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THE SPECTRUM OF WEATHER ECHOES

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1. INTRODUCTION

When the term “spectrum” arises, radar meteorologists usually think of the Doppler spectrum of the weather echo from a given point in the atmosphere. However, the spectrum of the echo from a given transmitted pulse, as received in range-time, is the one of concern in designing receiver filters to enhance discrimination of signal from noise, and in determining the power loss in such filters. This paper presents a demonstration of the nature of that spectrum.

The echo from each individual particle, small compared to the wavelength, in the atmosphere is a miniature replica of the transmitted pulse, and ideally would have the sinx/x spectrum of a rectangular pulse. Simple superposition arguments suggest that the total echo from all of the particles contributing to the echo at each instant must also have the same spectrum as that of the transmitted pulse (assuming ideal conditions of a “uniform” distribution of precipitation along the beam axis and range-corrected echo intensity). Section 5.5.1 of Bringi and Chandrasekar (2001) shows that the autocorrelation function (ACF) for such a weather echo would be triangular (again assuming a rectangular transmitted pulse), and a triangular ACF corresponds to a rectangular pulse and a sinx/x spectrum.

However, the video envelope of weather echoes, as displayed for example on an A-scope, bears little resemblance to the transmitted pulse. Thus it is far from obvious that the spectra would be identical. A demonstration that the spectra are in fact the same would therefore be of interest. We set out to conduct such a demonstration, first by performing computer simulations of an idealized weather echo (allowing us to control the properties of the simulated signal) and computing its spectrum, and then by analyzing some data collected with the CSU-CHILL radar (which are now available in the form of digitized IF data with sub-pulse-duration sampling rates).

2. SIMULATION PROCEDURE

The statistical characteristics of the random-phase weather echoes are well known (e.g., Marshall and Hitschfeld 1953). The echo can be represented by Gaussian distributions in both the in-phase (I) and quadrature (Q) channels. As the echo power in the two channels is equal and the spectra must be identical, we need only consider one of the channels in the simulation. To depict the echo variation in range-time with a view to later comparisons with CHILL data, we noted that the radar transmits a 1 μs pulse and the digital receiver sampled the 10 MHz IF signal at 40 MHz (2003 situation). Thus there would be 40 samples within the duration of a single pulse. This means the echo amplitude at discrete intervals of 25 ns can be simulated by taking a running sum of 40 independent, identically-distributed Gaussian components (Figure 1). At some instant t_1 this sum runs from element n_1 to n_{40} of the Gaussian array; the next sample at $t_2 = (t_1 + 25$ ns) runs from element n_2 to n_{41} of the array; and so on. Figure 2 shows a sample of the synthetic weather echo generated in this fashion.

3. AUTOCORRELATION FUNCTION

Following Bringi and Chandrasekar (2001), we examined the ACF of the simulated echo, with the initial results shown in Figure 3. This plot represents the ACF for a Gaussian array with 2^{12} = 4096 values, but as the running sum encompassed 40 of the values, there were only 4057 data points in the ACF calculation. This would correspond to just over 100 times the pulse duration, or about 15 km in range along the beam. While it is evident that the ACF is roughly triangular, substantial ripples remain. With the Gaussian array expanded to 2^{16} = 65,536 elements, corresponding to more than 1600 times the pulse duration or about 245 km along the beam, the triangular nature of the ACF becomes clearer (Figure 4). At the same time, this suggests one of the potential problems in trying to demonstrate the correspondence between the echo spectrum and the transmitted spectrum with actual data: even a 15 km path through uniform precipitation will not be easy to find.

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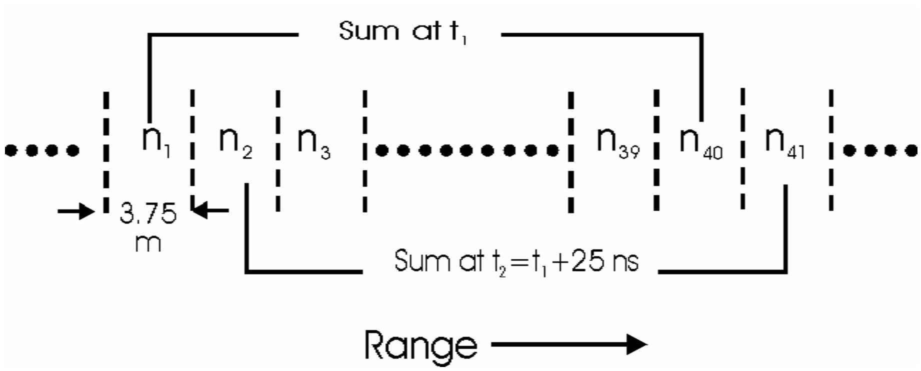

Fig. 1: Illustration of the simulation procedure for the amplitude of precipitation echoes from a 1 μs transmitted pulse, with the echo sampled at 25 ns intervals (40 MHz sampling rate). At each 25 ns time increment the echo comprises the sum of 40 random Gaussian components, each representing the contribution from a 3.75 m slice of the atmosphere.

