FORWARD AND BACKSCATTERING MEASUREMENTS OF RAINFALL OVER A PATH AT THE GPM FREQUENCIES

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1. INTRODUCTION

This paper describes the theoretical basis and conceptual design of a dual-frequency radar and microwave link (radar/link) system capable of measuring simultaneously forward and backscatter from rainfall. The forward and backscattering radar/link system will enable small-scale rain studies essential in understanding the rainfall process, and make significant contributions to the investigation of radar inversion algorithms for the estimation of precipitation parameters that include liquid water content, median mass diameter, number concentration, and rainfall rate. The system will operate at the frequencies of 13.6 GHz and 35.5 GHz enabling the testing and validation of established radar retrieval algorithms such as those used by the Tropical Rain Mapping Mission (TRMM) single-frequency radar, and those proposed for the Global Precipitation Mission (GPM) dual-frequency radar.

The dual frequency radar/link system will be deployed at the NASA/Wallops Island microwave link facility [1], where it will operate in conjunction with a network of disdrometers, tipping buckets, and optical rain gauges to validate the measurements and test the rain retrieval techniques [1]. This configuration will also serve as an ideal validation site for radar rainfall retrievals such as the GPM over flight rainfall estimates.

2. THEORY

The radar/link dual-frequency measurements permit the application of inversion techniques to estimate parameters of drop size distribution (DSD). In turn, the DSD obtained from the inversions can be used to compute important rainfall parameters. The following inversion methods are based on the non-Rayleigh scattering characteristics in the interaction of microwaves with raindrops exhibited when the signal wavelength is comparable to the size of the raindrops. In these methods, at least one of the frequencies must be in the Mie scattering regime so that the signal attenuations, or reflectivity factors, at two frequencies are linearly independent.

2.1. Microwave link Dual-wavelength Inversion technique

The microwave link analysis assumes that the path-average DSD can be represented by a two-parameter gamma distribution given by

$$\bar{N}(D_e) = \frac{1}{L}\int_0^L N(D_e, z')dz' = \bar{N}_o D_e^{2} \exp(-\bar{\Lambda} D_e) \ , \qquad (2.1)$$

where $\bar{N}_o$, and $\bar{\Lambda}$ are path-average parameters of the DSD, and D_e is the equi-volumetric drop diameter.

The following approach makes use of attenuation at 13.6 and 35.5 GHz to solve for $\bar{N}_o$ and $\bar{\Lambda}$.

The path-average attenuation, A (dB), for a signal propagating through rain can be expressed as [1]

$$A_\nu = 8.686 \lambda_\nu L \bar{N}_o \int_{D_{\min}}^{D_{\max}} \mathrm{Im}[f_\nu(D_e)] D_e^{2} \exp(-\bar{\Lambda} D_e) dD_e \ , \qquad (2.2)$$

where ν denotes the signal frequency, λ_ν is the wavelength, L is the signal path-length (m), Im[$f_\nu(D_e)$] (m) is the imaginary components of raindrop forward scattering amplitude, $\bar{N}$ (m $^{-4}$) is the path-average drop size distribution, and D_{max} and D_{min} are the maximum and minimum drop sizes in the distribution.

Using (2.2) and taking the ratio of attenuation at 13.6 and 35.5 GHz yields

$$\begin{aligned} &\frac{A_{35}}{A_{13}} \frac{\lambda_{13}}{\lambda_{35}} \int_{D_{\min}}^{D_{\max}} \mathrm{Im}[f_{13}(D_e)] D_e^{2} \exp(-\bar{\Lambda} D_e) dD_e \\ &- \int_{D_{\min}}^{D_{\max}} \mathrm{Im}[f_{35}(D_e)] D_e^{2} \exp(-\bar{\Lambda} D_e) dD_e = 0 \end{aligned} \ . \qquad (2.3)$$

Equation (2.3) is a transcendental equation in $\bar{\Lambda}$ which can be solved using standard numerical techniques. Since the A_{13} and A_{35} are linearly independent, the roots of this equation yield the value of $\bar{\Lambda}$ for a given attenuation ratio A_{35} / A_{13}. The forward scattering amplitudes f_{13} and f_{35} for non-

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spherical raindrops need to be computed numerically. The numerical method employed would depend on the assumption of raindrop shape. For oblate raindrop shapes, the T-matrix approach is commonly employed [2].

