1M.6 STATISTICAL CONVERGENCE IN SIMULATED MOIST ABSOLUTELY UNSTABLE LAYERS

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1. INTRODUCTION

Deep precipitating convection (i.e., a thunderstorm) has a scale of order 1-10 km. Consequently, grid spacing of at least *O* (1 km) grid spacing is required to simulate the basic structure of deep moist convective clouds. The utility of *O* (1 km) grid spacing for this purpose has been demonstrated clearly over the past several decades, given that numerical simulations have undoubtedly enhanced our knowledge of convection and its impacts on mesoscale processes.

Relatively recently, computing resources have become available for researchers to explore the effects of even higher resolution. Studies have shown that simulated clouds become turbulent once grid spacing is of *O*(100 m) (e.g., Droegemeier et al. 1997; Petch et al. 2002; Bryan et al. 2003). The dynamical explanation for this behavior is straightforward: when several scales are permitted in the simulation, an energy cascade can occur, wherein kinetic energy is transferred from the large-eddy scale (i.e., the thunderstorm scale) to smaller scales (i.e., the sub-cloud scale).

Bryan et al. (2003; hereafter referred to as BWF) demonstrated that simulated deep moist convection could begin to have an inertial subrange when grid spacing was less than approximately 250 m. However, one problem remained in their study: the simulated squall lines were still not converged statistically when comparing results from the two highest resolution simulations (using grid spacings of 250 and 125 m). Statistical convergence is expected for turbulent flows when the Reynolds number is high (e.g., Tennekes and Lumley, 1972, p. 6). This suggests that a truly converged, resolution-independent result has not been obtained yet. An investigation using higher resolution, with grid spacing less than 100 m, is warranted.

Unfortunately, computing resources are limited. Since the BWF study, we have been able to perform a simulation of a squall line with 62.5 m grid spacing (i.e., twice the resolution of the best case in BWF). Results from this simulation will be shown later in this paper; they do not indicate convergence, either. This simulation raises concerns about the theory for convergence in precipitating turbulent flows, and about the fidelity of numerical schemes in modern cloud models.

The issue of convergence goes beyond the short-term question of whether it is possible. We feel a converged solution would be valuable to the mesoscale processes community, because it would provide guidance in (at least) two areas: 1) a converged solution would reveal the biases that might be inherent in simulations using *O*(1 km) grid spacing, which is a commonly used grid spacing at present; and 2) a converged solution would help guide the development of model configurations appropriate for *O*(1 km) grid spacing. Although these two points seem similar, they are quite separate. For the first point, the use of grid spacing greater than 1 km means that individual clouds will always be too large. In our simulations using $\Delta < 1$ km, the convective clouds tend to be 2-4 km wide. Thus, using $\Delta > 1$ km guarantees that clouds will be anomalously large in most cases. To understand the consequences of clouds being too large, we need a series of simulations with results that are independent of resolution to use as "ground truth." The second point, concerning numerical schemes, is highlighted in several recent studies (e.g., Adlerman and Droegemeier 2002; Takemi and Rotunno 2003; Bryan 2005). When conducting simulations that explore the sensitivity of convection to numerical schemes, it becomes unclear what numerical configuration is best. A converged solution could be used as a benchmark to select a preferred numerical configuration.

2. METHODOLOGY

2.1 Rationale

To explore numerical convergence for moist convection, we require a computationally tractable problem. Our recent squall line simulations have several advantages for this purpose, such as the existence of many turbulent eddies (i.e., convective cells) along the line. This is advantageous for the calculation of statistical measures, which can be analyzed for statistical convergence. However, the squall line simulation requires a mesoscale domain (100s of km across the line), and must be integrated for 2-4 simulated hours. Similarly, other problems that have useful statistical properties, such as radiative-convective equilibrium studies, also require long integration times.