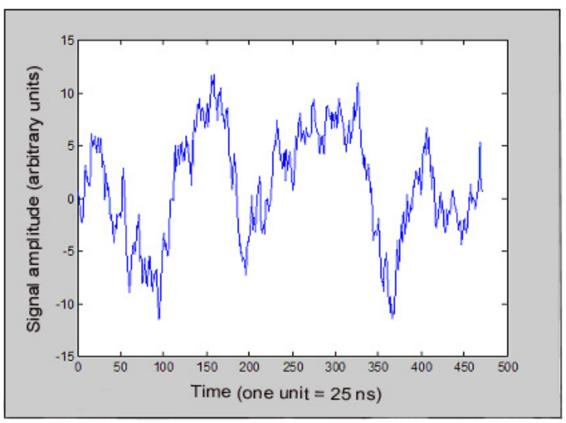

Fig. 2: First 473 values (covering about twelve times the pulse duration) of the simulated weather echo.

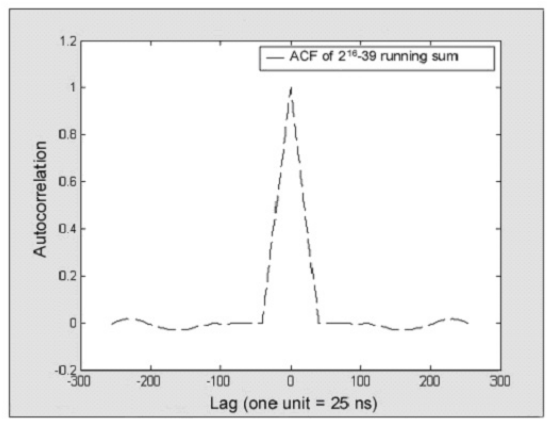

Fig. 4: Autocorrelation function of the simulated echo, based on 65,497 data points.

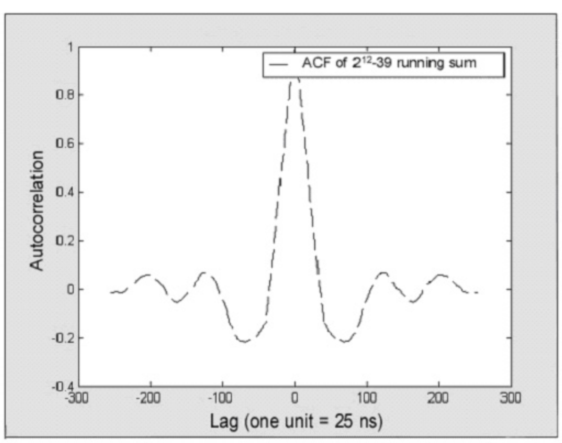

Fig. 3: Autocorrelation function of the simulated echo, based on 4057 data points.

4. THE SPECTRUM OF SIMULATED ECHOES

A straightforward digital Fourier transform (DFT) of the simulated echo yields the desired spectrum; the amplitude of the power spectrum at any given frequency is the square of the magnitude of the DFT. Figure 5 shows the result for $(2^9 - 39) = 473$ data points, or about 1.8 km along the beam axis, corresponding to the example in Fig. 2. The basic $(\sin x/x)^2$ pattern can be seen in the plot, but the description is quite noisy. Two possibilities would be available for obtaining a clearer portrayal:

- Increase the length of record; easily done in the simulations, more difficult to achieve in actual data
- Average multiple spectra; also easily done in the simulations, and doable with actual data

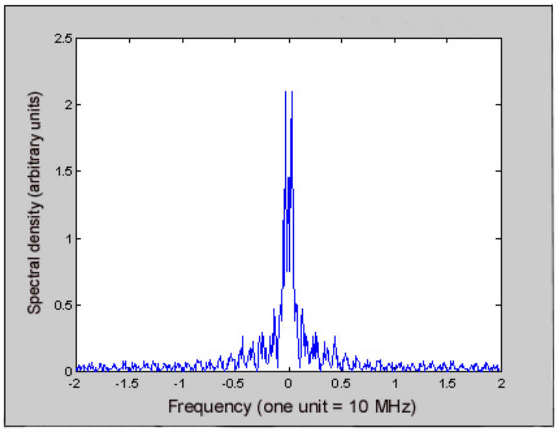

Fig. 5: Power spectrum of simulated echo, based on 473 data points.

