15R.6 PROFILER DSD RETRIEVALS: WHAT IS THE IMPACT OF AN ADDITIONAL PROFILER?

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1. INTRODUCTION

Radar profilers are excellent tools for validation of radar rainfall retrieval algorithms. The profiling drop-size distribution (DSD) retrieval algorithms employ Doppler spectra measurements. Often it is considered that retrievals using two different profilers operating at distinct frequencies are superior to single profiler ones. However, it is not always clear what is the added advantage of employing two profilers.

This study is dedicated to quantifying uncertainties in profiling DSD retrievals and to the development of an optimum DSD retrieval algorithm that uses spectral observations carried out by two profilers. As the basis for this study we assume that DSD retrieval algorithm is based on spectral observations by S-band profiler and the second profiler can be used to constrain the retrievals. It is also assumed that DSD can be approximated by a modeled distribution and therefore DSD retrieval algorithm is a non-linear least square solution for the parameters (Williams 2002; Moisseev et al. 2005), σ and v_0, as well as that of the DSD model. For example for a gamma model DSD parametersN_w, D_0 and μ are estimated.

As a starting point of this study we use sensitivity analysis to quantify uncertainties in the retrieval of the DSD parameters using an S-band profiler. From this study we determine which parameters contribute the most to the uncertainty in the retrieval. Than it is discussed how second frequency spectral observations can be incorporated to reduce errors in the DSD retrievals and what is an advantage of using two profiler DSD retrieval scheme.

2. DSD RETRIEVAL METHOD

We have used DSD retrieval technique based on S-band measurements of Doppler spectra, as the basis for this study. In this case retrieval procedure can be formulated as a minimization problem of sum squared residuals (SSR_{ds}) of a fit of the modeled power spectrum to the measured one (Williams 2002; Moisseev et al. 2005). Where, SSR_{ds} is defined as:

$$SSR_{ds}(N_w, D_0, \mu, v_0, \sigma_b) = \tag{1}$$

$$= \sum_{v=-v_{\max}}^{v_{\max}} \left(\begin{array}{c} \log(S_{hh,mod}(v, N_w, D_0, \mu, v_0, \sigma_b)dv) - \\ - \log(S_{hh,meas}(v)dv) \end{array} \right)^2,$$

where $S_{hh,\mathrm{mod}}(N_w, D_0, \mu, v_0, \sigma_b)$ is given by (2). Here we have assumed that DSD follows gamma distribution and is described by three parameters, N_w, D_0 and μ (Bringi and Chandrasekar 2001). And the modeled spectrum is given as (Moisseev et al. 2005):

$$\begin{aligned} S_{hh,mod}(v) &= S_{broad}(v) * S_{hh}(v) = \\ &= \frac{1}{\sqrt{2\pi}\sigma_b} \int \exp\left[-\frac{(v-\widetilde{v})^2}{2\sigma_b^2}\right] S_{hh}(v) d\widetilde{v}, \end{aligned} \tag{2}$$

where

$$S_{hh}(v)dv = \frac{\lambda^4}{\pi^5 |K_r|^2} \sigma_{hh}(D(v)) N(D(v)) \frac{dD(v)}{dv} dv$$

where σ_b is the width turbulent / cross-wind spectrum broadening kernel.

In (Moisseev et al. 2005) it was shown that this 5 parameter non-linear optimization problem can be simplified to 3 parameter nonlinear problem. This simplification is based on the observation that DSD intercept parameter, $N_{w,}$ and wind velocity, v_0, are scaling parameters of Doppler power spectra and can be estimated separately using more direct methods. Therefore the DSD parameters retrieval can reformulated as the following optimization problem:

$$\min_{D_0,\mu,\sigma_b} \min_{N_w} \min_{v_0} SSR_{ds}(N_w, D_0, \mu, v_0, \sigma_b) \tag{3}$$

3. SENSITIVITY ANALYSIS

Now, if additional measurements are available, such as measurements using second profiler operating at a different frequency, one would need to combine

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output	μ	D_0	σ_b
logN_w	0.01	0.51	0.48
μ	-	0.56	0.44
D_0	0.03	-	0.97
v_0	0.01	0.51	0.48
σ_b	0.02	0.98	-
β	0.12	0.45	0.43

Table 1: Results of the variance decomposition of the retrieved parameters. This table shows dependence of the RMS values of DSD parameters, v_0 and σ on uncertainties in D_0, μ and σ. One can see that uncertainties in D_0 and σhave a largest impact on the retrievals. Uncertainties in μ, on the other hand, have a very small impact.

these measurements in an optimum way to obtain a better DSD retrieval. To investigate the effect of data from second profiler operating at different frequency (or polarization) we have performed a sensitivity analysis of the optimization problem (3). Using variance decomposition method, total effect indices (Saltelli et al. 2000) of D_0, μ, σ_b on RMS errors of the retrieved parameters were calculated. The total effect indices shows effect of a factor, i.e. D_0, on the output variance. In Table 1 the total effect indices for RMS values of the retrieved parameters are given. It can be seen that uncertainties in D_0 and σ_b have large effect on the the retrieval accuracy. Uncertainties in μ, on the other hand, do not have much of impact. The RMS values of the retrieval are given in the Table 2.

