BISTATIC INTERFEROMETRY TO MEASURE CLEAR AIR WIND

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1. INTRODUCTION

The wind field can be measured either by Doppler or interferometry techniques. Weather radars such as WSR-88D measure the Doppler velocity (i.e., the radial component of wind) (Doviak and Zrnic, 1993). The BIstatic radar NETwork (BINET) measures the Doppler wind component perpendicular to the surface of constant phase (Wurman, 1994). NCAR's Multiple Antenna Profiler Radar (MAPR), a Spaced Antenna (SA) system, measures the cross-beam wind component (i.e., the component parallel to the baseline connecting two receivers) using interferometry (Cohn et al). Meteorological applications of interferometry have been limited to nearly co-located transmitter/receivers, i.e., a bistatic system with extremely close transmitter receiver spacing (i.e., typically less than the transmitting antenna diameter). In clear air sensing, however, a bistatic configuration with a long TR baseline is desirable to increase radar detection of echoes from Bragg scatterers. Doppler measurements of horizontal wind have poor accuracy at small angles. Furthermore, vertical resolution of the horizontal wind is determined by the beamwidth whereas bistatic interferometry has a vertical resolution determined by bandwidth. This could allow wind measurements, especially if atmospheric plankton is absent, and possibly the depth and growth of the mixed layer. The NSF-funded Collaborative Adaptive Sensing of Atmosphere (CASA) project offers the possibility for bistatic radar deployment.

In this paper, we propose and illustrate bistatic interferometry (BI) to measure the horizontal wind component perpendicular to the transmitter/receiver baseline by jointly processing two sets of radar signals. We propose to use the interferometry technique in a bistatic configuration in the CASA network. This could allow the measurement of mesoscale convergence throughout the mixed layer and accurate initialization for numerical weather model.

2. CONCEPTUAL DESCRIPTION OF BISTATIC INTERFEROMETRY

Radar interferometry obtains information of the scattering medium by jointly processing two (or more) sets of signals. The two sets of signals can be obtained in a bistatic configuration of spaced receivers or by Angular Interferometry (AI) (Zhang et al., 2003b). Therefore, the BI technique includes (a) Bistatic Spaced Antenna Interferometry (BSAI) and (b) Bistatic Angular Interferometry (BAI) as shown in Fig. 1. Fig. 1a shows the -

Corresponding author address: School of Meteorology, University of Oklahoma, Norman, OK 73019; Email: guzhangl@ou.edu principle of BSAI for transverse wind measurement with a pair of receiver antennas that allows interferometric signal processing (i.e., cross-correlating the two signals). If there is wind (i.e., red arrow) parallel to the R_1R_2 baseline



Figure 1: Configuration sketch of bistatic interferometry to measure transverse wind. (a) bistatic spaced antenna interferometry, (b) bistatic angular interferometry.

there would be a time delay for the signal received by R_2 to match that received by R_1 (i.e., the cross-correlation peak shifts away from zero time lag). Therefore the wind component can be estimated from the cross-correlation function (Doviak et al., 1996; Zhang et al., 2003). In the BAI (Fig. 1b), a single antenna in a bistatic mode collects signals from two overlapped bistatic resolution volumes and jointly processes them. Both BSAI and BAI measure cross-beam wind. It is expected that interferometry with a long TR baseline performs better than one with a short TR baseline (e.g., MAPR) because of the increased effective wavelength and consequently correlation time. Normally, the BSAI applies to broad beams whereas BAI requires fine angular resolution to perform well.

3. BSAI FORMULATION

We formulate the BSAI technique based on wave scattering from refraction index fluctuations, and derive the cross correlation function to estimate wind. BSAI consists of one transmitter and two receivers as shown in Fig. 2.



Figure 2: Scattering configuration for BSAI.