We have recently developed simple yet robust techniques that allow us to explore the *turbulent transition* of thermodynamically unstable layers. The physical problem is actually analogous to the convective region of a squall line, where air saturates, becomes unstable, and overturns as deep moist convection. In this new study, we specify, as an initial condition, a statically unstable environment in hydrostatic balance. This state is perturbed by analytic temperature perturbations with a specified spectrum. In the cases studied herein, we specify "white noise" perturbations, wherein *all scales* are perturbed with equal amplitude. By perturbing the

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statically unstable layer, the entire layer overturns turbulently. This problem requires much smaller scales (only a few km need be simulated) and shorter time scales (of order 30 min); thus, the problem is computationally tractable, and can be studied with ridiculously high resolution.

2.2 Simulation design

For all simulations, we use the numerical model of Bryan and Fritsch (2002). The three-dimensional domain is a cube of 2 km on all sides. We use 6 different grid spacings (Δ) of 125, 62.5, 31.25, 15.625, 7.8125, and 3.90625 m. For all runs, the horizontal grid spacing equals the vertical grid spacing.

The domain and setup is guided by the highest resolution simulation in BWF. In their squall line simulations, a moist absolutely unstable layer (i.e., MAUL) forms above the squall line cold pool. This saturated statically unstable layer had a vertical scale of about 2 km, which motivated the scale of the domain for these new simulations. It also led us to explore the issue of turbulent transition of statically unstable layers, because this is what actually happens at the leading edge of squall lines. It is also important to note that the highest resolution in BWF is the coarsest resolution for this new study.

The initial sounding is saturated, contains 5 g kg^{-1} of liquid water, and has a constant value for moist Brunt-Vaisala frequency (N_m^2) of 1×10^{-4} s^{-2}. The only microphysical processes are condensation and evaporation of cloud water; that is, the moist thermodynamics are strictly reversible for these preliminary simulations. This simplifies the design and interpretation of the results. There is sufficient cloud water specified at all heights in the initial conditions to maintain saturation at all grid points and at all times.

The initial temperature perturbation field is generated analytically, following the technique of Bryan et al. (2005). The field is "white noise" in three-dimensions in this study, and the perturbations have maximum amplitude of ± 0.2 K. The perturbations are specified on the highest resolution grid; that is, on the domain having $512 \times 512 \times 512$ grid points. For domains with fewer grid points, this field is truncated in spectral space using a wave-cutoff filter, and then reverse-transformed to the new grid. With this technique, all simulations have identical initial perturbation fields for the scales that are resolved.

3. RESULTS

3.1 Statistical convergence

Instantaneous fields of vertical velocity exhibit the expected result. The coarsest resolution simulation has smooth, laminar cells (Fig. 1a), because only one scale can be resolved with this grid. With 4 times the resolution, the field is beginning to look turbulent (Fig. 1b). With 4 times more resolution, the field is definitely turbulent (Fig. 1c). All simulations clearly capture the correct overall physical process – i.e., overturning by upward-moving warm air and downward-moving cool air. However, the turbulent nature of the flow is not captured with Δ = 125 m.

Of course, the coarsest resolution simulation cannot be expected to produce all details of the highest resolution simulation. Thus, a fairer comparison involves filtering a high-resolution simulation onto the grid used for a coarse-resolution simulation. To this end, we use a spectral filter that produces the same spectral response as the coarse resolution runs. That is, we totally remove scales not resolvable on the grid (with a wave cutoff filter), and then gradually ramp-up the filter from the 2Δ scale to the 8Δ scale; the reason for this ramp-up is because the numerical model is diffusive at small scales, and does not affect structures with a scale greater than ~8Δ. The results of this filtering process indicate that the Δ = 125 m simulation handles some aspects well, but has a clear bias (Fig. 2a). For example, the scale of the large eddies is reproduced accurately. This is expected, however, because the size of the domain will dictate the largest eddy possible – and this scale is resolved, even with Δ = 125 m. However, the filtered high-resolution vertical velocity is much weaker in amplitude ($w_{max} \sim$ 4-6 m s^{-1}, Fig. 2a) compared to the coarse resolution simulation (w_{max} > 8 m s^{-1} in several locations, Fig. 1a).