From this analysis one can conclude that if additional measurements are available, they should be used to reduce uncertainties in D_0 and/or σ_b.

4. SPECTRAL DIFFERENTIAL REFLECTIVITY CONSTRAINT

The CSU-CHILL radar can carry out dual-polarizations spectral measurement. If these measurements are taken with elevation angles lying in the interval between 30 and 70 degrees, one can retrieve both DSD parameters of precipitation and mean shapes of raindrops (Moisseev et al. 2005). Moreover, one can consider using spectral differential reflectivity measurements, $Z_{dr}(v)$, to add an additional constraint to the DSD retrieval. The spectral differential reflectivity is defined as the ratio of Doppler power spectra measured at hh and vv polarizations. In absence of the spectral broadening, $\sigma_b = 0$, the spectral differential reflectivity is directly related to raindrop shapes. Nonetheless, when spectral broadening is present, due to finite beamwidth and turbulence, spectral Z_{dr} becomes dependent on DSD parameters and σ_b, as can be seen in Figure 1. One can observe that D_0 and σ_b have a noticeable effect on $Z_{dr}(v)$. The other important parameter that affects spectral differential reflectivity is raindrop size-shape relation. In this study this relation was assumed to follow a linear relation to the drop equivolume diameter, after (Gorgucci et al. 2000):

$$\frac{a}{b} = 1 - \beta D. \tag{4}$$

By assuming such form of the raindrop-size shape relation one can take into account variability of raindrops due to oscillations (Gorgucci et al. 2000).

Therefore by incorporating $Z_{dr}(v)$ in the retrieval of DSD parameters one would add one extra unknown, but would be able to introduce an additional constraint. In this case the retrieval procedure can be formulated as solution of (3) with constraint $Z_{dr}^{\mathrm{mod}}(\mathbf{v}, \beta, D_0, \mu, \sigma_b) = Z_{dr}^{meas}(\mathbf{v})$.

5. RESULTS

To evaluate performance of the retrieval technique the optimization procedure was applied to simulated Doppler spectra. Using (2) and applying procedure described by Chandrasekar et al. (1986) realizations of hh and vv Doppler power spectra were created. For these simulations the input parameters were selected randomly from the intervals:

$$\begin{aligned} \log N_w &\in [1, 6] \\ D_0 &\in [0.5, 3.5] \quad (\mathrm{mm}) \\ \mu &\in (-1, 5] \\ \sigma &\in [0, 1] \quad (\mathrm{m\,s^{-1}}) \\ v_0 &\in [-5, 5] \quad (\mathrm{m\,s^{-1}}) \\ \beta &\in [0.02, 0.09] \quad (\mathrm{mm^{-1}}). \end{aligned} \tag{5}$$

Furthermore, the simulated measurements were constrained to have reflectivity values larger than 10 dBZ and smaller than 55 dBZ. Prior to the retrieval, fifteen realizations of the hh and vv Doppler power spectra for a given set of input parameters were calculated and averaged to obtain an estimate of the true spectrum. The velocity resolution of 0.3 m/s and 128 spectral lines were used for this simulation. The copolar correlation coefficient was assumed to be equal to 0.98. Then, the estimated spectrum is reduced to the spectral lines where $S_{hh}(v)dv$ is larger than -20 dBZ and are no more than 30 dB below the peak spectral density. This step simulates clipping of observed spectra to remove spectral lines

