the two retrievals.

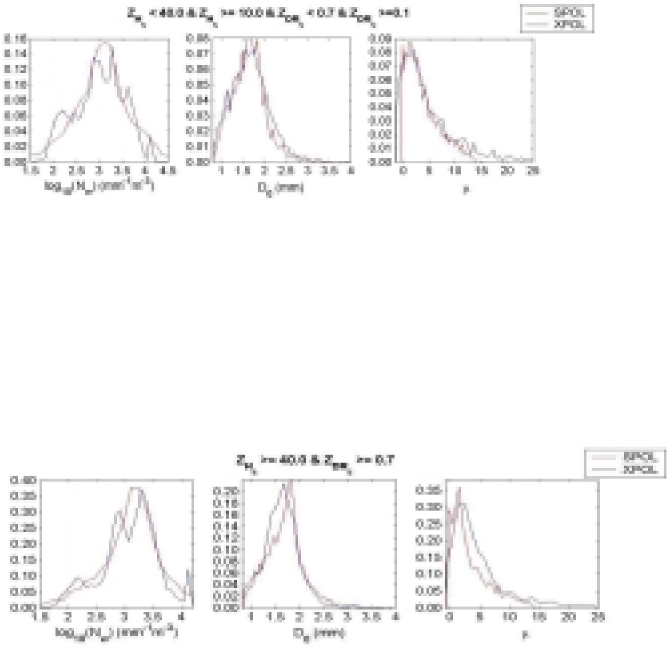


Fig. 3. Histograms of *DSD* parameters retrieved from *XPOL* corrected measurements (black line) and *S-POL* (blue line) observations for different $Z_H$ and $Z_{DR}$ thresholds.

Figure 4 shows scatter plots of *XPOL* versus *S-POL DSD* parameter retrievals for rainfall rates grater than 1 mm/hr. We note moderate scale, good correlation, and low bias between *XPOL* and *S-POL* retrievals.

The second dataset uses coincident *X-POL* and measured *DSD* spectra from an *in situ* (10-km radar range) 2-D video disdrometer. Figure 5 illustrates a time series comparison of *XPOL* attenuation-corrected horizontal and differential reflectivity and raw *XPOL* data with those simulated from the measured *DSD* spectra in the Ionia Sea Rainfall Experiment (*ISREX*) during the 12th of March 2004. This time series profile will be used to assess the *XPOL DSD* retrieval parameters. Figure 6 shows histograms of the retrieved (from *XPOL*) and calculated *DSD* parameters from the same time period.

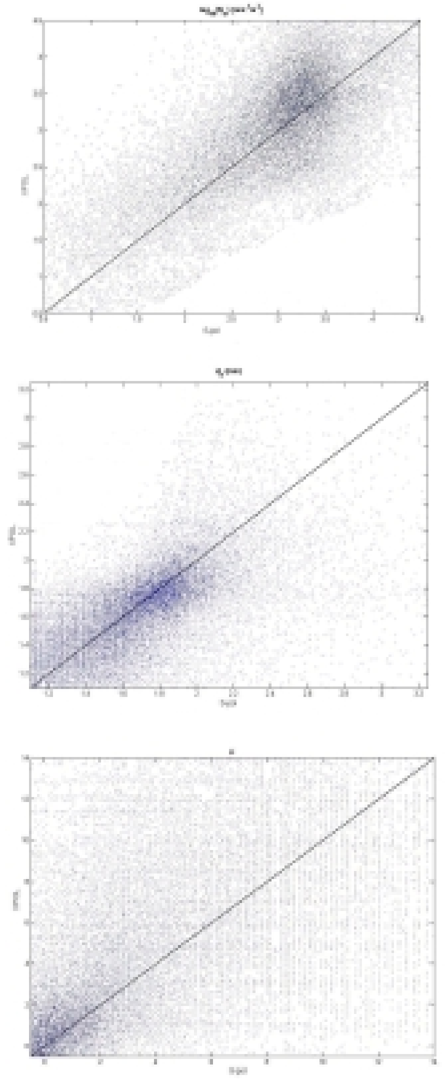


Fig. 4. Scatter plots of $N_w$, $D_0$ and $\mu$ values retrieved from *XPOL* attenuation-corrected versus *S-POL* observations. All matched rays were used here.

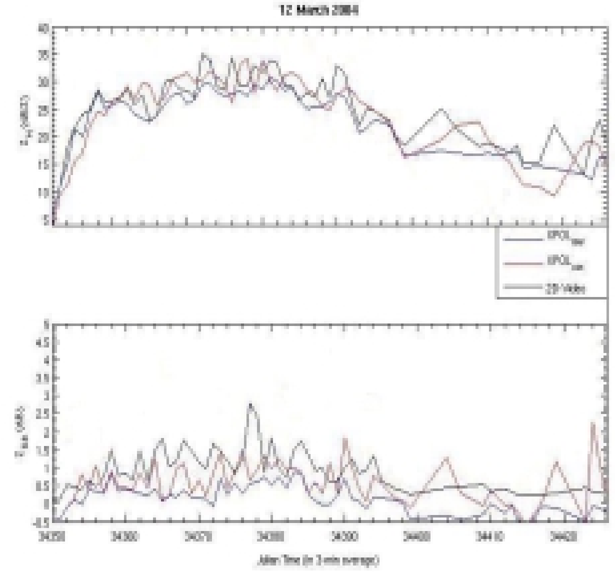


Fig. 5.Time series of *XPOL* radar observed and attenuation-corrected measurements compared to *X*-band simulations from the *in situ* (10-km radar

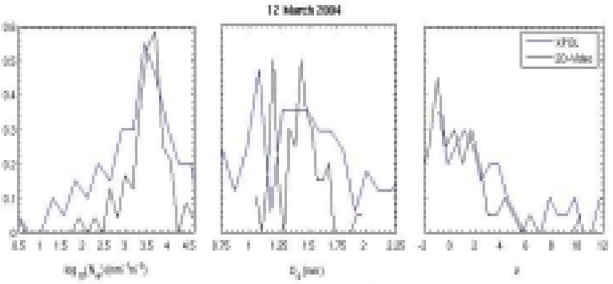


Fig. 6. Histograms of the three *DSD* parameters retrieved from *XPOL* and derived from the 2D-video disdrometer associated with the event shown in Figure 5.

## 7. SUMMARY AND CONCLUSION

The long-lasting goal of dual-polarimetric radar has been the rainfall estimation and the estimation of the parameters of the raindrop size distribution. *X*-band radars even though are associated with significant rain-path attenuation are more sensitive in low-to-moderate rain rates in favour of *S*-band radars. In this paper we exploited an algorithm for the estimation of the three-parameter gamma *DSD* model, $D_0$, $N_w$, and $\mu$ from *XPOL* observations $Z_H$, $Z_{DR}$, and $K_{DP}$. The *XPOL DSD* algorithm uses look up tables in order to relate the shape parameter $\mu$ to differential reflectivity and the 'normalized' intercept parameter $N_w$ to horizontal reflectivity and $\mu$. Coincident ray plots of *XPOL*/*S-POL* and time series comparisons with a 2D-video disdrometer showed good agreement between retrieved *DSD* parameters from radar observations and disdrometer spectra derived from S-POL and measured *in situ* by the disdrometer.

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