On the other hand, the simulation using Δ = 31.3 m produces a qualitatively correct result (Fig. 2b). Specifically, the patterns of vertical velocity have many structural similarities (c.f., Figs. 1b and 2b), and the velocity magnitudes are similar.

To evaluate the results quantitatively, we compute three-dimensional spectra of the vertical velocity, and then present the results one-dimensionally by integrating around shells in three-dimensional Fourier space. The results (Fig. 3) confirm the conclusions drawn qualitatively. That is, the simulation using Δ = 125 m contains too much power at small wavenumbers – i.e., large-scale updrafts are too intense. The simulation using Δ = 62.5 m also indicates too much power at low wavenumbers. In these two coarsest simulations, the only scales that can be resolved are the largest ones; thus, they are forced to overturn the unstable layer at only these scales. Consequently, the larger updrafts are too intense. In contrast, the higher resolution simulations (with $\Delta \leq$ 31.3 m) can resolve the smaller eddies.

Furthermore, an inertial subrange is apparent for simulations using $\Delta \leq$ 31.3 m. These simulations all exhibit a $\kappa^{-5/3}$ behavior, with nearly equal amplitude. (The falloff in power at ~8Δ occurs because of numerical diffusion in the model. As discussed in BWF, the important portion of the spectra to compare is the scales greater than ~6-8Δ.)

In conclusion, an energy cascade is not being resolved in the Δ = 125 and 62.5 m simulations. BWF showed

