On plume rise – matching Daysmoke with Briggs Equations for industrial stacks

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The Briggs Equations for plume rise from industrial and power plant stacks have been extensively validated with stack plume observations. Daysmoke, a plume rise model developed for the simulation of smoke plumes from prescribed burns, was matched with the Briggs Equations for a wide range of plume diameters and for stack-level wind speeds ranging from 2.5 m sec^{-1} to 10 m sec^{-1}. Daysmoke plumes match Briggs plume centerlines almost perfectly when Daysmoke entrainment coefficients are adjusted for the degree of plume "bentoverness" and wind speed. The results show that Daysmoke obeys the so-called 2/3 law for plume rise even though the 2/3 law is not explicitly formulated in Daysmoke.

1. INTRODUCTION

Southern forest land managers understand that prescribed fire is the most economical way to reduce fuels, remove nutrient-competing species, and manage for threatened and endangered species. Because Southern forests are dynamic ecosystems characterized by rapid growth – hence rapid deposition of fuels - within a favorable climate, the fire-return interval (3-5 years) is among the highest in the Nation. Approximately 6-8 million acres (2-3 million ha) of forest and agricultural lands are burned in the South each year (Wade et al., 2000).

The importance of prescribed fire in air quality and forest health makes it critical that air quality models incorporate the impacts of smoke accurately. One requirement is that the human element – how the burn is engineered and when the burn is conducted – must be part of air quality modeling. Prescribed fires are not simple ground sources of smoke. These fires, particularly the larger fires that contribute most to the fire emissions inventories of the South, are designed to place as much smoke as possible above ground – often above the planetary boundary layer.

With the goal of accurately representing forestry activities in air quality models, the Smoke Management Team of the U.S. Forest Service Southern Research Station has developed a coupled prescribed fire-air chemistry modeling framework called the Southern Smoke Simulation System (SHRMC-4S, Achtemeier et al. 2003). SHRMC-4S simulates and predicts chemical concentrations of smoke components and assesses their effects on regional air quality by using the EPA Models-3 Community Multi-Scale Air Quality (CMAQ) modeling system (Byun and Ching, 1999), with some modifications for time and location for point source fire emissions. A key component of SHRMC-4S is Daysmoke, a plume model designed to model prescribed burns in the manner they are engineered by southern land managers.

Daysmoke has been tested with CMAQ in SHRMC-4S and the results have been sufficiently encouraging to warrant a thorough re-derivation of the model for expanded use (Achtemeier, et al. 2005). DAYSMOKE consists of the following four models: (a) Entraining Turret Model. The plume is assumed to be a succession of rising turrets. (b) Detraining Particle Model. Movement within the plume is described by the horizontal and vertical wind velocity within the plume, turbulent horizontal and vertical velocity within the plume, and particle terminal velocity. Detrainment occurs when particles are found beyond plume boundaries. (c) Large Eddy Model. Eddies are two-dimensional and oriented normal to the axis of the mean layer flow. (d) Relative Emissions Model. Plume pathways from the entraining turret model are assembled like pipes of a pipe organ. Particles from the detraining particle model are passed through the pathways at specified time intervals to approximate the growth and demise of prescribed burns.

This paper reports on a comparison of plume pathways developed by the Entraining Turret Model with plume centerlines produced by the Briggs equations (Briggs, 1965).

THE DAYSMOKE ENTRAINING TURRET MODEL (ETM)

The purpose of the Entraining Turret Model is to define the "plume pathway", a three-dimensional volume swept out by a rising cylinder of heated air that expands through entrainment of surrounding air as it ascends (Figure 1).

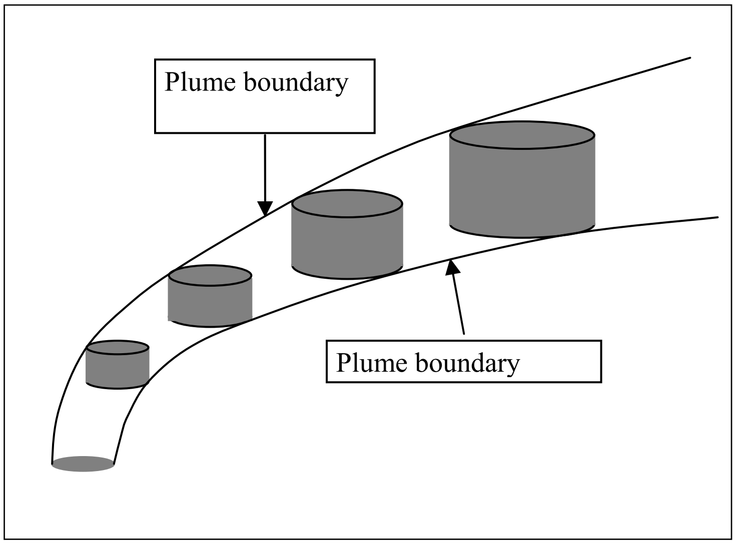

Figure 1. A schematic showing the boundaries of a smoke plume defined by rising, entraining turrets.

