P12R.2 ANTENNA SYSTEM REQUIREMENT FOR DUAL POLARIZATION RADAR DESIGN IN HYBRID MODE OF OPERATION

Yanting Wang [1*], V. Chandrasekar [1] and D.J. McLaughlin [2]

[1] Colorado State University
1373 Campus Delivery
Fort Collins, CO 80523, USA

[2] University of Massachusetts
College of Engineering
Amherst, MA 01003, USA

1. INTRODUCTION

Dual-polarization radar measurements are usually acquired in a pair of orthogonal polarization states. Currently, two operation modes are usually implemented, namely the alternate mode where the transmit polarization states are switched alternately, or hybrid mode where both polarization states are transmitted and received simultaneously. Alternating transmission mode of operation can measure the full covariance matrix when radar is configured with two receivers, whereas the hybrid mode can only measure the co-polar subset of the covariance matrix. The co-polar radar parameters have extensive applications in radar meteorology (see Bringi and Chandrasekar, 2001 for a summary of references).

For dual-polarization radar design, the hybrid mode of operation is becoming popular due to system simplicity compared to the polarization switching methodology. In the linear horizontal / vertical (H/V) polarization basis, the alternate mode operates at the eigen polarization states of precipitation whereas the hybrid mode does not. Doviak et al. (2000) presented a detailed comparison between these two polarization modes. The received samples in the hybrid mode at the two polarization states (H and V) are not intrinsic co-polar responses, and because of cross polarization coupling, this may lead to retrieval errors when radar measurements are used to estimate the co-polar radar parameters.

The cross polarization coupling can be induced by the precipitation medium, or the radar systems, or both. Wang and Chandrasekar (2005) characterized the hybrid mode measurements using the covariance matrix formulation and evaluated the accuracy over the full space of intrinsic radar parameters. The cross polarization coupling due to the precipitation medium is usually low in the linear H/V basis (as indicated by low linear depolarization ratio, LDR), and the perturbations in estimating the co-polar radar parameters are mostly negligible. However, the impact of the polarization errors of the antenna may be important. As a special case, Doviak et al. (2000) showed the perturbations in hybrid mode related to angular alignment of the antenna feed, which affects the measurements as if the hydrometeors are canted along the propagation path. Wang and Chandrasekar (2005) further illustrated the retrieval errors due to antenna polarization errors, with which the cross polarization coupling can continuously increase. In this paper, the measurement accuracy due to antenna polarization limitations is quantitatively evaluated. The bounds of retrieval biases are analyzed and compared between the alternate mode and the hybrid mode. Based on the typical dual-polarization parameters of different precipitation types, the evaluations are further converted to the specification of system requirements.

2. DUAL POLARIZATION OBSERVATIONS OF PRECIPITATION

In the linear horizontal/vertical (H/V) polarization basis, the polarimetric scattering properties of hydrometeors can be described by the backscattering matrix **S**, as

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \quad (1)$$

where the first subscript denotes the receiving polarization state while the second subscript denotes the transmitting polarization state. In the back scatter alignment (BSA) convention, the matrix **S** is symmetric for monostatic scattering of reciprocal targets (van Zyl and Ulaby 1991) and a reduced set of three-element "feature" vector

$$\Omega = \begin{bmatrix} S_{hh} & \sqrt{2}S_{hv} & S_{vv} \end{bmatrix}^t \quad (2)$$

can be introduced to describe the polarization response (Tragl 1990). In the alternating H/V polarization mode, both co-polar and cross-polar components can be

* *Corresponding author address*: ytwang@engr.colostate.edu.

measured explicitly and the radar variables are defined as the outer product of $\mathbf{\Omega}$ (commonly termed as covariance matrix) (Bringi and Chandrasekar 2001). In the hybrid mode, however, the polarization response to simultaneous H/V transmission of equal power can be described by a two-element vector

$$\tilde{\Omega} = [S_{hh} + S_{hv} \quad S_{vv} + S_{hv}]^t \qquad (3)$$

where and hereafter the superscript '~' is used to differentiate the variables in the hybrid mode from those in the alternate mode. The radar measurables, still defined as the outer product of $\tilde{\Omega}$, are neither co-polar nor cross-polar parameters in the presence of cross polarization coupling.

