2.7 ADAPTIVE OBSERVATION STRATEGIES WITH THE LOCAL ENSEMBLE TRANSFORM KALMAN FILTER

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1. Introduction

"Targeted" or "adaptive" observation strategies to select the optimal location for observations added to the standard observing system are an important area of research (e.g., Snyder 1996; Palmer et al. 1998). The effectiveness of some adaptive strategies has been tested in field experiments, such as FASTEX (Snyder 1996; Joly et al. 1997; Emanuel and Langland 1998), NORPPEX (Langland et al. 1999a), Winter Storm Reconaissance Program (Szunyogh et al. 2000) and Atlantic TOST/TReC. There are two basic types of adaptive strategies. One is based on the use of the adjoint model, such as the sensitivity to initial conditions and singular vectors to identify the sensitive regions in which additional observations will be taken (Palmer et al, 1998). The other is based on ensembles such as the ensemble spread technique (Lorenz and Emanuel 1998; Morss 2002), the Ensemble Transform Kalman Filtering (ETKF) technique proposed by Bishop et al. (2001) and implemented by Majumdar et al. (2001) in the FASTEX experiment, the quasi-inverse technique of Pu et al. (2001), and the breeding of the analysis/forecast system of Trevisan and Uboldi (2002). In this paper we explore ensemble-based adaptive strategies, and propose a more efficient method based on Ensemble Kalman Filtering.

The assimilation scheme subjects adaptive observations to a dynamical process, so that even with the same model and standard observations, different assimilation schemes can result in different results. Using direct insertion, Lorenz and Emanuel (1998, referred as LE98 hereafter) concluded that multiple breeding (an ensemble based technique) is more efficient than sensitivity-based singular vector technique for the 40-variable Lorenz model. However, using a 1024 member Ensemble Kalman filtering (EnKF), Hansen and Smith (2000, referred as HS00 hereafter) found that the singular vector approach is competitive with multiple breeding. The Local Ensemble Kalman Filtering (LEKF), recently proposed by Ott et al. (2004) does the assimilation on the eigenvector space of background error covariance. In this way, the analysis increment is projected onto the unstable space of the background as in Trevisan and Uboldi (2004, referred as TU04 hereafter), but more than one unstable vector is available at the adaptive observation time. The LEKF belongs to the class of Ensemble Square Root Filters (Whitaker and Hamill, 2002, Anderson 2001, Bishop et al, 2002, Tippett et al, 2003), but unlike other methods that gain efficiency from assimilating observations one after the other, LEKF does the assimilation in a small local patch around each grid point. This makes it very parallel and allows the simultaneous assimilation of many observations. Recently, a more efficient version of LEKF, Local Ensemble Transform Kalman Filtering (LETKF) has been proposed (Hunt, 2005) that avoids the need for explicit singular value decomposition at each grid point. Except for its higher speed, the LETKF yields the same results as the LEKF. It is being tested on the NCEP GFS, on the SPEEDY primitive-equation global model (Molteni, 2003), and on the NASA finite volume General Circulation Model (fvGCM), and is under consideration for operational implementation by Brazil and other countries. In this paper we consider the application of LETKF to adaptive observations. Section 2 describes the model and analysis system. Section 3 describes the formulation of three adaptive strategies, and their results are presented in Section 4, together with a comparison with previously published studies using the same experimental setup. Section 5 is a summary and conclusions. We follow the notation of Ide et al. (1997).

2. Descriptions of model, observations and analysis scheme

As in LE98, HS00 and TU02, the experiments of this study are based on a simulation in which "truth" is a long model integration started from an

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arbitrary state, and “observations” are obtained by adding random noise to the true values.

2.1 Forecast model

The 40-variable model first used by Lorenz and Emanuel is a low-dimensional system whose error growth shares some common characteristics with operational NWP systems. The model is governed by the following equation:

$$\frac{d}{dt}x_j = (x_{j+1} - x_{j-2})x_{j-1} - x_j + F \qquad (1)$$

The variables (x_j , j=1…J) represent a meteorological variable on a “latitude circle” with periodic boundary conditions. As in LE98, J is equal to 40, the time step is 0.05, which corresponds to about a 6-hour interval for the atmosphere. F is the external forcing, F=8 is considered as the “true” model. A model error is introduced by setting the forcing equal to 7.6 when we do forecasts.

2.2 Observations

We follow the experimental set-up of LE98, observing every “land” grid point (from *j*= 21 to 40) each observation time, with an error standard deviation of $\sigma = F/40 = 0.2$. A single adaptive observation point is picked from one of the points over “ocean” (grid points 1-20). The analysis is the combination of the 6-hour forecast and both standard observations and the adaptive observation. The optimality of this additional observation is evaluated by the analysis error at the observation time and 10-day forecast from this time.