Buoyancy and entrainment are the two physical constraints imposed on the rising turret. Buoyancy of the heated air accelerates the turret to faster vertical velocities as the turret ascends. Buoyancy is countered by entrainment which acts to retard turret rise in two ways. First, mixing with outside air decreases turret temperature and reduces buoyancy. Second, mixing of air with zero vertical velocity decreases the vertical velocity of the turret.

The ETM is not a time-dependent model. A plume pathway remains fixed once derived. (Time-dependency enters through the Detraining Particle Model and the Relative Emissions Model.) The change in the volume of a rising turret by entrainment of ambient air as it passes from height z-1 to z is,

$$V_z = V_{z-1} + \Delta V$$
$$where \qquad (1)$$
$$\Delta V = \pi(r+\Delta r)^2(h+\Delta h) - \pi r^2 h$$

The change in turret volume may be visualized with the aid of Figure 2 as taking place in three parts. Thus,

$$\Delta V = annulus1 + cylinder2 + annulus3$$
$$where$$
$$annulus1 = \pi h(r+\Delta r)^2 - \pi h r^2 \qquad (2)$$
$$cylinder2 = \pi r^2 \Delta h,$$
$$annulus3 = \pi \Delta h(r+\Delta r)^2 - \pi r^2 \Delta h$$

Entrainment of heat and momentum (horizontal and vertical) is a function of the slope of the plume. For example, if the horizontal wind speed is zero (the plume rises vertically), all of the material entrained through the bottom and top of the turret is plume air while entrainment of ambient air takes place through the sides. As the plume tilts in the presence of wind, more ambient air is entrained through the bottom and top of the turret until, if the plume blows horizontally, all air entrained into the turret is ambient air.

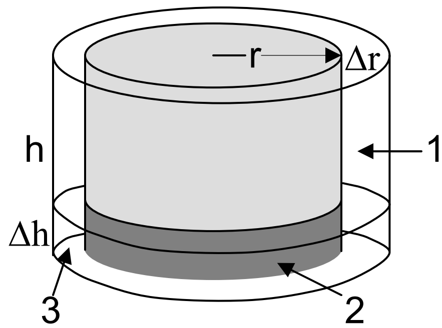

Figure 2. A breakdown of an expanding cylinder into three components: (1) an annulus around the original cylinder, (2) a cylinder added to the original cylinder, and (3) an annulus around the added cylinder.

Let Q_e represent an ambient constituent, and Q_{z-1} represent the plume constituent surrounding the turret at z-1. The constituent Q_z resulting from the combination of the original turret with the entrained volume of mixed constituents is,

$$Q_z V_z = Q_{z-1} V_{z-1} + Q' \Delta V \qquad (3)$$

Where Q' is found by weighting annulus1 by Q_e, weighting cylinder2 by $a_1Q_{z-1} + a_2Q_e$, and weighting annulus3 by $0.5(a_1Q_{z-1} + a_2Q_e + a_3Q_e)$. This definition for Q' requires that all air entrained into annulus 1 carries the ambient constituent; air entrained into cylinder 2 is a weighted sum of ambient and plume constituents; and air entrained into annulus 3 is the average of the mixture entrained into cylinder 2 and the constituent entrained into annulus 1. The three constants are chosen so that $a_3 = a_1 + a_2 = 1$. Furthermore,

$$a_2 = \frac{2}{\pi} A\tan\left|\frac{u}{w}\right|; (w \neq 0) \quad a_1 = 1 - a_2 \qquad (4)$$

From equation (4), if the horizontal wind is zero, the plume stands vertically, $a_2 = 0$, $a_1 = 1$, and entrainment from below is plume air. If the vertical velocity approaches zero, the plume drifts horizontally, $a_2 = 1$, $a_1 = 0$, and entrainment from below is ambient air. If u = w, the plume bends over to a 45 degree orientation, $a_2 = 0.5$, $a_1 = 0.5$, and entrainment is equally divided between ambient and plume air.

