P13R.2

Z TO R RELATIONSHIPS FOR SNOWFALL-EVENTS IN AUSTRIA, DETERMINED BY MODELLING WINTERLY PRECIPITATION PARTICLES REALISTICALLY

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1. INTRODUCTION

Radar measurements of precipitation critically depend on the relationship between radar reflectivity factor Z and precipitation rate R. The relationship has the general form $Z = a \cdot R^b$, where a and b are empirical parameters. A great many of such relationships with varying parameters exist for different precipitation types and different regions (Gray & Hale, 1981). In the present study the Z-R-relationships of three solid precipitation events in the mountains of Austria – each with a prevalent type of precipitation particles (dry snowflakes, ice pellets, rimed graupel) – were determined by realistically modelling a variety of different single precipitation particles and rigorously calculating their radar cross sections at 5.6 GHz frequency.

2. AVAILABLE DATA

Precipitation particles occurring in the three events were recorded by means of the 2D-Video-Distrometer (2DVD), a ground based imaging precipitation gauge. The 2DVD provides time, size, outline shape, and falling velocity information of every single precipitation particle (http://www.distrometer.at). An imaging distrometer as the 2DVD provides only sparse information on the inner structure of winterly precipitation particles and, as a consequence, on the precipitation rate. Therefore fall speed information and a co-sited heated tipping bucked rain gauge were used to determine the equivalent liquid precipitation rate from data provided by the 2DVD. Table 1 lists the details of the observed precipitation events.

Table 1: Studied precipitation events

Dry snowflakes	Site: Mt. Präbichl, Austria Date: 15/12/2000 Time: 3:45 - 4:15 a.m. Average temp.: –0.7 °C # recorded particles: 2.764
Ice pellets	Site: Mt. Präbichl, Austria Date: 26/01/2001 Time: 11:00 - 12:00 a.m. Average temp.: –1.3 °C # recorded particles: 3.088
Rimed graupel	Site: Mt. Präbichl, Austria Date: 26/01/2001 Time: 2:30 - 3:00 p.m. Average temp.: –1.5 °C # recorded particles: 27.115

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3. METHODOLOGY

For the reflectivity determination first the number of particles per cubic meter air volume was extracted from the 2DVD data. The whole spectrum of recorded particles was discretised in up to ten particle-size-classes with corresponding quantity. Figures 1 - 3 show these spectra for the three precipitation events averaged over the whole duration of the event.

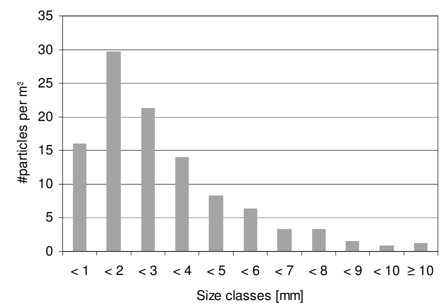

Figure 1: Number of snowflakes per m^3. (Broken down into size classes.)

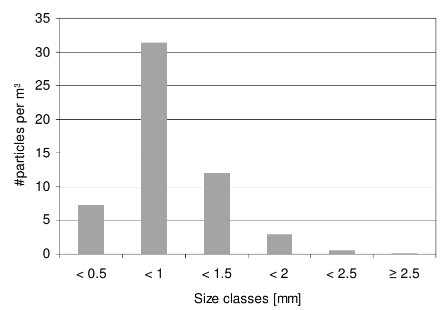

Figure 2: Number of ice pellets particles per m^3.

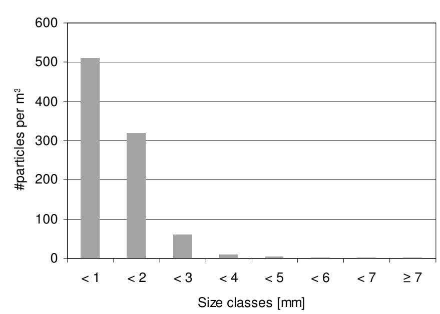

Figure 3: Number of rimed graupel particles per m^3.

Generally the particle distribution spectrum changes within an event. The duration of each precipitation event was therefore divided into six time intervals. For each interval the averaged particle distribution spectrum was calculated.

For each size class and each event the radar cross section (RCS) of a "characteristic" particle was determined. Together with the quantity information of the particles of each size class within a unit volume, the reflectivity factor Z was calculated. Consequently a value for the reflectivity factor of each of the six time intervals of the precipitation event was obtained. For each time interval also the average precipitation rate R was determined from the 2DVD and the heated rain gauge data. The Z and the corresponding R values of each event were the inputs of consequent regression analyses of the type $Z = a \cdot R^b$.