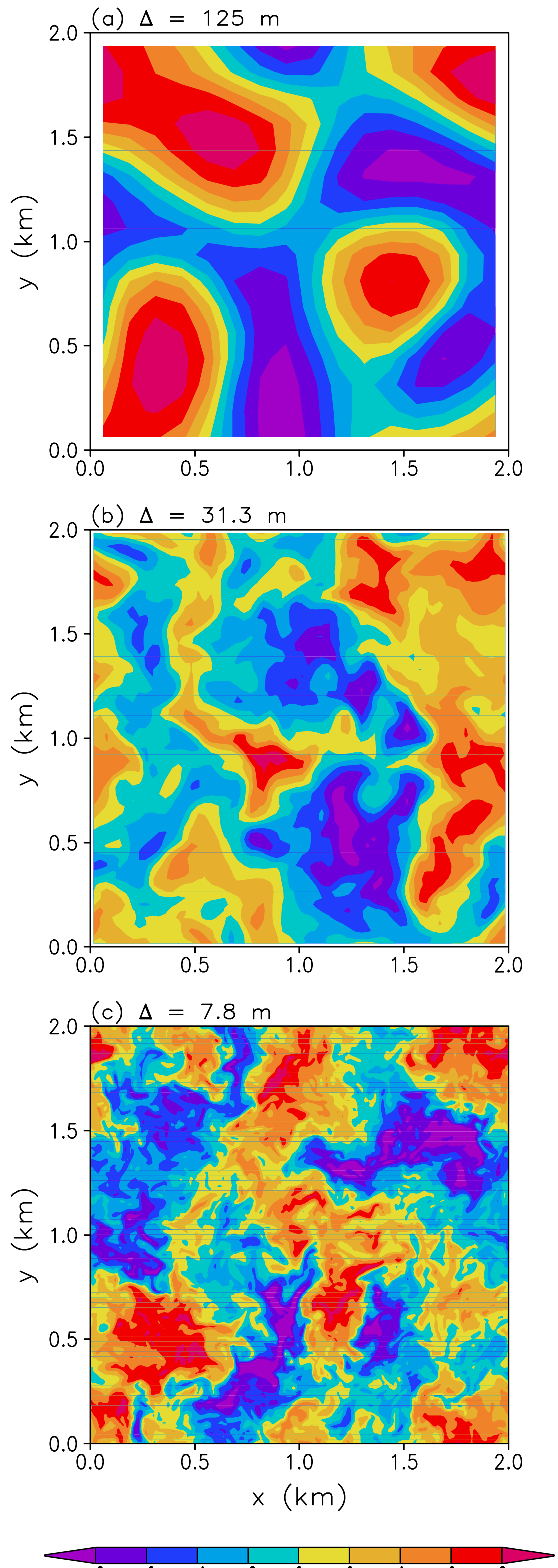

Fig. 1. Vertical velocity (m s^{-1}) at z = 1 km and t = 20 min from simulations using (a) Δ = 125 m, (b) Δ = 31.3 m, and (c) Δ = 7.8 m.

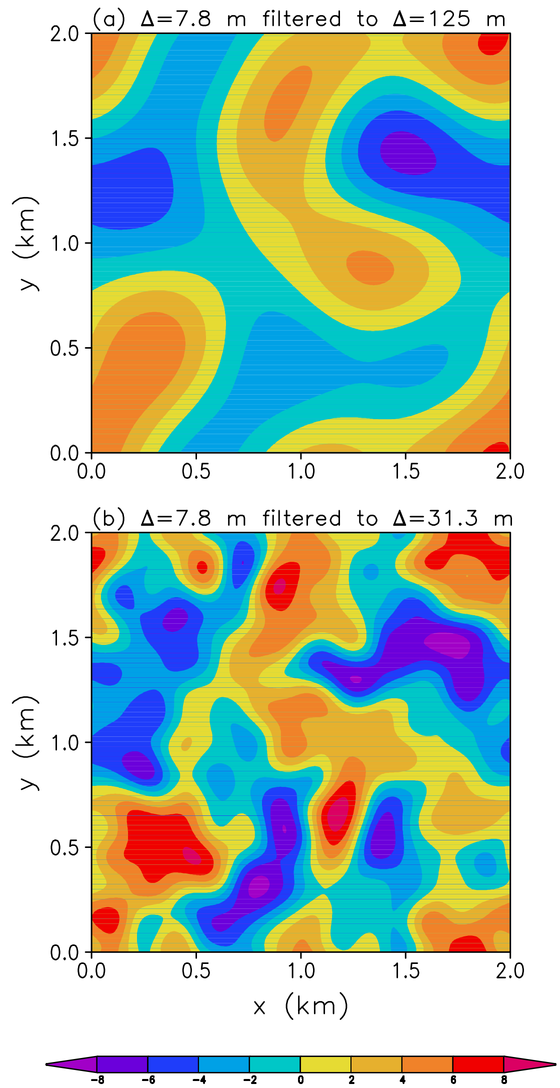

Fig. 2. The same as Fig. 1, except for the simulation using Δ = 7.8 m, and where the results have been filtered to (a) Δ = 125 m, (b) Δ = 31.3 m.

that adequate resolution for resolving an inertial subrange requires a grid spacing about two-orders of magnitude smaller than the large eddy scale. For this problem, the large eddy scale is approximately 2 km (the domain size), so, theoretically, grid spacing of O(20 m) is required to resolve an energy cascade. The results of these simulations agree nicely with the theoretical result, because the first simulation with an inertial subrange uses Δ = 31.3 m. Furthermore, the inertial subrange structure is nearly identical between all simulations using Δ ≤ 31.3 m. This means that *statistical convergence is possible*, and occurs roughly when theory says it should.