The ETM is subjected to two assumptions needed to make the derivation tractable. First, the changes in volume will be equally distributed between deepening and expanding the turret. Second, the changes in volume will be functions of the rate of turret rise. Therefore,

$$\Delta h = \Delta r = e\ w_{z-1} \Delta t \qquad (5)$$

where e is an entrainment coefficient. The second assumption constrains turret expansion so that a rapidly rising turret entrains rapidly and a slowly rising turret entrains slowly. Inclusion of (5) into equations (1), (2), and (3) gives a general algorithm for turret growth and for the evolution of constituents within the turret,

$$Q_z V_z = Q_{z-1} V_{z-1} + \pi \left\{ \begin{array}{l} (e_p w_{z-1} \Delta t)[r^2 (a_1 Q_{z-1} + a_2 Q_e) + 2rhQ_e] + (e_p w_{z-1} \Delta t)^2 [2r(a_1 Q_{z-1} \\ + a_2 Q_e + Q_e)/2 + hQ_e] + (e_p w_{z-1} \Delta t)^3 (a_1 Q_{z-1} + a_2 Q_e + Q_e)/2 \end{array} \right\} \quad (6)$$

Setting Q = T (temperature) in (6) yields an equation for turret temperature for buoyancy calculations. Setting Q = (u, v, w) (any of the velocity components) yields an equation for plume drift. Setting Q = 1 yields an equation for the volume growth of the turret.

Solving Equation (6) is an initial value problem. Initial conditions for plume temperature, rise rate, and volume start the plume rising through a veering/shearing horizontal wind field within a stratified atmosphere. Once the initial conditions are specified, (6) is solved numerically to yield the three-dimensional structure for the plume boundary as shown in Figure 1 and thus the plume pathway. In addition, the rise rate, once the plume temperature is gotten through (6), is adjusted for buoyancy through

$$w_z = w_{z-1} + g \frac{T_z - T_e}{T_e} \Delta t \quad (7)$$

The ETM carries four parameters that must be specified for the calculations. These are the entrainment coefficient (e), the effective plume diameter (D), the initial plume vertical velocity (w), and the initial difference between plume and ambient temperatures (ΔT).

<u>*Comparison between the Entraining Turret Model and Briggs Theory*</u>

In Daysmoke the conceptual initial plume behavior and the initial plume conditions for the ETM are not unlike those developed for power plant stacks by Briggs (1965) and others. Therefore the well-documented Briggs equations can be used to calibrate the entrainment coefficient in Daysmoke. This study used a version of the Briggs formulation provided by Stern (1968), and modified by Fisher, et al. (2001).

$$h = \left(\frac{3}{2\beta^2} \right)^{1/3} F^{1/3} x^{2/3} / U$$

$$where \quad (8)$$

$$F = \frac{g \Delta T w d^2}{4 T_s}$$

Here,
x = the horizontal distance along the plume centerline measured downwind from the stack.
U = the mean horizontal wind speed (m sec^{-1}) for the layer of the atmosphere containing the plume.
ΔT= the difference in temperature (C) between the plume and the environment at the stack.
w = the stack gas ejection speed (m sec^{-1}).
d= the internal exit diameter (m) of the stack.
T_s= the temperature of the plume at the stack (K).
h= the height of the plume axis above the source (stack) (m).
β=the entrainment coefficient (conventionally, 0.6) for the Briggs model, dimensionless.

D was substituted for d in Equation (8) while knowing that the effective plume diameter in Daysmoke (already an approximation of noncircular combustion areas) may be dynamically different from the internal exit diameter of the stack. The mean horizontal wind speed, U, was assigned throughout the depth of a neutral stability layer calculated from a National Weather Service rawinsonde sounding taken on 1200 GMT, 6 March 2002, at Peachtree City, Georgia, and modified by substituting for the surface temperature the late afternoon temperature (18C) observed at Macon, Georgia, and requiring the minimum lapse rate of temperature to be no less than the dry adiabatic lapse rate.

With the entrainment coefficient assigned at 0.6, and the plume-ambient temperature difference and the vertical velocity coefficients defined as w = ww = ΔT= tt = D/10, Daysmoke was compared with the Briggs equation as functions of the effective plume diameter, D. The results revealed that Daysmoke consistently underestimated the plume rise, meaning that Daysmoke plume pathways were more “bent-over” relative to Briggs plume centerlines calculated from Equation (8). However, the Briggs equations are empirically derived whereas Daysmoke is a dynamical model. Entrainment in Daysmoke is a function of the “bent-overness” of the plume. Equation (4), designed to account for the fraction of constituents entrained into each turret, does not account for this functionality.