Typically, a train of pulses are transmitted and the covariance of the received signal vector is estimated to retrieve the properties of precipitation media. Fig.1 sketches the common pulsing scheme for both the polarization modes in the linear H/V basis. As shown in Fig.1(a), radar reflectivities Z_h ($<|S_{hh}|^2>$) and Z_v ($<|S_{vv}|^2>$), and the differential reflectivity Z_{dr} (Z_h/Z_v), can be estimated directly on the alternate mode. To estimate the co-polar correlation R_{hv} ($<S_{hh}^* S_{vv}>$), however, special processing is necessary since the co-polar signals are not sampled simultaneously (Bringi and Chandrasekar 2001; Liu et al. 1994). With two receivers, the cross-polar returns can be measured, hence the cross-polar reflectivity Z_x ($<|S_{vh}|^2>$), the linear depolarization ratio (LDR), the cross-polar correlations R_{xh} ($<S_{vh}^* S_{hh}>$), and R_{xv} ($<S_{hv}^* S_{vv}>$) can be measured. On the contrary, it can be seen from Fig.1(b) that in the hybrid mode there is cross talk coupling, between the co-polar and cross-polar returns at both H and V polarization states. In this mode, the radar measurables can be expressed as linear combinations of the intrinsic radar variables. The observed radar reflectivity in the H channel ($\tilde{Z}_h$) and that in the V channel ($\tilde{Z}_v$) can be respectively written as (Wang and Chandrasekar 2005)

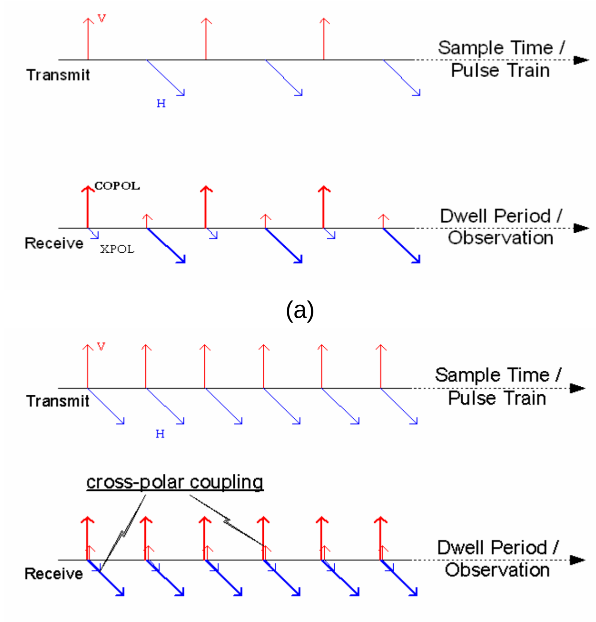
$$\tilde{Z}_h = Z_h + Z_x + 2\Re(R_{xh}) \qquad (4)$$

$$\tilde{Z}_v = Z_v + Z_x + 2\Re(R_{xv}) \qquad (5)$$

The ratio $\tilde{Z}_h / \tilde{Z}_v$ gives the differential reflectivity ($\tilde{Z}_{dr}$) between these two orthogonal channels. In addition, the correlation between H and V channels ($\tilde{R}_{hv}$) can be expressed as

$$\tilde{R}_{hv} = R_{hv} + Z_x + R_{xh}^* + R_{xv} \qquad (6)$$

The correlation coefficient is then computed by definition: $\tilde{\rho}_{hv} e^{j\tilde{\Psi}_{dp}} \equiv \tilde{R}_{hv} / \sqrt{\tilde{Z}_h \tilde{Z}_v}$.



(b)

Figure 1: Diagrams of pulsing scheme and received samples at a fixed range in the linear H/V polarization basis, on (a) alternate mode and (b) hybrid mode.