These results probably explain why we have not seen convergence in our squall line simulations. In most cases, the deep moist convection in the squall lines has

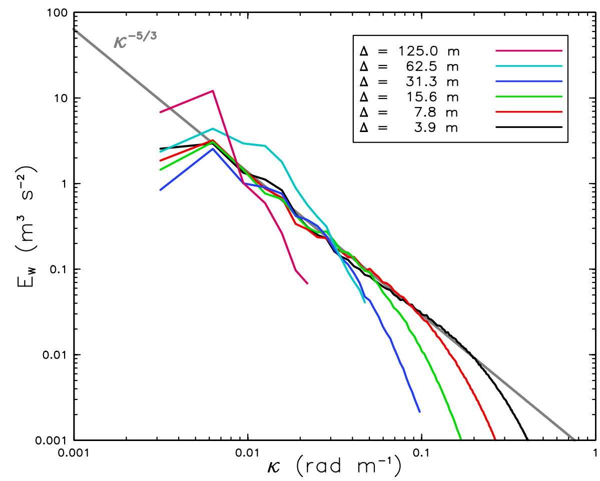

Fig. 3. Vertical velocity spectra at z = 1 km and t = 20 min from all simulations. The thick grey line is a $\kappa^{-5/3}$ spectrum.

a scale of ~3 km (e.g., Fig. 10 of BWF). We have not been able to simulate an entire squall line with grid spacing of ~30 m, which apparently would be required for convergence. Furthermore, the grid spacing has to be reduced lower than 30 m to know that convergence has been reached. For example, in these new simulations, it is not obvious that convergence has occurred unless a comparison is made to the simulations using $\Delta \leq 15.6$ m.

3.2 The consequence of inadequate resolution

These simulations reveal some of the consequences of inadequate resolution. First, as already shown, there is too much power at (relatively) large scales. This means that total vertical mass flux is greatly overestimated, because the largest eddies carry most of the flux in a turbulent flow. Thus, when the largest eddies are too intense, they carry too much total vertical flux. This effect is apparent in plots of vertical turbulent flux (e.g., Fig. 4). In this case, the simulation with Δ = 125 m overpredicts the vertical potential temperature flux by a factor of 5. As a consequence, the coarser resolution simulations *overstabilize* the layer. That is, they transport too much warm and moist air upwards, and too much cool and dry air downwards. This conclusion is verified by analyzing the final distributions of passive fluid tracers that are included in the simulations (not shown).

Thus, it seems likely that simulations with inadequate resolution produce convection that is too intense. This conclusion is supported by a series of squall line simulations using very high resolution. Specifically, we simulate a squall line in an environment without shear, and with small-amplitude perturbations in the environment [see Bryan and Rotunno (2004) for details]. Results using Δ = 250, 125, and 62.5 m confirm that convection becomes weaker as grid spacing decreases (Fig. 5). In this case, line-averaged simulated reflectivities lower from ~40 dBZ using Δ = 250 m to only ~25 dBZ using Δ= 62.5 m. Furthermore, maximum cloud tops lower

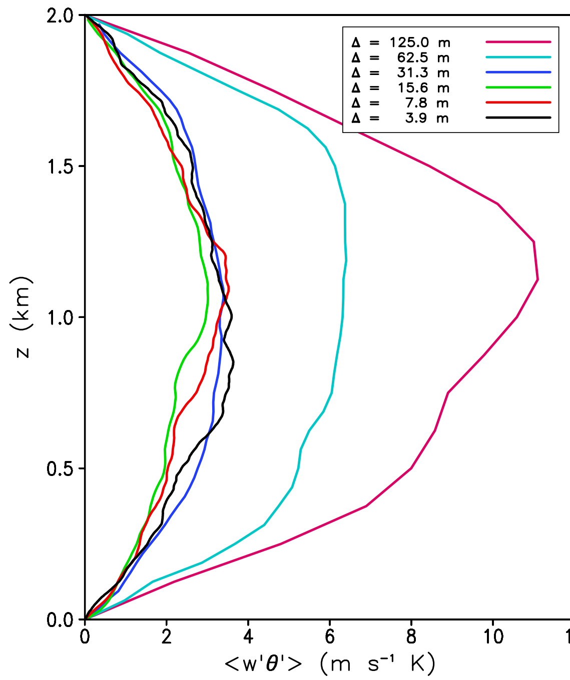

Fig. 4. Vertical profiles of vertical temperature flux from all simulations at the time of maximum flux.