Daysmoke and Briggs plumes can be matched by multiplying $\beta$by a fraction. Figure 3 shows the distribution of multipliers of the entrainment coefficient required to match Daysmoke and Briggs plume centerlines. The matching was done at the top of the neutral stability layer (1700m) or at a horizontal distance of 3 km, whichever came first. The calculations were done for three stack-level mean wind speeds, U, and for effective plume diameters ranging from 15 m (w = 1.5 m sec^{-1}; ΔT = 1.5C) to 200 m (w = 20.0 m sec^{-1}; ΔT= 20.0C). These plume diameters should represent a range from small stacks to large power plant stacks and also cover the range of wildland fires for all except the largest prescribed fires and large wildfires. The range of windspeeds covers the most frequent meteorology for both stacks and fires. High-end exceptions are tall stacks in fast winds and wind-driven wildfires. Low-end (near calm) exceptions become ill-posed by Equation (8) as U–>0. The plume becomes vertical and any choice for the entrainment coefficient will match Daysmoke and Briggs centerlines.

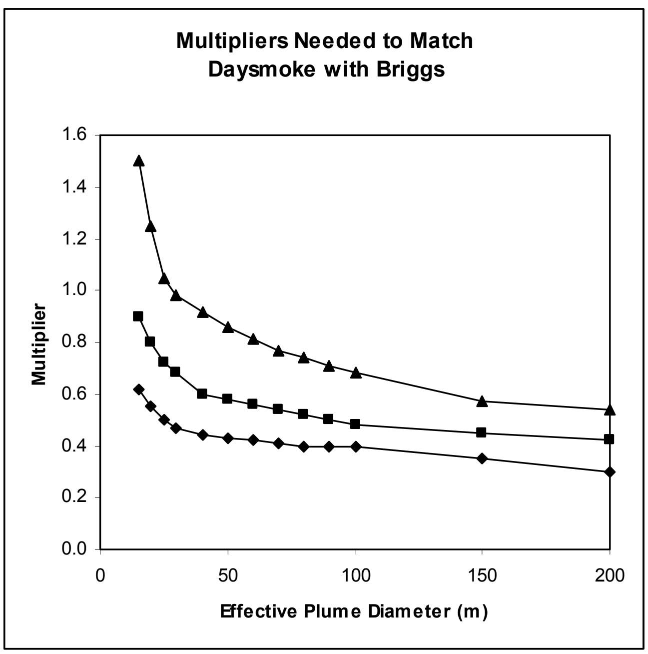

Figure 3. Entrainment coefficient multipliers required to match Daysmoke and Briggs plume centerlines. Mean boundary layer winds are, respectively, 2.5 m sec^{-1} (diamonds), 5.0 m sec^{-1} (squares), and 10.0 m sec^{-1} (triangles).

Figure 3 shows that the entrainment coefficient multipliers range from 0.30 for the slightly bent-over plume (D = 200 m) under light winds (2.5 m sec^{-1}) to 1.50 for the highly bent over and small plume (D = 15 m) under strong winds (10.0 m sec^{-1}). This produces entrainment coefficients for Daysmoke ranging from 0.18 to 0.90. Briggs (1975) surveyed the available literature to find entrainment coefficients for vertical plumes in calm air in neutral conditions to range from 0.080

for “jets” (high momentum plumes of low buoyancy) to 0.155 for buoyant plumes. For bent-over plumes, Briggs reported entrainment coefficients in the range from 0.52 to 0.66. From Figure 4, with the exception of the two most highly bent-over plumes with 10 m sec^{-1} wind speeds, all of the entrainment coefficients calculated for Daysmoke fell within the range of entrainment coefficients reported in the literature.