Having obtained $\bar{\Lambda}$, the value of $\bar{N}_o$ can be found from (2.2). The DSD obtained from this technique can be used to compute path-average rainfall parameters such as liquid water content, median mass diameter, number concentration, and rainfall rate.

2.2. Radar dual-wavelength Inversion technique

This approach makes use of measured reflectivity factors and path-integrated attenuation at 13.6 and 35.5 GHz to solve for N_o and Λ.

The radar analysis assumes that the DSD can be represented by a two parameter gamma distribution

$$N(D_e, r) = N_o(r) D^2 \exp(-\Lambda(r) D_e) \,, \qquad (2.4)$$

where N_o and Λ are the free parameters of the DSD, which are functions of the range *r*.

The effective reflectivity factor, Z^e, from a volume of raindrops at a distance *r* from the transmitter can be expressed as [3]

$$Z^e_\nu(r) = C_\nu N_o(r) \int_{D_{\min}}^{D_{\max}} \sigma_{b\nu}(D_e) D_e^{\,2} e^{-\Lambda(r) D_e} dD_e \,, \qquad (2.5)$$

where $C_\nu = \lambda^4 / (\pi^5 |K_\nu|^2)$, $K = (\varepsilon_\nu - 1) / (\varepsilon_\nu + 2)$, ε_ν is the dielectric constant of a raindrop, and σ_b is the backscattering cross-section.

The measured reflectivity factor, Z^m, is related to Z^e by

$$Z^m_\nu = A_\nu(r) Z^e_\nu(r) \,. \qquad (2.6)$$

Using (2.5) and (2.6), and taking the ratio of measured reflectivity factors at 13.6 GHz and 35.5 GHz yields

$$\begin{aligned} &\frac{Z^m_{35}}{Z^m_{13}} A_{13}(r) \int_{D_{\min}}^{D_{\max}} \sigma_{b13}(D_e) D_e^{\,2} e^{-\Lambda(r) D_e} dD_e \\ &- A_{35}(r) \int_{D_{\min}}^{D_{\max}} \sigma_{b35}(D_e) D_e^{\,2} e^{-\Lambda(r) D_e} dD_e = 0 \end{aligned} \,. \qquad (2.7)$$

Equation (2.7), which is similar to (2.3), is a transcendental equation in Λ which can be solved using standard numerical techniques for a given ratio of reflectivity factors Z_{35} / Z_{13}. However, in order to solve (2.7), the attenuations A_{13} and A_{35} must be known for any given *r*.

One approach to solve (2.7) is by measuring the attenuations at the farthest point of the radar range of interest, and then estimating the DSD progressively in each cell for all range gates using a backward recursion algorithm [3]. However, measuring the attenuation with space borne radars (the same applies to ground based radars) cannot be accomplished directly, and one must resource to ancillary techniques, such as the surface reference technique, to estimate the path-average attenuation [4].

As in the microwave link case, the backscattering cross-section and the scattering amplitudes in (2.7) must be computed numerically for non-spherical raindrops.

3. SYSTEM DESCRIPTION

The dual-frequency radar/link system will measure the forward and the backscattered signals from rain simultaneously using a combination of radar and microwave link techniques. This approach will allow to solve equation (2.7) can be solved readily with out the introduction of uncertainties from estimates of attenuation.

The radar/link system will be deployed at the NASA/Wallops Island microwave link facility where it will operate in conjunction with a ground network of impact disdrometers, tipping buckets, and optical rain gauges that provide the DSD spectra, rainfall accumulation, and rainfall rate directly under the microwave signals path. The ground network will not only help validating the measurements and testing the rain retrieval techniques, but it will also help evaluate any differences between elevated-path-averages estimates and point ground measurements, and to study the DSD time evolution and spatial variability over the microwave path.

A block diagram of the radar and the link transmitter is shown in figure 1, and the microwave link receiver is shown in figure 2. The line-of-sight, microwave link system will consist of a two-channel transmitter and receiver system characterized by coherent operation at the frequencies of 13.6 GHz, and 35.5 GHz. The microwave link will share the radar transmitter (fig. 1), and a use a coherent receiver (fig. 2) synchronized to the incoming radar signals. The system will measure path-integrated attenuation ($A_{13.6}$ and $A_{35.5}$) and path integrated frequency phase-shift ($\Phi_{35.5\text{-}13.6}$).