The coordinate origin is chosen at the center of the radar resolution volume. The received signal is from scatterers in the resolution volume, and it can be obtained by integrating, the refraction index perturbations $\Delta n(\vec{r}', t)$ with proper weighting and phase shifts (Doviak et al., 1996). Assuming Gaussian weighting for both beam and bistatic range, we have

$$E(\vec{r},t) \approx A \frac{g_{T0}^{1/2} g_{R0}^{1/2}}{r_{T0} r_{R0}} \int \Delta n(\vec{r}',t) \exp\left(-\frac{|\hat{s}_{T} - \hat{s}_{T0}|^{2}}{4\sigma_{\theta T}^{2}} - \frac{|\hat{s}_{R} - \hat{s}_{R0}|^{2}}{4\sigma_{\theta R}^{2}}\right) \\ \times \exp\left(-\frac{(r_{T} + r - r_{T0} - r_{0})^{2}}{4\sigma_{R}^{2}} - jk(r_{T} + r - r_{T0} - r_{0})\right) d\vec{r}'$$
(1)

where A is a constant depending on radar parameters; g_{T0} and g_{R0} are gains of transmitting and receiving antennas, respectively; r_{T0} is the distance from the origin to transmitter and r_{R0} is that to the receiver; r_T and r are the distances to the scattering element; \hat{S}_T and \hat{S}_{T0} are unit vector from the transmitter to the scattering element and to origin; \hat{S}_R and \hat{S}_{R0} are these from the receiver; $\sigma_{\theta T}$ and $\sigma_{\theta R}$ are standard deviations of the one-way radiation patterns. The prime defines the location of $\Delta n(\vec{r}', t)$. The first term in the exponent is the angular weighting function, the second term is the range weighting function, and $k(r_T + r - r_{T0} - r_0)$ is the phase difference between the path through $\Delta n(\vec{r}', t)$ and that through the origin.

The cross-correlation function is the ensemble average of the product, $E(\vec{r_1}, t_1)E^*(\vec{r_2}, t_2)$, in which $E(\vec{r_1}, t_1)$, $E(\vec{r_2}, t_2)$ are signals from receivers R₁, R₂. The crosscorrelation function of the received signals is

$$C(\vec{r}_{1},t_{1};\vec{r}_{2},t_{2}) \approx |A|^{2} \frac{g_{T0}g_{R0}}{r_{1}^{2}\sigma_{R0}^{2}} \iint \langle \Delta n(\vec{r}_{1}',t_{1})\Delta n(\vec{r}_{2}',t_{2}) \rangle d\vec{r}_{1}' d\vec{r}_{2}'$$

$$\cdot \exp\left(-\frac{|\hat{s}_{T1} - \hat{s}_{T0}|^{2} + |\hat{s}_{T2} - \hat{s}_{T0}|^{2}}{4\sigma_{\theta T}^{2}} - \frac{|\hat{s}_{R1} - \hat{s}_{10}|^{2} + |\hat{s}_{R2} - \hat{s}_{20}|^{2}}{4\sigma_{\theta R}^{2}}\right)$$

$$\cdot \exp\left(-\frac{(r_{T1} + r_{1} - r_{T0} - r_{10})^{2}}{4\sigma_{R}^{2}} - \frac{(r_{T2} + r_{2} - r_{T0} - r_{20})^{2}}{4\sigma_{R}^{2}}\right)$$

$$\cdot \exp\left(-jk\left(r_{T1} + r_{1} - r_{T2} - r_{2}\right)\right) \qquad (2)$$

It is assumed that (1) the field of $\Delta n(\vec{r}',t)$ is statistically homogeneous and wide sense stationary; (2) radar beams are narrow and elevation angle is small; (3) the phase terms can be expanded up to second order along the transmitter/receiver center baseline (TR); and (4) the resolution volume size is small compared with distance between transmitter and receivers. Using above assumptions and making coordinate transformation with center and difference coordinates, we complete the integrations in (2) and keep important terms to arrive at

$$C(\vec{r}_{d},\tau) \approx |A|^{2} \frac{g_{T0}g_{R0}}{r_{T0}^{2}r_{R0}^{2}} \frac{\pi^{3}\ell^{3}}{a_{x}a_{y}a_{z}} \sigma_{n}^{2} \exp\left(-\frac{1}{2}a_{x}^{2}(v_{x}\tau)^{2}\right)$$

$$\cdot \exp\left(-\frac{k^{2}}{2a_{y}^{2}}\left(\frac{y_{d}}{2L_{R}} - \left(\frac{1}{2L_{T}} + \frac{1}{2L_{R}}\right)v_{y}\tau\right)^{2}\right)$$

$$\cdot \exp\left(-\frac{k^{2}}{2a_{z}^{2}}\left(\frac{y_{z}}{2L_{R}} - \left(\frac{1}{2L_{T}} + \frac{1}{2L_{R}}\right)v_{z}\tau\right)^{2}\right)$$

$$\cdot \exp\left(-2k^{2}\beta^{2}\left(\frac{H}{2L_{T}} + \frac{H}{2L_{R}}\right)^{2} - 2jk\left(\frac{H}{2L_{T}} + \frac{H}{2L_{R}}\right)v_{z}\tau\right)$$