4.1 *Increased length of record*

Gossard and Strauch (1983) demonstrate that the signal-to-noise ratio (SNR) in an analyzed echo spectrum will show little improvement after the record length extends beyond about ten times the signal decorrelation time. In these simulations the decorrelation time is of the same order as the pulse duration, so this means spectra computed from records longer than about 400 points will display similar SNR characteristics. (To be sure, spectra from longer records would have finer resolution in frequency, but that is of little advantage here.) Figure 6 shows the result of extending the simulation to (2^{12} - 39) = 4057 data points, or about 15 km along the beam – corresponding to the ACF in Fig. 3. Here the $(\sin x/x)^2$ pattern is no more clearly elucidated.

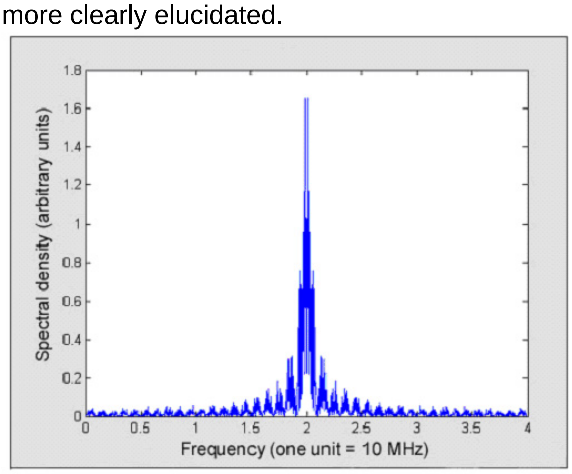

Fig. 6: Power spectrum of simulated echo, based on 4057 data points (center frequency offset to 20 MHz).

4.2 *Averaging spectra*

Averaging of multiple spectra obtained with shorter lengths of record provides a more promising approach; the noise fluctuations are reduced by the square root of the number of spectra included in the average. Figure 7 shows the result of averaging just 16 spectra from data segments each of length (2^9 – 39) = 473 points, or about 1.8 km along the beam. The total record used is thus (2^4) (2^9 – 39) = 7568 data points, or about twice that for the ACF illustrated in Figure 3 or the spectrum in Fig. 6. Thus averaging multiple spectra from shorter data segments in this way brings out the nature of the echo spectrum more clearly.

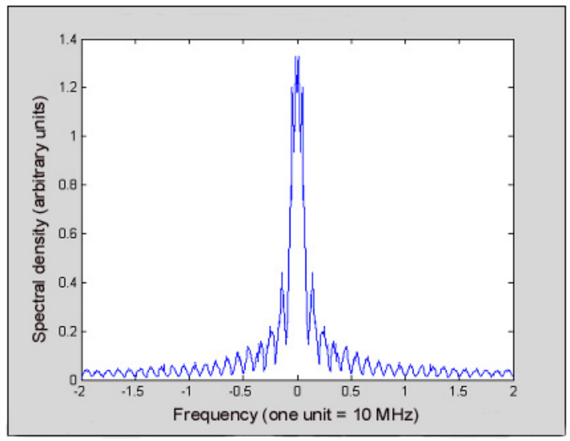

Fig. 7: Average of 16 power spectra, each based on 473 data points from the simulated echo.

5. THE SPECTRUM OF ACTUAL WEATHER ECHOES

As indicated above, we used sample echo data from the CSU-CHILL radar, collected (with the antenna stationary) in stratiform precipitation, in efforts to demonstrate the behavior noted in the simulations. Two issues complicate the use of actual data for this purpose.

(1) The (1/r) variation of echo amplitude, present even if the precipitation were uniform along the beam. This could be handled by applying range normalization to the digitized IF signal before calculating the DFT.
(2) The inherent variation of the precipitation, specifically its reflectivity, along the beam. This could be handled by normalizing the digitized signal to represent the same re-

flectivity value at each point along the beam.

Only a small dataset was acquired and we made only cursory attempts to apply these corrections.

5.1 The transmitted spectrum

The transmitted pulse is rarely, if ever, an ideal rectangle, even in a quality research radar like CHILL. Moreover, the limited digitizing rate restricts the resolution available in the corresponding spectra. Figure 8 shows the computed spectrum of the CHILL transmitted pulse, derived from IF data also digitized at the 40 MHz rate, to provide an idea of what might be expected from DFT analysis of the echo signal. The basic $(\sin x/x)^2$ character is evident in the first lobe or two around the center frequency, but (as the pulse surely lacks sharp leading and trailing edges) the further-out parts of the spectrum are more diffuse.