It is evident from (4)-(6) that inherent errors exist when the radar measurables in the hybrid mode are used to retrieve the co-polar radar parameters. The retrieval errors are defined respectively for the radar reflectivity, the differential radar reflectivity, the co-polar correlation coefficient, and the differential propagation phase, as follows:

$$\Delta Z_h \equiv 10\log_{10}(\tilde{Z}_h / Z_h) \qquad (7a)$$

$$\Delta Z_{dr} \equiv 10\log_{10}(\tilde{Z}_{dr} / Z_{dr}) \qquad (7b)$$

$$\Delta\rho_{hv} \equiv \tilde{\rho}_{hv} / \rho_{hv} - 1 \qquad (7c)$$

$$\Delta\Psi_{hv} \equiv \tilde{\Psi}_{dp} - \Psi_{dp} \qquad (7d)$$

where Z_h and Z_{dr} are given in linear scale, and $\rho_{hv} e^{j\Psi_{dp}} \equiv R_{hv} / \sqrt{Z_h Z_v}$. Note that the slope of Ψ_{dp} relative to range (r), or the specific differential phase K_{dp}, is of particular interest. The specific differential phase in the hybrid mode can be further evaluated as

$$\tilde{K}_{dp} = \frac{d\Psi_{dp}}{dr} + \frac{d\Delta\Psi_{dp}}{d\Psi_{dp}} \frac{d\Psi_{dp}}{dr} = K_{dp}\left(1 + \frac{d\Delta\Psi_{dp}}{d\Psi_{dp}}\right) \qquad (8)$$

It shows that the slope of (7d) with respect to Ψ_{dp} can be defined as the relative bias in K_{dp} (ΔK_{dp}). Therefore, ΔK_{dp} will be studied on the following instead of $\Delta\Psi_{dp}$.

When the H and V polarizations are actually close to the eigen polarization states of precipitation media, both the cross-polar correlations (R_{xh} and R_{xv}) are negligible (Nghiem et al. 1992). The retrieval errors in the hybrid mode are shown negligible for all typical precipitations (Wang and Chandrasekar 2005).

3. IMPACT OF ANTENNA POLARIZATION ERRORS ON POLARIMETRIC OBSERVATION

3.1 *Hard Targets*

The dual-polarization performance of a radar system can be described by the error matrix $\boldsymbol{\chi}$ (Bringi and Chandrasekar 2001):

$$\boldsymbol{\chi} = \begin{bmatrix} i_h & e_v \\ e_h & i_v \end{bmatrix} \tag{9}$$

where $|i_h|^2 + |e_h|^2 = |i_v|^2 + |e_v|^2 = 1$. The system polarization errors can distort the polarization states on both transmission and receiving. Accordingly, the retrievals of radar parameters will be biased.

Under polarization errors $\boldsymbol{\chi}$, the backscattering matrix is modified as

$$\mathbf{S}' = \boldsymbol{\chi}^t \mathbf{S} \boldsymbol{\chi} \tag{10}$$

where the superscript '/' is used to differentiate the measurements contaminated with polarization errors from those by ideal systems. Expanding this modified backscattering matrix leads to a complicated system with cross polarization coupling due to both backscatter media and system errors. However, it is known that $|S_{hv}|^2 << |S_{hh}|^2$ or $|S_{vv}|^2$ for most precipitation types, and $|e_h|^2 << |i_h|^2$; $|e_v|^2 << |i_v|^2$ for reasonably good radar systems. Therefore the expansion of (10) can be simplified by discarding the higher order cross polarization coupling terms. Eliminating the terms above the 2nd order, the co-polar radar observables in the alternate mode can be determined as,

$$Z_h^/ \approx |i_h|^4 Z_h + 2\Re\{i_h^{*2} e_h^2 R_{hv}\} \tag{11a}$$

$$Z_v^/ \approx |i_v|^4 Z_v + 2\Re\{i_v^2 e_v^{*2} R_{hv}\} \tag{11b}$$

$$R_{hv}^/ \approx i_h^{*2} i_v^2 R_{hv} + i_h^{*2} e_v^2 Z_h + i_v^2 e_h^{*2} Z_v \tag{11c}$$

Similarly, retaining only the polarization errors up to the 1st order, the polarimetric observables in the hybrid mode can also be determined, and the expressions are not presented for brievity.