(3)

where ℓ is the correlation length of $\Delta n(\vec{r}',t)$, σ_n^2 is the variance of refraction index fluctuation, $\beta^2 = \ell^2 + \sigma_t^2 \tau^2$, σ_t is the standard deviation of radial component of turbulence, v_x , v_y , and v_z are wind components along x, y and z directions, respectively. The first three exponential terms are the decorrelation caused by the receiver separation and mean wind. The scale factors a_x , a_y , and a_z are

$$a_x^2 = \frac{\sin\theta_T}{4\sigma_{\theta T}^2 r_{T0}^2} + \frac{\sin\theta_R}{4\sigma_{\theta R}^2 r_{R0}^2}$$
(4a)

$$a_{y}^{2} = \frac{1}{4\sigma_{\theta T}^{2}r_{T0}^{2}} + \frac{1}{4\sigma_{\theta R}^{2}r_{R0}^{2}}$$
(4b)

$$a_z^2 = \frac{1}{4\sigma_{\theta T}^2 r_{T0}^2} + \frac{1}{4\sigma_{\theta R}^2 r_{R0}^2} + \frac{1}{4\sigma_R^2}.$$
 (4c)

Assuming the range resolution σ_R (elliptical shell) is much smaller than the beam size $\sigma_{\theta T} r_{T0}$ and $\sigma_{\theta R} r_{R0}$, we have $a_x << a_y << a_z$. Neglecting small contribution terms, we obtain the cross-correlation function of signals from the two receivers as

$$C(y_d, \tau) \approx S \exp\left(-\frac{k^2}{2a_y^2} \left(\frac{y_d}{2L_R} - \left(\frac{1}{2L_T} + \frac{1}{2L_R}\right)v_y\tau\right)^2\right)$$
$$\cdot \exp\left(-2k^2\sigma_t^2\tau^2 \left(\frac{H}{2L_T} + \frac{H}{2L_R}\right)^2 - 2jk\left(\frac{H}{2L_T} + \frac{H}{2L_R}\right)v_z\tau\right) \quad (5)$$

Eq. (5) shows that the signal correlation depends on the receiver separation y_d , cross-beam wind parallel to the baseline of the receivers, and turbulence. The phase term accounts for the Doppler phase in the bistatic configuration, which can be used to estimate the Doppler wind (v_z) in the scattering plane. The transverse wind component (v_y) perpendicular to scattering plane can be estimated from the cross-correlation function magnitude using the cross-correlation ratio (CCR) method (Zhang et al., 2003a) to obtain



Figure 3. Standard deviation of wind estimates as a function of turbulence for BSAI. $v_y = 10 \text{ m/s}$. Other parameters are $\lambda = 0.03$ m, $T_d = 1 \text{ s}$, $\sigma_{\theta T} = \sigma_{\theta R} = 2.0^{\circ}$, $\theta_T = \theta_R = 10^{\circ}$.

The estimation error for the wind component, calculated for the CCR method, is shown in Fig. 3. The results for short TR baseline (e.g., MAPR) using radar with the same characteristics are shown for comparison. As expected, the standard deviation of wind estimates with BSAI is smaller than that for the short TR baseline case, especially for large turbulence.

4. DISCUSSIONS

The cross-correlation function (3) and its simplified form (5) for BSAI have the similar form as that for conventional short TR baseline interferometers (e.g., Eq. (38) of Doviak et al., 1996). The correlation length of interference pattern is of the order of the antenna size, the same as for the short baseline case. But, the correlation time for the long baseline is increased by a factor of $L_{T,R}/H$, a factor of ten or more increase because of the increased effective wavelength in the bistatic configuration. The

increased correlation time allows better coherence of signals for wind estimation with reduced error. The wind estimation in long baseline BSAI is affected less by turbulence than in the short baseline case. Therefore, BSAI has two advantages in transverse wind measurement over the short baseline interferometric technique: (1) better sensitivity of clear echo and (2) better wind measurement accuracy.

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