15R.6 PROFILER DSD RETRIEVALS: WHAT IS THE IMPACT OF AN ADDITIONAL PROFILER?

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1. INTRODUCTION

Radar profilers are excellent tools for validation of radar rainfall retrieval algorithms. The profiling drop-size distribution (DSD) retrieval algorithms employ Doppler spectra measurements. Often it is considered that retrievals using two different profilers operating at distinct frequencies are superior to single profiler ones. However, it is not always clear what is the added advantage of employing two profilers.

This study is dedicated to quantifying uncertainties in profiling DSD retrievals and to the development of an optimum DSD retrieval algorithm that uses spectral observations carried out by two profilers. As the basis for this study we assume that DSD retrieval algorithm is based on spectral observations by S-band profiler and the second profiler can be used to constrain the retrievals. It is also assumed that DSD can be approximated by a modeled distribution and therefore DSD retrieval algorithm is a non-linear least square solution for the parameters (Williams 2002; Moisseev et al. 2005), as well as that of the DSD model. For example for a gamma model DSD parameters and are estimated.

As a starting point of this study we use sensitivity analysis to quantify uncertainties in the retrieval of the DSD parameters using an S-band profiler. From this study we determine which parameters contribute the most to the uncertainty in the retrieval. Than it is discussed how second frequency spectral observations can be incorporated to reduce errors in the DSD retrievals and what is an advantage of using two profiler DSD retrieval scheme.

2. DSD RETRIEVAL METHOD

We have used DSD retrieval technique based on S-band measurements of Doppler spectra, as the basis for this study. In this case retrieval procedure can be formulated as a minimization problem of sum squared residuals of fit of the modeled power spectrum to the measured one (Williams 2002; Moisseev et al. 2005). Where, SSR is defined as:

\[
SSR_{ds}(N_w, D_0, \mu, v_0, \sigma_b) = \sum_{v=-v_{\text{max}}}^{v_{\text{max}}} \left( \log(S_{hh,\text{mod}}(v, N_w, D_0, \mu, v_0, \sigma_b)) - \log(S_{hh,\text{meas}}(v)) \right)^2,
\]

where \(S_{hh,\text{mod}}(N_w, D_0, \mu, v_0, \sigma_b)\) is given by (2). Here we have assumed that DSD follows gamma distribution and is described by three parameters, \(N_w, D_0\) and \(\mu\) (Bringi and Chandrasekar 2001). And the modeled spectrum is given as (Moisseev et al. 2005):

\[
S_{hh,\text{mod}}(v) = S_{\text{broad}}(v) * S_{hh}(v) = S_{hh}(v)dv,
\]

where

\[
S_{hh}(v)dv = \frac{\lambda^4}{\pi^3 |K_r|^2} \sigma_{hh}(D(v)) N(D(v)) \frac{dD(v)}{dv} dv
\]

where \(\sigma_b\) is the width turbulent / cross-wind spectrum broadening kernel.

In (Moisseev et al. 2005) it was shown that this 5 parameter non-linear optimization problem can be simplified to 3 parameter nonlinear problem. This simplification is based on the observation that DSD intercept parameter, \(N_w\), and wind velocity, \(v_0\), are scaling parameters of Doppler power spectra and can be estimated separately using more direct methods. Therefore the DSD parameters retrieval can reformulated as the following optimization problem:

\[
\min_{D_0, \mu, \sigma_b} \min_{N_w, v_0} SSR_{ds}(N_w, D_0, \mu, v_0, \sigma_b)
\]

3. SENSITIVITY ANALYSIS

Now, if additional measurements are available, such as measurements using second profiler operating at a different frequency, one would need to combine...
when spectral broadening is present, due to finite  
directly related to raindrop shapes. Nonetheless,  
polarizations. In absence of the spectral broaden-  
of Doppler power spectra measured at  
S

spectral differential reectivity is dened as the ratio  
am using spectral differential reflectivity. In this  
ad/dv, the spectral differential reectivity becomes  
dependent on DSD parameters and \( \sigma_b \), as can  
be seen in Figure 1. One can observe that \( \sigma_b \)  
and \( \sigma_d \) have a noticeable effect on \( \hat{Z}_{dr} \). The  
other important parameter that affects spectral  
differential reectivity is raindrop size-shape relation. In  
this study this relation was assumed to follow a linear  
relation to the drop equivolume diameter, after  
(Gorgucci et al. 2000):  
\[
\frac{a}{b} = 1 - \beta D.  
\]  

By assuming such form of the raindrop-size shape  
relation one can take into account variability of rain- 
drops due to oscillations (Gorgucci et al. 2000).  

Therefore by incorporating \( \hat{Z}_{dr}(v) \) in the retrieval  
of DSD parameters one would add one extra unknown,  
and be able to introduce an additional constraint. In this  
case the retrieval procedure can be formulated as solution  
of (3) with constraint \( Z_{dr}^{mod}(v, \beta, D_0, \mu, \sigma_b) = Z_{dr}^{meas}(v) \).  

5. RESULTS  
To evaluate performance of the retrieval technique  
the optimization procedure was applied to simulated  
Doppler spectra. Using (2) and applying procedure  
described by Chandrasekar et al. (1986) realizations  
of hh and vv Doppler power spectra were created.  