For given polarization errors e_h and e_v, it can be shown that the hybrid mode measurements are affected by first order errors whereas the alternate mode measurements are affected by second order errors. Therefore, for co-polar parameter retrievals, the hybrid mode of operation is more sensitive to the system polarization errors than the alternate mode of operation.

3.2 *Distributed targets*

For distributed targets such as precipitation media, the beam-filling effect by antenna patterns should be taken into account to describe the radar measurements. The "modified" backscattering response with non-ideal antenna polarization characteristics can be explicitly expressed as a convolution equation,

$$\mathbf{S}^/(\theta,\phi,r) = \iint \boldsymbol{\chi}^t(\theta^/ - \theta, \phi^/ - \phi)\mathbf{S}(\theta^/, \phi^/, r)\boldsymbol{\chi}(\theta^/ - \theta, \phi^/ - \phi) d\Omega^/ \tag{12}$$

where θ and ϕ are respectively referred to the elevation angle and azimuth angle of the antenna on-axis beam. Expanding (12) and applying the incoherent property of meteorological scattering, the radar parameters for distributed targets can be derived.

For simplicity, precipitation is usually regarded as homogeneous within resolution volume. Then, the co-polar radar measuables in the alternate mode can be written as,

$$Z_h^/ \approx Z_h \iint |i_h|^4 \, d\Omega + 2\Re\{R_{hv} \iint i_h^{*2} e_h^2 d\Omega\} \tag{13a}$$

$$Z_v^/ \approx Z_v \iint |i_v|^4 \, d\Omega + 2\Re\{R_{hv} \iint i_v^{*2} e_v^2 d\Omega\} \tag{13b}$$

$$R_{hv}^/ \approx R_{hv} \iint i_h^{*2} i_v^2 d\Omega + Z_h \iint i_h^{*2} e_v^2 d\Omega + Z_v \iint i_v^2 e_h^{*2} d\Omega \tag{13c}$$

and the radar measurables in the hybrid mode can be written as,

$$\begin{aligned} \tilde{Z}_h^/ &\approx Z_h \iint |i_h|^4 \, d\Omega + Z_x \iint |i_h i_v|^2 \, d\Omega \\ &+ 2Z_h \Re\{\iint |i_h|^2 \, i_h e_v^* d\Omega\} + 2\Re\{R_{hv} \iint i_h^{*2} i_v e_h d\Omega\} \end{aligned} \tag{14a}$$

$$\begin{aligned} \tilde{Z}_v^/ &\approx Z_v \iint |i_v|^4 \, d\Omega + Z_x \iint |i_h i_v|^2 \, d\Omega \\ &+ 2Z_v \Re\{\iint |i_v|^2 \, i_v e_h^* d\Omega\} + 2\Re\{R_{hv} \iint i_v^2 i_h^* e_v^* d\Omega\} \end{aligned} \tag{14b}$$

$$\begin{aligned} \tilde{R}_{hv}^/ &\approx R_{hv} \iint i_h^{*2} i_v^2 \left(1 + \frac{e_h}{i_v} + \frac{e_v^*}{i_h^*}\right) d\Omega + Z_x \iint |i_h i_v|^2 \, d\Omega \\ &+ Z_h \iint |i_h|^2 \, i_h^* e_v d\Omega + Z_v \iint |i_v|^2 \, i_v e_h^* d\Omega \end{aligned} \tag{14c}$$

Note that the retrieval biases come from various "integrated" error terms, coupling with co-polar patterns, over the radar beam.