4. CHOICE OF A "CHARACTERISTIC" PARTICLE

The "characteristic" particle of each size class was chosen due to following consideration: For spherical particles the RCS and the reflectivity factor are proportional to the sixth power of the particle's diameter for particles small compared to the wavelength (Rayleigh scattering region). Therefore e.g. if, for a certain size-class, a particle is modelled whose diameter exactly equals the average diameter of all the n particles in this class, its reflectivity would not represent the average. A more representative value can be achieved if the diameter equals D_{AR}.

$$D_{AR} = \sqrt[6]{\frac{1}{n} \cdot \sum_{i=1}^{n} D_i^6} \tag{1}$$

Where n is the number of particles of one size-class and D_i are the diameters of the precipitation particles of one size-class.

This diameter is henceforth called average reflectivity diameter (D_{AR}). Since a particle of this diameter represents the average reflectivity of all n particles. In other words: the reflectivity of a volume that contains n differently sized particles can be approximated by a volume that contains n notional particles with the average reflectivity diameter D_{AR}.

Below it is shown that this idea is not only valid for spherical particles but also for arbitrarily shaped hydrometeors, e.g. for modelled snowflakes from 2 to 4 mm in diameter. It is shown what reflectivity five differently sized snowflakes alone and together have. Further is calculated what reflectivity results from five equally sized snowflakes whose diameter is the average reflectivity diameter D_{AR}.

Table 2: RCS and Z for modelled snowflakes at 5.6 GHz

Diameter [mm]	RCS [m^2]	RCS [dBm^2]	Z [mm^6 m^{-3}]	Z [dBZ]
2.0	7.34E-11	-101.34	11.62	10.65
2.5	2.51E-10	-96.00	39.78	16.00
3.0	8.69E-10	-90.61	137.57	21.39
3.5	2.34E-09	-86.31	370.43	25.69
4.0	4.38E-09	-83.59	693.37	28.41
		Total:	1252.76	30.98
D_{AR}: 3.342	1.68E-09	-87.74	266.27	24.25
		5 x:	1331.33	31.24

The assumption of n "characteristic" particles with an average reflectivity diameter seems to be an acceptable method to represent the reflectivity of n differently sized particles if the size difference is not substantial.

5. MODELLING OF SINGLE PRECIPITATION PARTICLES

For each precipitation type, each size class, and for each of the six time intervals a "characteristic" precipitation particle was chosen that approximately has the average reflectivity diameter of all particles of that size class. With the 2DVD data of this particle it was as good as possible modelled. For spherical particles (e.g. graupel) the RCS was calculated with the exact Mie-solution, Mie (1908), while for particles of more complex shape (e.g. dry snowflakes) it was determined my means of the Finite Element Method. In both cases the material was assumed to be inhomogeneous (ice with air-locks) and its permittivity was calculated with the mixing-formula of Maxwell-Garnett (1904). The volume fraction of the inclusions was determined by comparison of the volume that is determined by the 2DVD-data and by the water-volume measured by a co-sited heated tipping bucked rain gauge.

6. RESULTS

For the three precipitation events Z-R relationships were determined by means of regression analyses. For the relationships of the form $Z = a \cdot R^b$ either the reflectivity factor Z or the equivalent reflectivity factor Z_e are frequently used.

The radar reflectivity factor is defined as

$$Z = \frac{\lambda^4}{\pi^5 \cdot |K_0|^2} \cdot \sum_{i=1}^{N} \sigma_i \tag{2}$$

where

σ_i RCS of single particle

λ wavelength

$|K_0|^2$ dielectric factor 0.17 for ice, 0.93 for water

The summation takes place over a unit volume of size 1 m³.

The conventional unit of Z is in $mm^6\ m^{-3}$.

The equivalent reflectivity factor Z_e may be estimated from measurements of the radar reflectivity η of precipitation and is defined by

$$Z_e = \frac{\lambda^4}{\pi^5 \cdot 0.93} \cdot \eta \qquad (3)$$

where

$$\eta = \sum_{i=1}^{N} \sigma_i \qquad (4)$$

In Table 3 the Z-R relationships for the three precipitation events are listed for the reflectivity factor Z and the equivalent reflectivity factor Z_e.

Table 3: Z-R relationships for different precipitation types.