The radar will employ Frequency Modulated Continuous Wave (FMCW) signals to measure rainfall backscatter, and provide reflectivity profiles along the path. Both 13.6 GHz and 35.5 GHz signals will be transmitted simultaneously (fig 1).

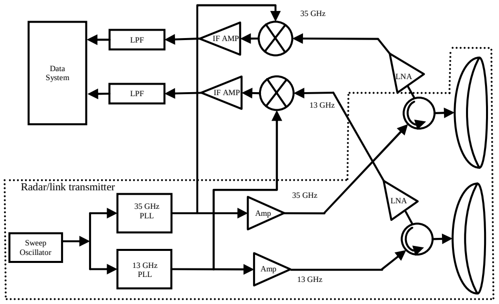

Figure 1. FMCW radar system / Microwave link transmitter

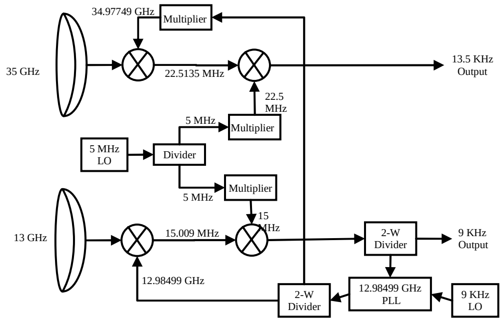

Figure 2. Microwave link receiver

The radar range resolution will be determined by the choice of three FMCW parameters. These are the sweep bandwidth, sweep duration, and IF filter bandwidth. The transmitter will generate a frequency sweep of bandwidth $B = f_{max} - f_{min}$ (Hz), during a time T_s (sec). Each sweep is repeated at interval T_s as shown in figure 1.

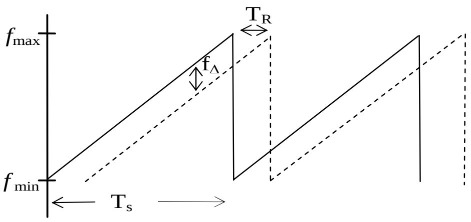

Fig. 3 Radar frequency: transmit (—) and receive (---)

The transmit output signal can be expressed as

$$s_{tx}(t) \sim \cos(2\pi f_o t + \pi \frac{B}{T_s} t^2) \ , \qquad (3.1)$$

where f_o is the transmitted signal frequency.

The signal scattered from a rain volume located at a distance r_1 (m) from the transmitter returns to the radar at a time $T_R = 2r / c$

$$s_r(t) \sim s_{tx}(t - \frac{2r_1}{c}) \sim \cos\left[2\pi f_o (t - \frac{2r_1}{c}) + \pi \frac{B}{T_s}(t - \frac{2r_1}{c})^2\right]$$

$$. \qquad (3.2)$$

Upon return, the signal is coherently detected by mixing it with the transmitter signal. After mixing, the signal is run through a low pass filter to get rid of the high frequency components. The output signal s_o is

$$s_o(t) \sim \cos(\frac{4\pi f_o r_1}{c} + \frac{4\pi B r_1 t}{cT_s} + \frac{4\pi B r_1^2}{c^2 T_s}) \ , \qquad (3.3)$$

which has a frequency

$$f_1 = \frac{2Br_1}{cT_s} \ . \qquad \text{(Hz)} \qquad (3.4)$$

Equation (3.4) can also be obtained by inspection from fig. 1 by noting that $B / T_R = f_\Delta / T_R$.
Similarly, the frequency of a signal return from a contiguous rain volume located a r_2 is $f_2 = {2Br_2}/{cT_R}$. The bandwidth of the IF filter, B_{IF}, at the receiver required two resolved the frequencies is

$$B_{IF} = f_2 - f_1 = \frac{2B\Delta r}{cT_s} \ . \qquad (3.5)$$

where $\Delta r = (r_2 - r_1)$ is the range resolution.

Re-writing (3.5) yields the FMCW range resolution,

$$\Delta r = \frac{cT_R B_F}{2B} \ . \qquad (3.6)$$

A comparison of the resolution between the FMCW radar and a pulse radar with an equivalent pulse width can be made using (4.6), and by noting that the IF filter bandwidth, B_F, should be always less than or equal to sweep bandwidth, B. Using this fact in equation (3.6) yields

$$\Delta r = \frac{cT_s B_F}{2B} <= \frac{cT_s}{2} = \frac{c\tau}{2} \qquad (3.7)$$

Hence by choosing B_F smaller than B the FMCW radar range resolution can be significantly improved from a pulse radar with am equivalent pulse width τ.