4. SYSTEM ISOLATION REQUIREMENTS

The difference between the pair of co-polar channels is basically a calibration issue, while only the cross polarization ratios are of our concern in this paper. For hard targets, a pair of complex polarization ratios can be defined to facilitate our study of antenna polarization performance (Hubbert and Bringi 2003),

$$\chi_h e^{j\phi_h} \equiv e_h / i_h \quad (15a)$$

$$\chi_v e^{j\phi_v} \equiv e_v / i_v \quad (15b)$$

Substituting (15) into (9), the polarization errors $\boldsymbol{\chi}$ can be expressed as follows,

$$\boldsymbol{\chi} = \begin{bmatrix} 1 & \chi_v e^{j\phi_v} \\ \chi_h e^{j\phi_h} & 1 \end{bmatrix} \begin{bmatrix} i_h & \\ & i_v \end{bmatrix} \quad (16)$$

The second matrix on the right hand side can be viewed as calibration constants and hence can be dropped without any loss of generality.

The evaluation parameters for the system polarization performance are different in the context of distributed targets. Typically, the co-polar characteristics are well matched (i.e. $i_h \approx i_v$), at least in the mainlobe. Then, in the alternate mode, the integrated cross polarization ratios can be defined as follows,

$$\chi_h^2 e^{j2\phi_h} \equiv \frac{\iint i_h^{*2} e_h^2 d\Omega}{\iint |i_h|^4 \, d\Omega} \quad (17a)$$

$$\chi_v^2 e^{j2\phi_v} \equiv \frac{\iint i_v^{*2} e_v^2 d\Omega}{\iint |i_v|^4 \, d\Omega} \quad (17b)$$

Similarly, in the hybrid mode, the integrated cross polarization ratios can be defined as,

$$\chi_h e^{j\phi_h} \equiv \frac{\iint i_h^{*2} i_v e_h d\Omega}{\iint |i_h|^4 \, d\Omega} \quad (18a)$$

$$\chi_v e^{j\phi_v} \equiv \frac{\iint i_v^{*2} i_h e_v d\Omega}{\iint |i_v|^4 \, d\Omega} \quad (18b)$$

With these definitions, the measurements can be approximately expressed by same set of parameters (χ_h, χ_v), for both hard target and distributed targets.

The retrieval errors in both polarization modes vary with the differential propagation phase. Accordingly, the bias profiles can vary widely. Therefore, to predict the envelopes of the retrieval biases will be more important in radar design. In addition, the envelopes of retrieval biases also depend on the specific values of ϕ_h and ϕ_v, since the error terms can accordingly sum up constructively or destructively. In the paper, the worst cases over the phase space {Ψ_{dp}, ϕ_h, ϕ_v} will be considered to draw system specification on polarization isolations.

There is no apparent reason that χ_h and χ_v could differ from each other significantly. The cross polarization level (CPL) can be defined as

$$CPL \equiv 20\log_{10}(\chi_h) \approx 20\log_{10}(\chi_v) \quad (19)$$

Considering that different levels of cross polarization coupling can occur due to backscattering of precipitations, several typical hydrometeor types are selected to represent for the precipitation medium, including heavy rain, light rain, hail and bright band. Their intrinsic polarimetric parameters are listed in Table.1. Fig.2 and Fig.3 respectively show the bias profiles at different isolation levels for the alternate mode and for the hybrid mode. The solid lines present the upper bound while the dash lines present the lower bound; and different markers label the different precipitation types for comparison.

Table 1: Typical polarimetric signatures for different precipitation types

Precipitation Types	Heavy Rain	Light Rain	Hail	Bright Band
Z_{dr} (dB)	3	1	0	3
LDR (dB)	-25	-32	-20	-17
ρ_{hv}	0.98	0.98	0.90	0.80

It is clearly shown that the accuracy of polarimetric measurements in the hybrid mode is much more sensitive to the cross polarization isolation, by comparing Fig.2 and Fig.3. Roughly the perturbation due to poor isolation level is one order larger in the hybrid mode than in the alternate mode (note that different scales are used in these two figures). All the biases are dominated by the cross polarization isolation. For different precipitation types, even though the cross polarization coupling of scattered returns varies, the retrieval bias bounds do not differ significantly except that of co-polar correlation coefficient. This is expected since, without the system artifacts, the retrieval errors are indeed rather small as H/V polarizations are close to the eigen polarization states of precipitations.

