P14R.11 INFERENCE OF MEAN RAINDROP SHAPES FROM DUAL-POLARIZATION DOPPLER SPECTRA OBSERVATIONS

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1. INTRODUCTION

Direct observations of effective raindrop shapes from dual-polarization radar measurements, are not straightforward. In order to interpret dual-polarization radar measurements one would need to know not only raindrop size-shape relation but also drop-size distribution (DSD). Goddard et al. (1982) have used disdrometer data to address this problem. Recently Gorgucci et al. (2000) have introduced an additional shape parameter, β, into polarimetric rain rate retrieval algorithm that represents the equivalent slope of the size-shape relationship. This approach uses reflectivity (Z_h), differential reflectivity (Z_{dr}) and specific differential phase (K_{dp}) measurements to estimate β. Use of K_{dp} restricts applicability of the proposed technique to moderate to heavy rain events. Moreover, use of K_{dp} implies reduction of the range resolution, and therefore results in range averaged estimates of β.

A number of studies have been presented in literature that deal with the retrieval of DSD parameters and ambient air velocity from Doppler power spectra. Hauser and Amayenc (1981) have shown that assuming no spectral broadening Doppler power spectra measurements of precipitation at vertical incidence can be used to retrieve vertical air motion and two parameters of exponential DSD. Nonetheless, it was acknowledged that spectral broadening would be one of the major sources of errors in this retrieval. Recently, Williams (2002) has shown that nonlinear least square optimization procedure can be used for a joint retrieval of three parameters of gamma DSD, vertical air motion and spectral broadening from vertical incident profiler Doppler measurements. Therefore, by combining Doppler spectra observations taken at a sufficiently high elevation angle with dual-polarization measurements of precipitation one should be able to retrieve both DSD information and raindrop size-shape relation on a short time and spatial scales.

To facilitate investigation of β on small time and spatial scales in this study we introduce a new retrieval method of raindrop size-shape relation that is based on dual-polarization spectral measurements of precipitation. In this method the drop-size distribution information, spectrum broadening and ambient air velocity are obtained from Doppler power spectra measurements and given this information the β parameter is obtained from differential reflectivity measurements. The measurements should be carried out at a sufficiently high elevation angle for correct DSD retrieval and at a rather low elevation angle for accurate dual-polarization measurements. Based on simulations we have studied sensitivity of the proposed retrieval method to the radar antenna elevation angle and have determined the window of suitable elevation angles. Moreover, the sensitivity analysis of the proposed technique to spectrum broadening is also performed. The performance of the method is illustrated using data collected by the CSU-CHILL radar (Brunkow et al. 2000).

2. RETRIEVAL METHODOLOGY

2.1. *Model*

For radar observations at non zero elevation angle the radial velocity of raindrop can be written as a function of raindrop equivolume diameter (Atlas et al. 1973)

$$v(D) = \left(\frac{\rho_0}{\rho}\right)^{0.4} [9.65 - 10.3\exp(0.6D)] \cdot \sin\theta + v_0, \quad (1)$$

where ρ_0 and ρ are the air densities at the sea level and the altitude of a considered range gate respectively, v_0 is the ambient air radial velocity, D is the equivolume raindrop diameter given in mm, and θ is the antenna elevation angle. Quite air Doppler spectrum measured at a sufficiently high elevation angle can be written as (Russchenberg 1992).

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$$S_{hh}(v)dv = \frac{\lambda^4}{\pi^5|K_r|^2}\sigma_{hh}(D(v))N(D(v))\frac{dD(v)}{dv}dv \tag{2}$$

where λ is the radar wavelength, K_r is the dielectric factor, D is the equivolume diameter, θ is the radar elevation angle, σ_{hh} is the backscattering cross-section, v is the radial velocity of a raindrop and $N(D)$ is the drops-size distribution. In this study, the drop-size distribution, $N(D)$, is assumed to be described by a gamma distribution (Bringi and Chandrasekar 2001).

In reality Doppler spectrum of precipitation is always broadened by turbulence, raindrop oscillations, cross-wind or wind shear (Doviak and Zrnic 1993). It is common to model the effect of spectral broadening as a Gaussian shaped convolution kernel (Doviak and Zrnic 1993), therefore the observed spectrum can be written as

$$\begin{aligned} S_{hh,mod}(v) &= S_{broad}(v) * S_{hh}(v) = \qquad (3) \\ &= \frac{1}{\sqrt{2\pi}\sigma_b}\int \exp\left[-\frac{(v-\widetilde{v})^2}{2\sigma_b^2}\right] S_{hh}(\widetilde{v})d\widetilde{v}, \end{aligned}$$

where $*$ is the convolution operator, and σ_b is the broadening kernel width.