2.3 Assimilation scheme

LETKF (Hunt, 2005) is a type of square-root ensemble Kalman filter. In Ensemble Kalman Filter, an ensemble of K forecasts

$$x_i^b = Mx_i^a,\ i = 1,\ldots,K \qquad (2)$$

is carried out in order to estimate the background error covariance. The ensemble mean is defined as

$$\overline{x}^b = \frac{1}{K}\sum_{i=1}^{K} x_i^b \ .$$

The background error covariance is written as

$$\mathbf{P}^b = \frac{1}{K-1}\mathbf{X}^b\mathbf{X}^{bT} \ . \qquad (3)$$

where $\mathbf{X}^b$ is a J x K matrix whose columns are the K ensemble perturbations $x_j^b - \overline{x}^b$, and J is the dimension of the state vector. The analysis mean state is

$$\overline{x}^a = \overline{x}^b + \mathbf{K}(y^o - HX^b)\ , \qquad (4)$$

y^o is the observation vector, *H* is the observation operator, and $\mathbf{K}$ is the Kalman gain matrix that can be written as

$$\mathbf{K} = \mathbf{X}^b[(k-1)\mathbf{I} + (\mathbf{E}^b)^T\mathbf{R}^{-1}(\mathbf{E}^b)]^{-1}(\mathbf{E}^b)^T\mathbf{R}^{-1}$$

where $\mathbf{E}^b = \mathbf{H}\mathbf{X}^b$. The matrix inversion is performed in the *KxK* space of the ensemble perturbations. The analysis perturbations are obtained by using Ensemble Transform Kalman Filter (ETKF) approach (Bishop et al, 2001, Hunt, 2005):

$$\frac{1}{K-1}\mathbf{X}^a\mathbf{X}^{aT} = \mathbf{X}^b\tilde{\mathbf{P}}^{\mathbf{a}}\mathbf{X}^{bT}, \qquad (5)$$

$\tilde{\mathbf{P}}^a = [(K-1)\mathbf{I} + \mathbf{E}^b\mathbf{R}^{-1}\mathbf{E}^{bT}]^{-1}$, a *KxK* matrix, represents the new analysis covariance in the *K*-space of the ensemble forecasts. The analysis ensemble perturbations are then obtained from:

$$\mathbf{X}^a = \mathbf{X}^b[(K-1)\tilde{\mathbf{P}}^a]^{1/2} .$$

The LETKF scheme does analysis in the local patch centered on each grid point. In our experiments, the local patch size is 9, i.e., it includes four grid points at each side of every grid point. The calculation is independent in each local patch, so the calculation can be done in parallel and because of the small patch size, the computations are very fast (Szunyogh et al, 2005). In order to allow for model errors and nonlinear error growth we use background error covariance inflation, but the inflation factor is determined adaptively (Miyoshi, 2005).

3. Adaptive strategies

The optimality of adaptive observation is measured by the RMS error at the analysis time, which is the reduction of the background error due to the assimilation of observations. The analysis error can become “optimal” only if the adaptive observation is made where the background error is large. When the true state is not known, the background uncertainty can be estimated by measures derived from the background error covariance, such as the ensemble spread or, equivalently, the trace of the background error covariance. Since it is easy to compute, we only consider the ensemble-spread method here. If the truth were known (as would happen only in a simulation), the true background error can be calculated, leading to the (unattainable) true optimal location. We denote this approach as the “true ensemble error variance”. The analysis error covariance can be calculated without knowing the actual observation value (Bishop et al, 2001). By calculating analysis error covariance before the actual analysis is performed, and minimizing the analysis error variance, one can find the “optimal” observation location. We refer to this approach (done locally) as the “local P^a” method discussed below.

3.1 Ensemble spread method

By picking the point with largest ensemble spread in the ensemble-spread method, the trace of the background error covariance is minimized. However, the accuracy of adaptive observation is not explicitly taken into account in this method. If the type of adaptive observation used happens to have large errors, the analysis will not be as improved as with an accurate observation.

Ensemble spread is calculated at each grid point according to the equation:

$$S_j = (K-1)^{-1}\sum_{i=1}^{K}(x_{i,j}^b - \overline{x}_j^b)(x_{i,j}^b - \overline{x}_j^b)^T \quad , \qquad j=1\ldots40. \tag{6}$$

$x_{i,j}^b$ is the background ensemble at each grid point *j*, k is the number of ensemble number, $\overline{x}_j^b$ is the ensemble mean state.

3.2 Local P^a method

Following the idea of Bishop et al. (2001), the local P^a method maximizes the analysis error variance reduction, which is equal to the difference between background error variance and analysis error variance. We call it local P^a method because the analysis error variance is calculated on a local patch in our implementation.