As an example, the supposed accuracies on Z_h, Z_{dr} and K_{dp} in this paper are chosen by simply considering the measurement accuracy (Bringi and Chandrasekar 2001) and they are summarized in Table.2. Also shown in the table are the corresponding requirement specification of the antenna polarization isolation, converted from the system bounds in Fig.2 and Fig.3. An isolation performance better than -20 dB can satisfy the supposed constraints in the alternate mode, but it needs to be improved to -30 dB or better in the hybrid mode, even to -40 dB when Z_{dr} is considered.

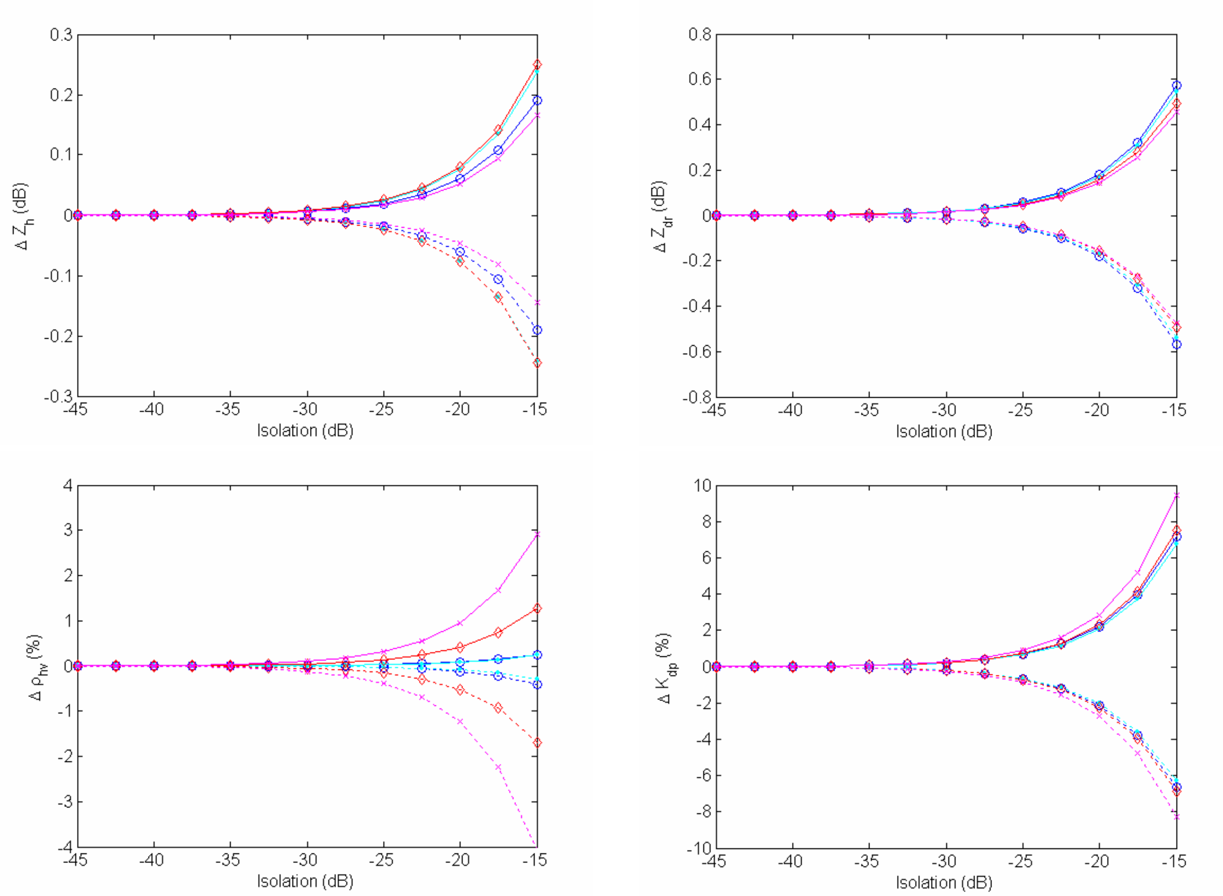


Figure 2: The bound profiles of susceptible polarimetric biases in the alternate mode with respect to isolation levels for different precipitation types: heavy rain ('o'), light rain ('•'), hail ('◊'), and bright band ('×'). The solid lines show the upper bound and the dash lines show the corresponding lower bound.