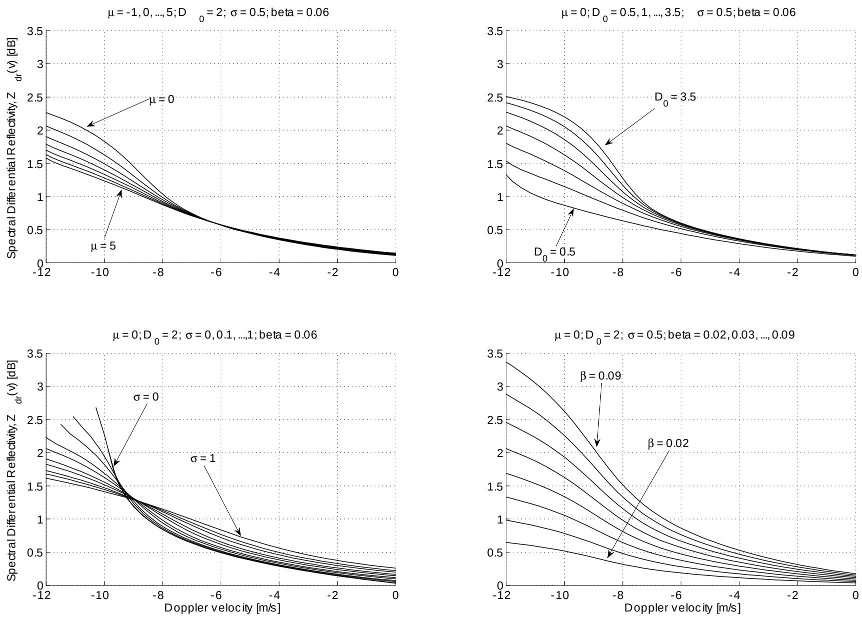

Figure 1: Dependence of $Z_{dr}(v)$ on μ, D_0, σ_b and β.

parameter	RMS	RMS (constr.)
logN_w	0.71	0.42
μ	1.7	1.5
D_0	0.5	0.27
σ	0.l	0.06
v_0	0.7	0.43
β	0.01	0.008

Table 2: Resulting RMS values for the retrieved parameters using constrained and not-constrained retrieval methods

affected by noise and spectral leakage. Then the optimization procedure was applied to the averaged hh spectrum to retrieve DSD parameters, spectral broadening and air velocity. For these simulations the antenna elevation angle of 45 degrees was used.

Using the simulated data we have compared method described by (Moisseev et al. 2005) and $Z_{dr}(v)$ constrained method. In Figure 2 an example of resulting fits using these methods is shown. We can see that performance of these two methods is similar, only the constrained method gives a slightly better result. During our simulations we have noticed that if non-constrained method gives good fit then the new method also performs well. Only in cases where the old method fails the constrained methods performs noticeably better.

Using 300 simulated spectra we have calculated RMS errors of the retrieved parameters for two methods, the results are shown in Table 2. One can see that use of $Z_{dr}(v)$ measurements as additional constraint improves retrievals. For example, RMS errors of D_0 are almost two times smaller for the new method.

6. CONCLUSIONS

In this study sensitivity analysis of DSD parameters retrievals using S-band spectral measurements was carried out. It is shown that uncertainties in D_0 and σ_b have the largest effect on the output values. Therefore, it is advantageous to use data from a second profiler to constrain retrievals of these two parameters. By the data from a second profiler we presume an additional measurements either carried out at a different frequency (or polarization) that bring new information into our retrieval. Using the results of the sensitivity analysis a new retrieval method was proposed. In this method dual-polarization spectral measurements are used for retrieval of DSD parameters. It was proposed to constrain the retrievals using spectral differential reflectivity measurements. It was shown that using this new method errors of the retrieved values can significantly be reduced.

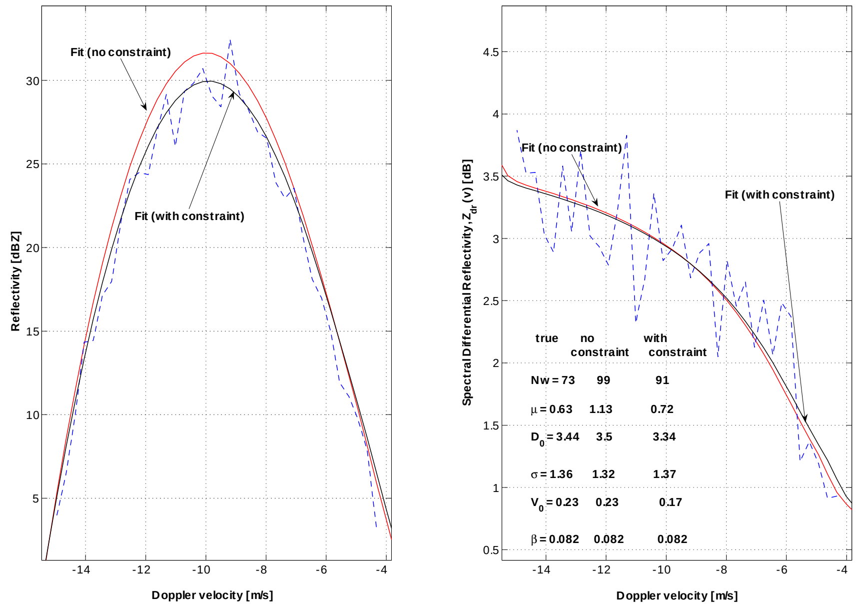

Figure 2: Examples of the fit using methods with and without $Z_{dr}(v)$ contraint .

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