Precipitation type	$Z_e = a \cdot R^b$	$Z = a \cdot R^b$
Snowflakes	$Z_e = 2009\ R^{1.73}$	$Z = 10987\ R^{1.73}$
Ice pellets	$Z_e = 229\ R^{1.44}$	$Z = 1250\ R^{1.44}$
Rimed graupel	$Z_e = 1060\ R^{1.57}$	$Z = 5798\ R^{1.57}$

Figures 4 - 6 plot the Z-R curves for the equivalent reflectivity factor Z_e and show the Z-R values that lead to these curves.

The analyses show that even in a single winterly precipitation event the Z-R relationship can change considerably. Therefore for the different precipitation events a wide spectrum of possible relations around those determined is thinkable. Nevertheless the Z-R relationships of the three different types of precipitation occupy totally different ranges. The relation for dry snowflakes corresponds remarkably with that of Gunn and Marshall (1958) $Z = 2000\ R^2$.

Not all Z-R relationships for frozen particles that can be found in the literature can be compared one-to-one. Often the radar reflectivity factor for frozen particles is estimated by

$$Z = \sum_{i=1}^{N} D_i^6 \qquad (5)$$

Where D_i is given by the water droplet that would result if the ice particle were to melt completely. In this study however the reflectivity was determined by calculating the RCSs of the observed particles.

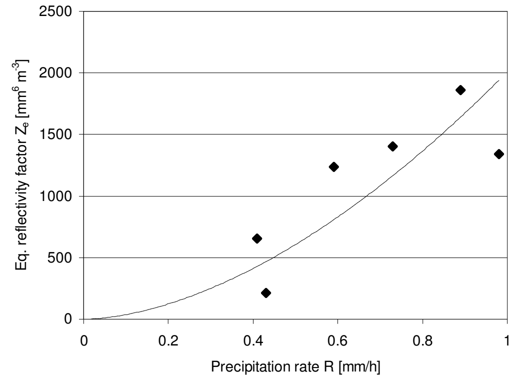

Figure 4: Z-R-relationship determined for dry snowflakes, $Z_e = 2009\ R^{1.73}$.

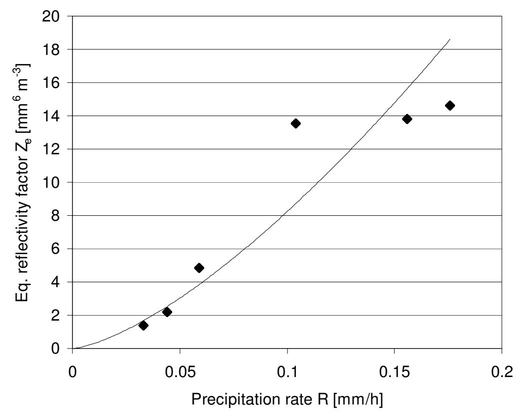

Figure 5: Determined Z-R-relationship for ice pellets $Z_e = 229\ R^{1.44}$.

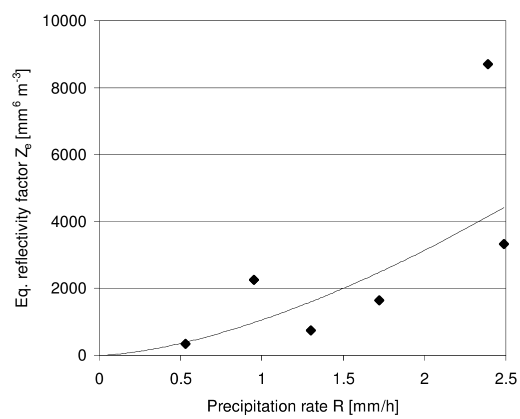

Figure 6: Z-R-relationship determined for rimed graupel, $Z_e = 1060\ R^{1.57}$.

7. EFFECT OF ERRORS OF THE MEASURED PRECIPITATION RATE

The calculation of the RCS of one particle and thus the calculated reflectivity critically depends on the measured precipitation rate by the tipping bucket rain gauge. Such rain gauges generally underestimate the actual precipitation. In this model the precipitation rate also affects the reflectivity since it determines the volume fraction of the air-locks of the notional particle material. The effects on the generated Z-R relationships are however marginal. If it is assumed that e.g. the rain gauge underestimates the precipitation rate by 24 % the reflectivity values determined for dry snowflakes would increase on the average by 36 %. Because of the exponential Z-R relationship this has no significant effect on the Z-R curve. The thus determined relations are $Z = 10875\ R^{1.73}$ and $Z_e = 1988\ R^{1.73}$.

8. REFERENCES

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