Table 2: The requirement specifications on *CPL* at given measurement accuracies.

Supposed Constraints	Requirements in alternate mode	Requirements in hybrid mode
$\lvert \Delta\rho_{hv} \rvert < 10\%$	-	*CPL* < -30 dB
$\lvert \Delta Z_h \rvert < 0.8$ dB	-	*CPL* < -27 dB
$\lvert \Delta Z_{dr} \rvert < 0.2$ dB	*CPL* < -20 dB	*CPL* < -40 dB
$\lvert \Delta K_{dp} \rvert < 10\%$	-	*CPL* < -28 dB

5. SUMMARY

Co-polar radar parameters can be acquired with the hybrid mode of operation, however, cross polarization coupling can perturb their retrievals from radar measurements. The cross polarization coupling can be induced by the backscattering of precipitation media, or by the antenna polarization errors, or both. On the backscattering side, the eigen polarization states of the precipitation medium are mostly close to H and V polarizations, and the associated cross polarization coupling has negligible effects on the measurement accuracy.

The impact of system polarization errors on polarimetric observations of precipitation was analyzed. Both alternate mode and hybrid mode of operation can be perturbed by the system polarization errors, however, the perturbations can be more outstanding in the hybrid mode. In the alternate mode, the contamination due to cross polarization errors is coupled to co-polar observations through their second order and higher, whereas the contamination is coupled through their first order and higher in the hybrid mode.