At the S-band frequencies the radar cross-section of raindrops can be estimated using Rayleigh-Gans calculations for oblate spheroids (Bringi and Chandrasekar 2001). Several axis ratio relations were used in this study. Following Gorgucci et al. (2000) the axis ratio of raindrops can be assumed to follow a linear relation to the drop equivolume diameter:

$$\frac{a}{b} = 1 - \beta D \tag{4}$$

where β is the slope parameter. Sometimes another linear relation (Matrosov et al. 2002; Ryzhkov and Schuur 2003) is used:

$$\frac{a}{b} = (1 + 0.05\beta) - \beta D. \tag{5}$$

In order to compare these two relations we have used both of them in our study.

Changes in canting angle distribution of raindrops have a similar effect on the co-polar backscatter cross-section as changes of the magnitude of β. Therefore changes in β would correspond to both changes in canting angle distribution and to oscillation of raindrops. Since raindrop size-shape relations are generally nonlinear, β values also depend on DSD parameters. In order to study whether observed raindrops have equilibrium shapes we also have used Beard and Chuang (1987) relation to compare our retrievals to, as discussed further on.

2.2. *Retrieval approach*

The model of a Doppler power spectrum depends on six parameters, i.e. three DSD parameters, spectrum broadening, ambient air velocity and slope of the drop size-shape relation. It can be seen that β has a negligible effect on a Doppler spectrum and therefore can be omitted from our considerations at this stage of the retrieval. Moreover, one can observe that changes in μ and D_0 would result in changes of the spectrum shape. The spectrum broadening widens and smooths the spectrum. The effect of v_0 and N_w on Doppler spectra is rather easy to imagine. The ambient air velocity, v_0, shifts the velocity axis according to (1). And changes in N_w would result in scaling along reflectivity axis.

Therefore, given a microphysical model of the Doppler spectra observations one can formulate the DSD parameters retrieval procedure as an optimization problem of fitting modeled spectra to the observed ones. Least-square is often used to solve such type of problems (Williams 2002).

$$\min_{N_w,D_0,\mu,v_0,\sigma_b} \sum_{v=v_a}^{v_b} \begin{pmatrix} \log(S_{hh,mod}(v,N_w,D_0,\mu,v_0,\sigma_b)dv)- \\ -\log(S_{hh,meas}(v)dv) \end{pmatrix}^2, \tag{6}$$

More details on the retrieval can be found in (Moisseev et al. 2005).

Given dual-polarization observations of precipitation and knowing corresponding DSD parameters, one can infer information about drop size-shape relation. For this purpose measurements of differential reflectivity, Z_{dr}, that is ratio of hh and vv reflectivities, can be used. The Z_{dr} values depend on DSD parameters, μ and D_0, as well as on β. Therefore, given D_0 and μ one can retrieve β value by solving the following minimization problem (Moisseev et al. 2005)

$$\min_{\beta}\left(Z_{dr}^{mod}(\beta)|_{D_0,\mu} - Z_{dr}^{meas}\right)^2, \tag{7}$$

where Z_{dr}^{mod} and Z_{dr}^{meas} are respectively modeled and measured differential reflectivities.

3. Error analysis

To evaluate performance of the retrieval technique the optimization procedure was applied to simulated Doppler spectra. Using (2) and (3) and applying procedure described by Chandrasekar et al. (1986) realizations of hh and vv Doppler power spectra were created. For these simulations the input parameters were selected randomly from the intervals:

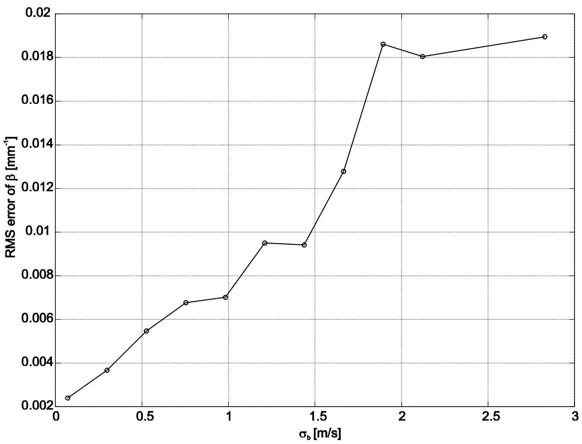

Figure 1: RMS errors of the retrieved β as a function of spectrum broadening kernel width.