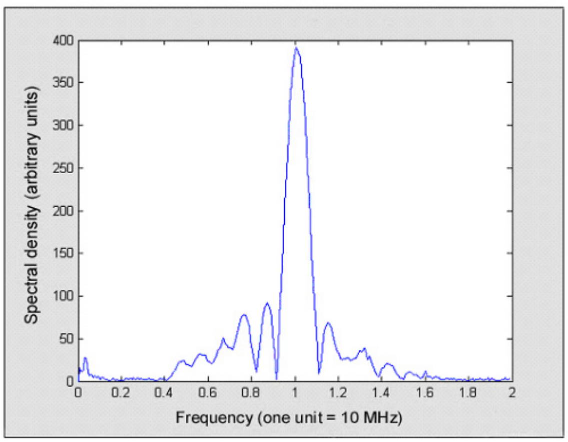

Fig. 8: Power spectrum of the CSU-CHILL transmitted pulse (sampled at the IF, with IF frequency 10 MHz).

5.2 The echo spectrum

Figure 9 shows a power spectrum for a single echo sample of $(2^{12} - 39)$ data points obtained from roughly uniform precipitation. Comparing it with Figure 8 (the transmitted spectrum) and Figure 6 (a corresponding simulated echo spectrum) shows that the noisy character of the data again makes the spectrum barely recognizable.

The data acquired from CHILL do not permit the luxury of examining records of much greater length. Even under favorable circumstances, finding a data array as large as even 2^{16} points (= 246 km along the beam) would be unlikely – and as Figure 6 suggests, that would not help to bring out clearly the nature of the spectrum.

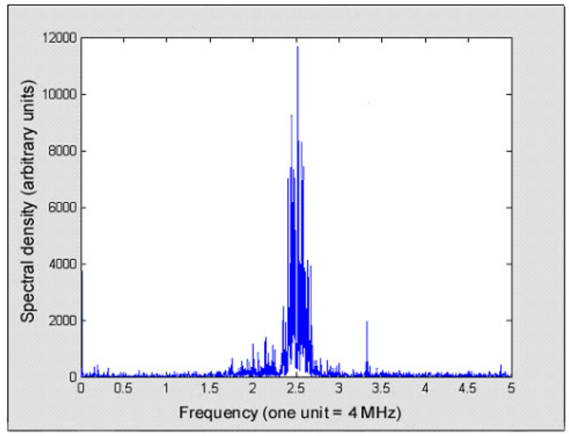

Fig. 9: Power spectrum of a single sample of 4057 data points from an echo from stratiform precipitation. (IF frequency = 10 MHz)

5.3 Averaged spectrum

At the time the CHILL data were acquired we had not recognized either the great record lengths that would be needed to bring out the ACF, or the full advantages of averaging multiple spectra. Consequently, only 14 usable “roughly uniform” segments of data, each with 2^{12} points, were available. We normalized these segments to have the same total energy (but did not otherwise correct for variations along the beam). This means the total record available was about 56,800 points, slightly less than that used in Figure 4. Then we computed the DFT for each segment and averaged the 14 resulting power spectra (we also removed some spurious spikes, apparently caused by a neighboring air traffic control radar). Figure 10 shows the resulting average spectrum. While the rendition is noisy, general features visible in the transmitted spectrum in Figure 8 are also evident here. Consequently, the basic objective of demonstrating that the (average) echo spectrum corresponds to that of the transmitted pulse was achieved.

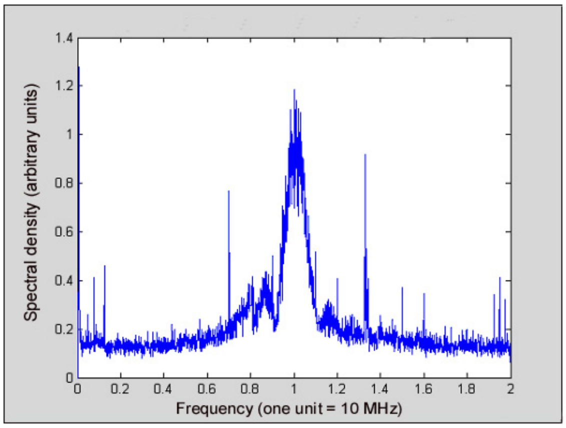

Fig. 10: Average of fourteen power spectra from different segments, each of length 4057 data points, of echo from stratiform precipitation. (IF frequency = 10 MHz.)

6. CONCLUDING REMARKS

This work merely confirms a known property of weather echoes. In this era of digital signal processing, the main utility of this property probably lies in the use of the transmitted spectrum to design digital IF filters and to calculate the loss in those filters. In reality, the echo has variations over and above the random-phase fluctuations, due to variations in reflectivity along the beam as well as the range effects. Thus the spectrum of the echo from any given pulse passing through the filters usually will differ from the transmitted spectrum. The transmitted spectrum surely provides a representation of the average echo spectrum, and hence is a useful guide to designing the filters and calculating the average filter loss. However, this analysis raises an interesting question: Do deviations from the average spectrum constitute an added source of significant uncertainty in the reflectivity estimates?

References

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