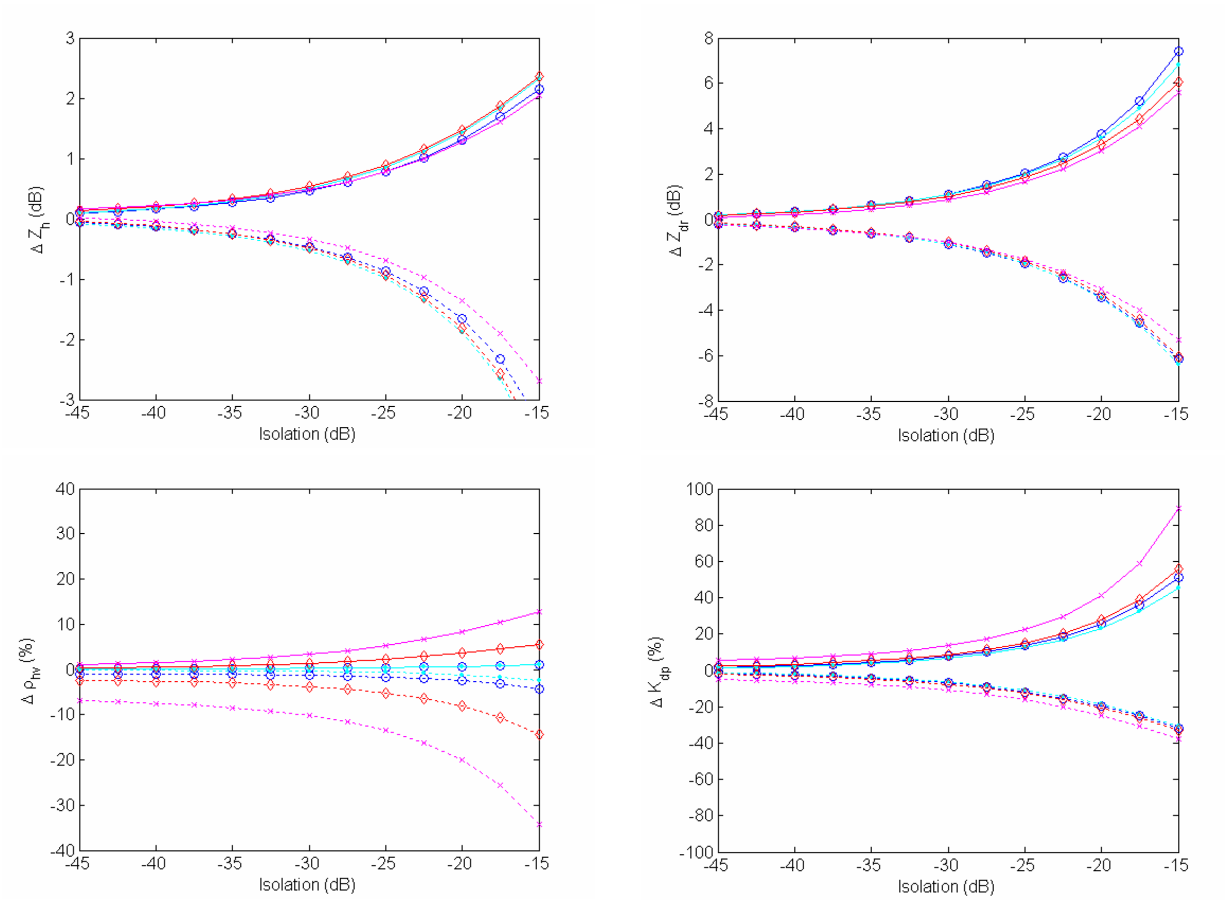


Figure 3: Same as Fig.4 except showing the error profiles in the hybrid mode of operation.

The worst cases are considered to determine the system bounds of retrieval biases. The system bounds are further used to derive the system requirements of cross polarization isolation for the hybrid mode of operation. Considering the antenna patterns, the system polarization performance should be evaluated using specific form in each polarization mode. The cross polarization level is mostly demanded by cross-polar radar parameters in the alternate mode (Bringi et al. 2002). In the hybrid mode, even though only co-polar radar parameters are available, however, the cross polarization level should not be compromised.

Acknowledgement This research is supported by the National Science Foundation via ATM-0313881 and the NSF/Engineering Research Center Program ERC-0313747.

References

Bringi, V., Y. Wang, and J. Hubbert, 2002: Polarimetric data evaluation in severe storms using two research dual-polarized radars. *Proceedings of 2nd European Conference on Radar Meteorology*, 73–77.

Bringi, V. N. and V. Chandrasekar, 2001: *Polarimetric Doppler Weather Radar: Principles and Applications*. Cambridge.

Doviak, R., V. Bringi, A. Ryzhkov, A. Zahrai, and D. Zrnic, 2000: Considerations for polarimetric upgrades to operational wsr-88d radars. *J. Atmos. Oceanic Technol.*, **17**, 257–278.

Hubbert, J. and V. Bringi, 2003: Studies of the polarimetric covariance matrix. part ii: Modeling and polarization errors. *J. Atmos. Oceanic Technol.*, **20**, 1011–1022.

Liu, L., V. Bringi, and V. Chandrasekar, 1994: Analysis of the copolar correlation coefficient between horizontal and vertical polarizations. *J. Atmos. Oceanic Technol.*, **11**, 950–963.

Nghiem, S., S. Yueh, R. Kwok, and F. Li, 1992: Symmetry properties in polarimetric remote sensing. *Radio Sci.*, **27**, 693–711.

Tragl, K., 1990: Polarimetric radar backscattering from reciprocal random targets. *IEEE Trans. Geosci. Remote Sensing*, **28**, 856–864.

Van Zyl, J and F.T. Ulaby, 1991: Scattering matrix representation for simple targets, *Radar Polarimetry for Geoscience Applications*, F.T. Ulaby and C. Elachi, Ed. Norwood, MA: Artech House, 17-50.

Wang, Y. and V. Chandrasekar, 2005: Performance of dual-polarization radar measurement in hybrid mode for Precipitation, *Proceedings of Intl. Geoscience and Remote Sensing Symposium*, IEEE, 5084–5085.