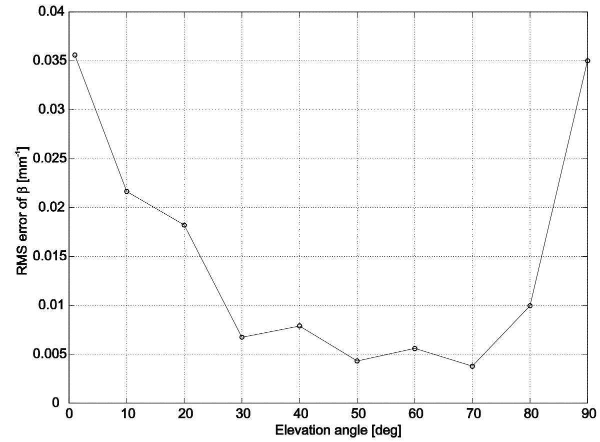

Figure 2: Root mean square errors of the retrieved values of β as a function of elevation angle.

$$
\begin{aligned}
N_w &\in [50, 15\cdot 10^5] \quad (\mathrm{mm^{-1}\,m^{-3}}) \\
D_0 &\in [0.5, 2.5] \quad (\mathrm{mm}) \\
\mu &\in (-1, 5] \\
\sigma &\in [0, 1] \quad (\mathrm{m\,s^{-1}}) \\
v_0 &\in [-1, 1] \quad (\mathrm{m\,s^{-1}}) \\
\beta &\in [0.03, 0.07] \quad (\mathrm{mm^{-1}}).
\end{aligned} \tag{8}
$$

3.1. *Effect of spectral broadening*

Part of the optimization problem (6) can be considered as an inversion problem of finding $S_{hh}(v)$ from $S_{broad}(v) * S_{hh}(v)$ or solving the integral equation (3). That is a Fredholm integral equation of the first kind. Since the spectral broadening kernel $S_{broad}(v)$, (3), is smooth and monotonic from $v = 0$ to ∞, one can expect increase in instability of the solution with the relative increase of the width of the kernel with respect to the width of $S_{hh}(v)$ (Twomey 1977). In our case this instability would translate into higher errors in the retrieved DSD parameters, and ambient air velocity values. Since retrieval of β requires knowledge of D_0 and μ it also would be affected by the instability of the inversion. In Figure1 the resulting root mean square errors (RMS) of retrieved β is shown for different values of σ_b,the simulations were carried for 45 degree elevation angle.

3.2. *Elevation angle dependence*

Dependence of the DSD parameters retrieval on the elevation angle is caused by two opposing effects. Firstly, lower the elevation angle smaller the difference between fall velocities radial projections of differently sized raindrops. And therefore spectrum broadening would have a larger effect on the retrieval at smaller elevation angles. Secondly, due to the same reason, a number of spectral lines available for the retrieval would be smaller at lower elevation angles if the same number of samples is used to calculate Doppler spectra. On the other hand, the influence of raindrop shapes on the Z_{dr} measurements is larger for smaller elevation angles. Therefore, accuracy of the β estimate for different elevation angles would be a trade off between accuracy of the retrieval of DSD parameters and influence of raindrop shapes on the Z_{dr} measurements. This effect is shown in the Figure 2. From this figure we can observe that useful elevation angles for this retrieval belong to the interval between 30 and 70 degrees.

4. Results

The CSU-CHILL data were collected during a thunderstorm event on July 23, 2004. The observed reflectivities were ranging between 50 and 55 dBZ. The antenna elevation angle was 30 degrees for this measurement. The range resolution was 50 m. Two minutes of time-series data were collected during this event. The measurements were carried out in alternating mode (Bringi and Chandrasekar 2001). The pulse repetition time of 1 ms was used for this measurement. The Doppler power spectra were estimated by averaging 15 spectra, where each spectrum was estimated from 64 samples using the periodogram approach with the hamming window. The resulting velocity resolution was 0.39 m/s. In the Figure 3 an example of the measured spectral density function and corresponding model fit is shown.