From figure 3, it can be seen that the maximum unambiguous time for a signal to return to the receiver is Ts. Hence, the maximum unambiguous range is $r_{max} = cTs / 2$. For a system deployment over a path of 2.3 km, such as the one at the NASA/Wallops Island microwave link facility [1], a desired unambiguous range to avoid aliasing in the time domain will be in the order of 50 km. This yields Ts = 33.3 ms. IF a range resolution in the order 250 meters is required (which is roughly the separation of validation sites at the Wallops facility), along with a sweep bandwidth of 5 MHZ (the microwave link receiver bandwidth), the IF filter bandwidth would be 250 Hz.

The IF filter will be implemented in the radar processor, where the return from each frequency will be coherently detected, Fast Fourier transformed, and gated to give the desired response.

4. TRMM AND GPM ALGORITHM VALIDATION

Using the dual-frequency radar/microwave link system, the single frequency TRMM Precipitation Radar (PR) algorithm, as well as, the proposed dual frequency GPM algorithm can be tested in an ideal setting. The present TRMM PR algorithm employs a combination of two methods depending on storm conditions. In light rain rates, a single frequency Hitschfeld-Bordan (HB) technique is used to correct the measured reflectivity factor for attenuation [5]. The HB is forward algorithm which becomes error prone as the rain intensity increases. In moderate to heavier rains, and a modified HB algorithm is employed which uses as a constraint, the path-integrated value of total attenuation. This is obtained by a surface reference technique [4].

The 13.6 GHz signal of the dual-frequency radar/microwave link system will be used to test the TRMM PR algorithm. The rain gauges under the propagation path will provide data on rain rates along the path for comparison with the estimated algorithm values. The total path integrated attenuation between the transmitter and receiver is available from the microwave link measurements at the same frequency for use in the algorithm. Two immediate advantages of the implementation are that the rain gauges are directly under the radar backscatter cells and the total attenuation values are reliably measured by the Link. Thus, the uncertainty of surface backscattering coefficients does not enter into the calculation. Understanding the TRMM PR algorithm under ideal conditions will help bound errors. Although the TRMM mission is near its completion, the GPM satellite will use the single frequency algorithms in the outer swath where the Ka-band data will not be available or in the inner swath when the Ka band attenuation leads to a loss of signal. It is also likely that single-wavelength processing at the Ku-band channel will continue in GPM so that a consistent long-term Ku-band data record will be available for investigators.

The GPM will employ a Dual-frequency Precipitation Radar (DPR) at the frequencies of 13.6 and 35.5 GHz. In light rains, the GPM measured reflectivity factors can be corrected for attenuation in a forward stepping mode as the DSD is progressively estimated in each cell. In heavier rains, a total path-integrated attenuation can be used so that the DSD is estimated in the backward direction from the surface to the top of the storm [4].

The dual frequency algorithms can be implemented by using the radar/link frequencies. Direct estimates of total path-integrated attenuation can be obtained for each of the frequencies from the Link's forward propagation measurements. The DSDs that are predicted along the Link path -by using the DPR algorithm- can be compared to the DSDs from the disdrometers on the ground. The measurements will help us understand the uncertainties of the method under near-ideal conditions and help us determine which parameterization of the DSD leads to the smallest error. These measurements will also be useful in determining when to employ the forward or backward algorithm. Since the forward algorithm does not require the total path-integrated attenuation, it is desirable to use it when this measurement is either unavailable or unreliable. More generally, multi-wavelength measurements of the backscattered power and path attenuation can be used to assess most of the proposed dual-wavelength techniques.

5. CONCLUSION

The dual-frequency radar/microwave link system presented here is capable of measuring simultaneously forward and backscatter from rainfall at 13.6 and 35.5 GHZ, by using a combination of radar and microwave link techniques. The radar/link measurements permit the implementation of inversion algorithms for the estimation of the drop size distribution, and are useful in the validation of rainfall retrieval algorithms such as those employed by TRMM and GPM. The radar/link concept will also enable small-variability rainfall studies of important rainfall parameters. This configuration will also serve as an ideal validation site for radar rainfall retrievals such as the GPM over flight rainfall estimates.

6. REFERENCES

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