For these simulations the input parameters were se- 
lected randomly from the intervals:  
\[
\begin{align*}
\log N_w & \in [1, 6] \\
D_0 & \in [0.5, 3.5] \quad (\text{mm}) \\
\mu & \in (-1, 5) \\
\sigma & \in [0, 1] \quad (\text{m s}^{-1}) \\
v_0 & \in [-5, 5] \quad (\text{m s}^{-1}) \\
\beta & \in [0.02, 0.09] \quad (\text{mm}^{-1}).
\end{align*}
\]

Furthermore, the simulated measurements were  
constrained to have reectivity values larger than 10  
dBZ and smaller than 55 dBZ. Prior to the retrieval,  
fifteen realizations of the hh and vv Doppler power  
spectra for a given set of input parameters were cal- 
culated and averaged to obtain an estimate of the  
true spectrum. The velocity resolution of 0.3 m/s and  
128 spectral lines were used for this simulation. The  
copolar correlation coefcient was assumed to be  
equal to 0.98. Then, the estimated spectrum is re- 
corded randomly from the intervals:  
\[
\begin{align*}
\log N_w & \in [1, 6] \\
D_0 & \in [0.5, 3.5] \quad (\text{mm}) \\
\mu & \in (-1, 5) \\
\sigma & \in [0, 1] \quad (\text{m s}^{-1}) \\
v_0 & \in [-5, 5] \quad (\text{m s}^{-1}) \\
\beta & \in [0.02, 0.09] \quad (\text{mm}^{-1}).
\end{align*}
\]

Table 1: Results of the variance decomposition of  
the retrieved parameters. This table shows dependence  
of the RMS values of DSD parameters, \( v_0 \) and  
\( \sigma \) on uncertainties in \( D_0, \mu \) and \( \sigma \). One can see that  
uncertainties in \( D_0 \) and \( \sigma \) have a large impact on  
the retrievals. Uncertainties in \( \mu \), on the other hand,  
have a very small impact.  

<table>
<thead>
<tr>
<th>output</th>
<th>( \mu )</th>
<th>( D_0 )</th>
<th>( \sigma_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log N_w )</td>
<td>0.01</td>
<td>0.51</td>
<td>0.48</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-</td>
<td>0.56</td>
<td>0.44</td>
</tr>
<tr>
<td>( D_0 )</td>
<td>0.03</td>
<td>-</td>
<td>0.97</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>0.01</td>
<td>0.51</td>
<td>0.48</td>
</tr>
<tr>
<td>( \sigma_b )</td>
<td>0.02</td>
<td>0.98</td>
<td>-</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.12</td>
<td>0.45</td>
<td>0.43</td>
</tr>
</tbody>
</table>

4. SPECTRAL DIFFERENTIAL REFLECTIVITY  
CONSTRAINT  
The CSU-CHILL radar can carry out dual- 
polarizations spectral measurement. If these  
measurements are taken with elevation angles lying  
in the interval between 30 and 70 degrees, one  
can retrieve both DSD parameters of precipitation  
and mean shapes of raindrops (Moisseev et al. 2005). Moreover, one can consider using spectral  
differential reectivity measurements, \( Z_{dr}(v) \), to add  
an additional constraint to the DSD retrieval. The  
spectral differential reectivity is deened as the ratio  
of Doppler power spectra measured at hh and vv  
polarizations. In absence of the spectral broadening,  
\( \sigma_d = 0 \), the spectral differential reectivity is  
directly related to raindrop shapes. Nonetheless,  
when spectral broadening is present, due to finite  
beamwidth and turbulence, spectral \( Z_{dr} \) becomes  
dependent on DSD parameters and \( \sigma_b \), as can  
be seen in Figure 1. One can observe that \( D_0 \)  
and \( \sigma_b \) have a noticeable effect on \( \hat{Z}_{dr}(v) \). The  
other important parameter that affects spectral differential  
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and be able to introduce an additional constraint. In this  
case the retrieval procedure can be formulated as solution  
of (3) with constraint \( Z_{dr}^{mod}(v, \beta, D_0, \mu, \sigma_b) = Z_{dr}^{meas}(v) \).
Using 300 simulated spectra we have calculated RMS errors of the retrieved parameters for two methods, the results are shown in Table 2. One can see that use of $Z_{dr}(v)$ measurements as additional constraint improves retrievals. For example, RMS errors of $D_0$ are almost two times smaller for the new method.

<table>
<thead>
<tr>
<th>parameter</th>
<th>RMS</th>
<th>RMS (constr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log N_w$</td>
<td>0.71</td>
<td>0.42</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>$D_0$</td>
<td>0.5</td>
<td>0.27</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.1</td>
<td>0.06</td>
</tr>
<tr>
<td>$v_0$</td>
<td>0.7</td>
<td>0.43</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.01</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Table 2: Resulting RMS values for the retrieved parameters using constrained and not-constrained retrieval methods.

6. CONCLUSIONS

In this study sensitivity analysis of DSD parameters retrievals using S-band spectral measurements was carried out. It is shown that uncertainties in $D_0$ and $\sigma_b$ have the largest effect on the output values. Therefore, it is advantageous to use data from a second profiler to constrain retrievals of these two parameters. By the data from a second profiler we presume an additional measurements either carried out at a different frequency (or polarization) that bring new information into our retrieval. Using the results of the sensitivity analysis a new retrieval method was proposed. In this method dual-polarization spectral measurements are used for retrieval of DSD parameters. It was proposed to constrain the retrievals using spectral differential reflectivity measurements. It was shown that using this new method errors of the retrieved values can significantly be reduced.
Figure 2: Examples of the fit using methods with and without $Z_{dr}(v)$ constraint.

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**References**