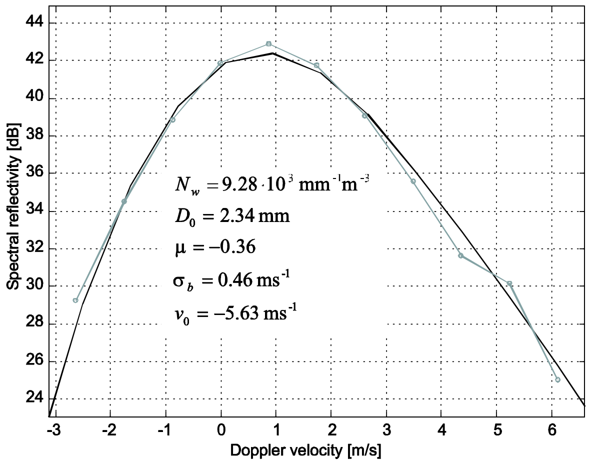

Figure 3: An example of Doppler power spectral density measured by CSU-CHILL. The spectrum was calculated from 64 samples. Fifteen spectra were averaged to obtain this plot. The grey line represent the measurement and the black line shows the fit to the data obtained by solving (6).

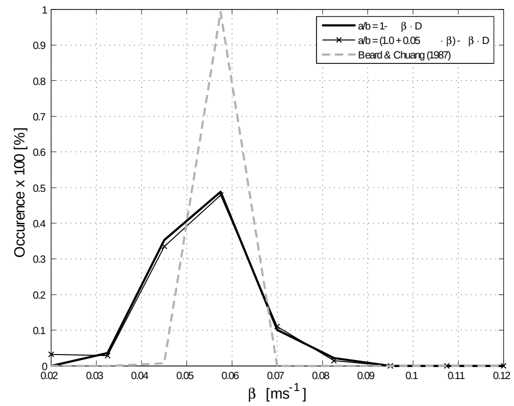

Figure 4: Histograms of the retirieved β values. Also the histogram of effective β values for equilibrium raindrop shapes (Beard and Chuang 1987) is shown.

Then, given the DSD parameters, the β values were retrieved by matching modeled and observed Z_{dr} values. Here two sets of β values were calculated one using Gorgucci et al. (2000) definition of β and the other using Matrosov et al. (2002) definition. The histogram of the resulting β values is shown in Figure 4. One can see that these two definitions of the effective slope of size-shape relation produce very similar results. Moreover, using retrieved DSD parameters and Beard and Chuang (1987) size-shape relation we have calculated Z_{dr} values, as if the raindrops have had equilibrium shapes. Applying β retrieval procedure (7) on these Z_{dr} values, instead of measured ones, we estimated effective slope of Beard and Chuang (1987) size shape relation for a given DSD. For this calculation we have followed definition (4) of β. The results of these calculations is also shown in Figure 4. We can see that two definitions of β show similar results here as well. Also we should note that equilibrium β values are larger than the observed ones. The mean observed β value is equal to 0.054 mm^{-1} while the mean equilibrium β value is 0.057 mm^{-1}. For this measurement mean retrieved D_0 value is 2.5 mm.

5. Conclusions

In this study a new approach to retrieve raindrop shape information was developed. It was shown that the slope of raindrop shape size relation can be retrieved from dual-polarization spectral analysis of time-series radar data. As a result a fine scale analysis of the precipitation microphysical properties can be carried out.

Sensitivity analysis of the proposed technique was also performed. It was shown that spectral broadening has a very strong influence on the retrieval accuracy. Nonetheless, for spectral broadening widths smaller than 1 ms^{-1} an accurate retrieval of DSD parameters, wind velocity and slope of the raindrop size-shape relation can be achieved. This is a general limitation applicable to most DSD parameters retrieval methods based on analysis of Doppler power spectra.

The effect of different radar elevation angles on the errors in the retrieved parameters was also studied. It was shown that for observations between 30 and 70 degrees, the slope of the raindrop size-shape relation can be retrieved with an absolute error of 0.007 mm^{-1} or less. It was also shown that for elevation angles larger than 30 degrees an accurate estimate of DSD parameters and ambient air velocities can be obtained.

It was also shown that observed raindrops are more spherical than raindrops in equilibrium. It was shown that for equilibrium drops mean β values would be equal to 0.057 mm^{-1} and the retrieved values assuming linear size-shape relation were found to be equal to 0.054 mm^{-1}.

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