The above argument may be quantified as follows. Let e = mβ where the multiplier, m, is given by,

$$m = a_1 + [a_2 + a_3(u - u_0)]\left(\frac{u}{w}\right)^{1/2} \quad (9)$$

where,

u = U is the horizontal wind speed at the initial (stack) level, (m sec^{-1}), but is not necessarily equal to the mean wind speed within the neutral layer.

u_0 = U_0 is a reference horizontal wind speed at the initial (stack) level, (5 m sec^{-1})

w = the plume vertical velocity at the initial (stack) level, (m sec^{-1})

a1 = 0.25, dimensionless

a2 = 0.33, dimensionless

a3 = 0.02, (m sec^{-1})$^{-1}$

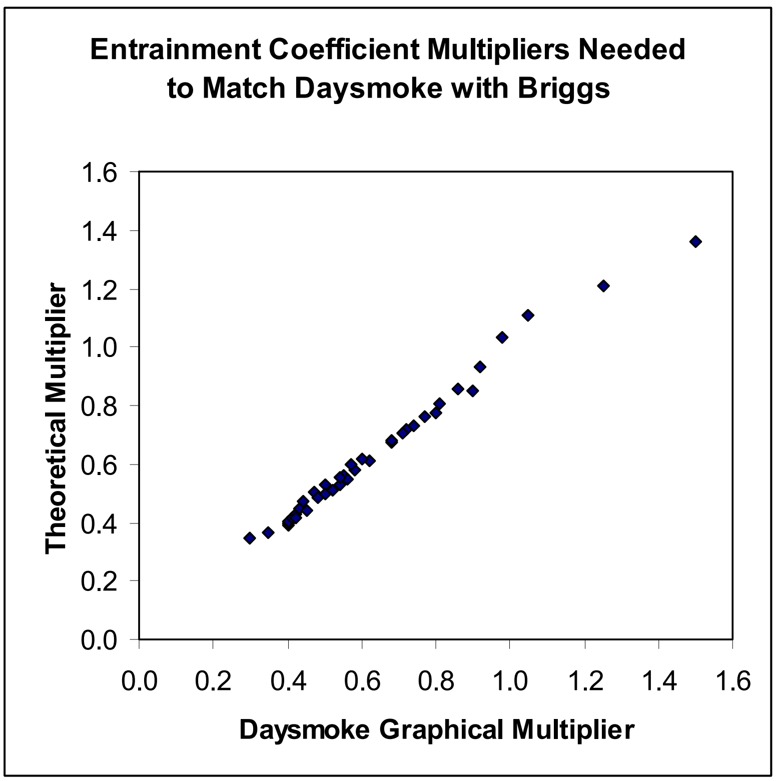

Figure 4. Comparison of entrainment coefficient multipliers obtained by Equation (11) with multipliers obtained by graphical matching of Daysmoke and Briggs plume centerlines.

Figure 4 shows how well Equation (9) reproduces the multipliers that were obtained through graphical matching of Daysmoke and Briggs plume centerlines for the range of effective plume diameters and initial (stack level) wind speeds given in Figure 4. The initial vertical velocity is the only plume parameter in Equation (9). Since the vertical velocity is specified as a function of D (although it does not have to be), then all four of the assignable parameters for Daysmoke plume pathways are functions of D and therefore, functions of initial plume volume flux from SHRMC-4S.

A second analysis matched the slopes of the Daysmoke and Briggs plume centerlines from origin to terminus. The Briggs plume centerlines fell entirely within Daysmoke plume pathways for all 39 plumes in Figure 3. Figure 5 shows the Daysmoke plume pathway for D=90 m as calculated for the neutral stability layer for the modified rawinsonde sounding of 6 March 2002. The Briggs plume centerline extends from the origin out to 3 km. Daysmoke plumes therefore closely follow the 2/3 law even though the 2/3 law is not explicitly formulated in the model.