From ~11 km using Δ = 250 m to ~6 km using Δ = 62.5 m. Also, total surface rainfall decreases by more than one order of magnitude from Δ = 250 to 62.5 m. This is clearly a substantial difference!

4. CONCLUSIONS

Evidence continues to mount showing that grid spacing of O(1 km) is insufficient to accurately simulate some aspects of deep moist convection. The simulated flow does not become turbulent unless Δ < 100 m (for convective clouds that are < 10 km in diameter). This means that an energy cascade is not resolved, and the simulation is forced to overturn at too large of a scale. Consequently, vertical mass fluxes are overestimated, and the simulation *overstabilizes* the atmosphere. Finally, our preliminary simulations of an entire squall line suggests that cloud tops can be systematically too high, and that precipitation can be overestimated.

The consequences for the mesoscale processes community are mixed, and depend greatly on the intended use of numerical simulations. In a broader sense, the qualitative physical response is reproduced correctly, even with very coarse resolution. For example, all of the squall line simulations produce relatively weak systems, with rain far behind the surface gust front (Fig. 5). Also, in the simple idealized simulations, warm air ascends and cool air descends in all simulations (Fig. 4). Thus, the broader questions concerning physical processes can be addressed with coarser resolution. However, projects seeking *quantitative* information from numerical simulations might need grid spacing < 100 m to obtain reliable results. For example, based on our

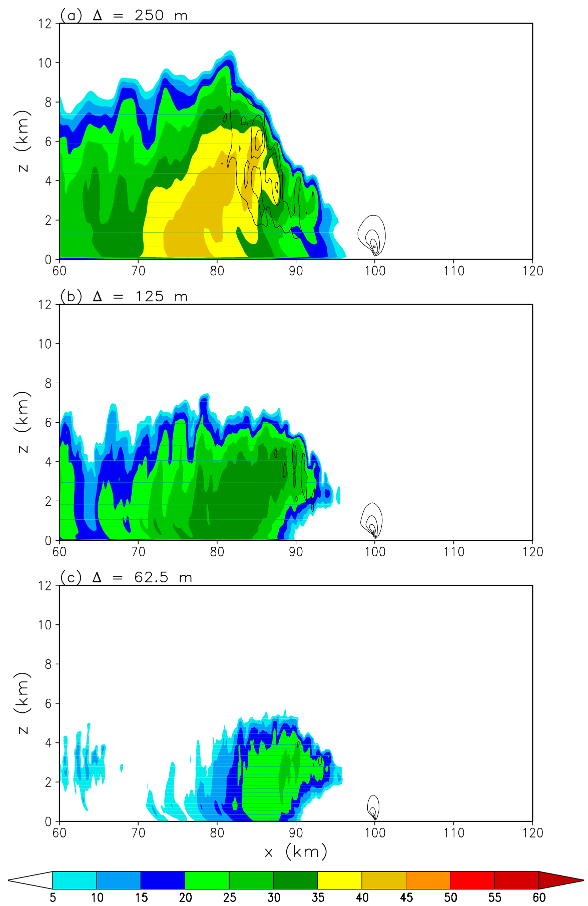

Fig. 5. Line-averaged vertical cross sections of simulated reflectivity (dBZ, shaded) and vertical velocity (m s^{-1}, contoured, positive values only, every 1 m s^{-1}) from squall line simulations using (a) Δ = 250 m, (b) Δ = 125 m, and (c) Δ = 62.5 m.

studies, quantitative precipitation amounts and vertical transport calculations should be interpreted with skepticism, if only *O*(1 km) grid spacing is being used.

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