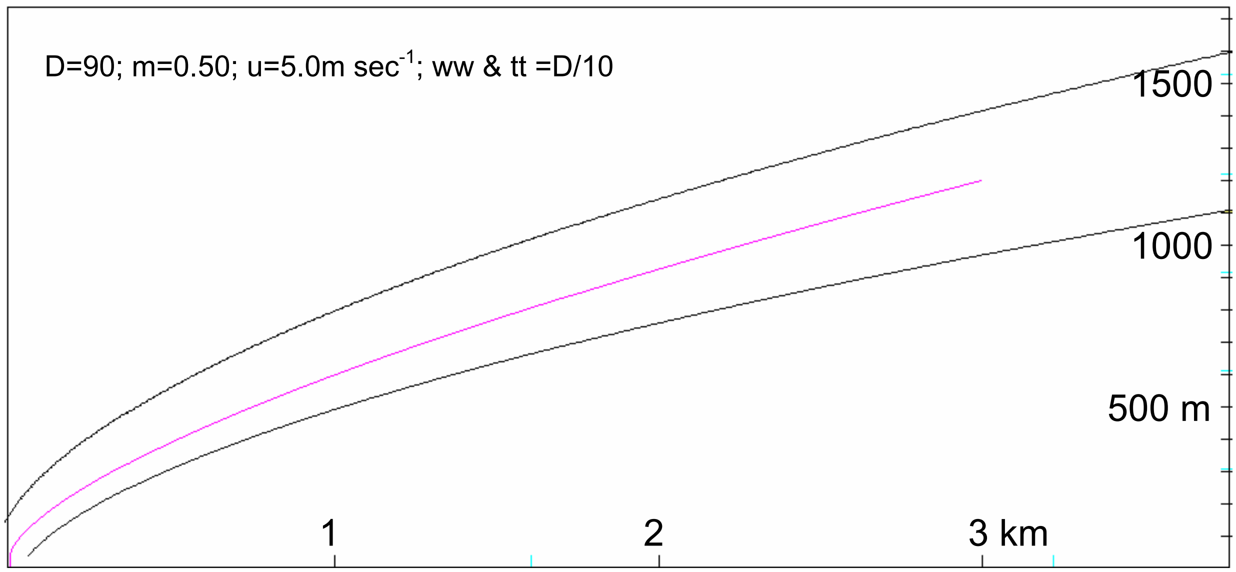

Figure 5. Daysmoke plume pathway and Briggs plume centerline to 3 km for modeled plume with effective plume diameter equal to 90 m.

It was convenient to define the Daysmoke initial temperature anomaly and vertical velocity as functions of the effective plume diameter to simplify the match with the Briggs equations. However, solutions exist for which Daysmoke does not match Equation (8) nor should it always be expected to do so. An example is a “jet” of near zero buoyancy air. Equation (8) yields zero plume growth for non-buoyant plumes regardless of the initial ejection velocity (although other equations do approximate the growth of non-buoyant plumes). Daysmoke calculates ejection velocity independently from buoyancy and therefore does yield a solution for the non-buoyant jet. Figure 6 shows the Daysmoke solution for D = 50, e = 0.255, and w = 10 m sec^{-1}. The initially vertical plume bends over rapidly within the first 300 m of transport then rises slowly thereafter as the plume momentum never goes to zero. The average vertical velocity from 1-3 km is 0.45 m sec^{-1}. This plume does not obey the 2/3 law. The solution from Equation (8) is the straight line from the origin to 3 km.

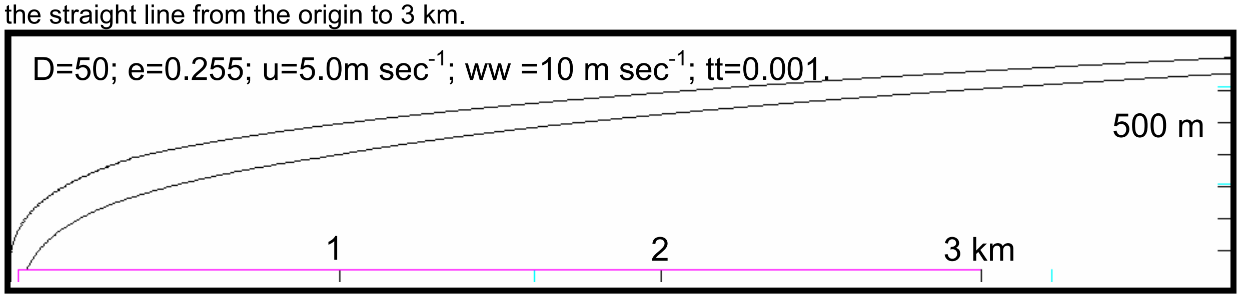

Figure 7. Daysmoke simulation for a non-buoyant jet.

SUMMARY

In summary, the ETM creates plume pathways via the above described theory subject to the values of four assigned variables. These are the entrainment coefficient (e), the effective plume diameter (D), the initial plume vertical velocity (w), and the initial difference between plume and ambient temperatures ($\Delta\tilde{T}$). Part of the above development has been devoted to finding ways to represent these four variables. As regards the entrainment coefficient, a theorem was developed that matches ETM entrainment coefficients through the Briggs equations. This development allowed the replacement of e by e_0 thus partially removing one degree of freedom. However, it is assumed that "bent overness" in smoke plumes from prescribed fires functions as does "bent overness" in plume from industrial stacks. Furthermore, the choice $e_0 = 0.6$ may not be applicable to smoke plumes from prescribed fires. More appropriate values for e will await validation of Daysmoke with observations of plume behavior. Finally, the Daysmoke solutions closely approximate the 2/3 law.

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