The analysis error covariance is calculated from equation (5). The variance of P^a in each local patch, which is the average of the diagonal values of P^a, is regarded as the variance of the center point. The global variance of P^a is the summation of the variance at each grid point. 20 different P^a's have to be calculated in order to determine the adaptive observation point that makes the magnitude of the global analysis error variance smallest. Unlike the ensemble-spread method, the local P^a method considers the adaptive observation error variance explicitly, which allows for adaptive observations of different types.

3.3 Combined method

Ensemble spread method is simple and almost cost-free. However, it may not be optimal due to the possible large observation error variance in the adaptive observations. Under this condition, the overall analysis error variance may not decrease substantially. The local P^a method allows for observation errors to get a better analysis, but it computationally much more expensive. A combined method proposed here combines the advantages of both methods. We compute first the ensemble spread, and choose the 5 grid points with largest ensemble spread from the 20 points over ocean. Then, the local P^a method is computed only for these 5 grid points. The grid point that makes the expected trace of analysis error covariance smallest is the adaptive observation point. In this way, the smallest expected analysis error covariance trace is guaranteed with the adaptive point. Moreover, this reduces substantially the computation time compared with local P^a method, an important consideration if carried out in operations with much larger dimension models.

3.4 True ensemble error variance

The true ensemble error variance method cannot be computed in operation, but is the standard that the other methods strive to reach. In this method, truth is supposed to be known. The formula of ensemble error variance is

$$V_j = (k-1)^{-1}\sum_{i=1}^{k}(x_{i,j}^b - x_j^t)(x_{i.j}^b - x_j^t)^T \quad , \qquad j=1\ldots40. \tag{7}$$

The adaptive point is the point with largest ensemble variance. The only difference between the calculation of this method and ensemble-spread method is that the truth is used instead of ensemble mean state.

4. Results

4.1 Comparison between different strategies

As in LE98, the Observation-Analysis-Forecast (OAF) routine has been carried out through a 90-day spin-up time and a 5-year testing period. The analysis RMS error is the average over the 5-years. The performance of these methods is estimated by 5-year average analysis RMS error.

Fig. 1 shows the analysis RMS error comparison between different methods. The RMS errors are very similar over land in every adaptive strategy (except for random picking not shown because it has much larger errors, see Table 1), and they are all below the observation standard deviation error. Two minimum RMS errors appear near the land boundaries, and we have verified that this is due to the systematic error in the forcing. Without the error in the forcing, the RMS error over land is essentially uniform. Over the ocean, the ideal adaptive strategy based on true ensemble variance, as expected, is the best and only slightly larger than the observation error with only one adaptive observation over the whole ocean region. The performance of the other three operational possible methods is very similar.

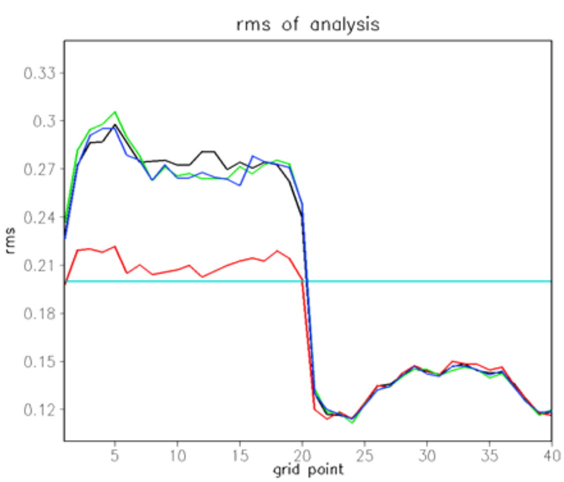


Fig.1. Five-year-average analysis RMS error for different methods (the straight line is the observation error standard deviation, the red line is RMS error from true ensemble variance method, the black line is the RMS error from ensemble spread method, the green line is the result of local P^a method, the blue line is the result of combined method).

The spatial average of the analysis RMS error is compared in the table 1. The results from all methods are much better than randomly choosing the adaptive location. Ensemble-spread method, local P^a method and combined method show similar performance in this idealized model, but it may show different results in a more complex model.

Since the combined method has the advantages of both ensemble-spread method and local P^a method, we will mainly discuss this method in the following. In order to compare with previous work (LE98, HS00, TU04), the 10-day forecast RMS error is calculated. Fig. 2a is the 10-day forecast RMS error from true ensemble variance method. It takes 4 days for the forecast RMS error to reach 1. Fig. 2b is the RMS error difference between combined method and true ensemble variance method. The combined method is worse initially over ocean, and transported eastward with time. The magnitude of the difference is small, initially about 20% of the RMS error and becoming smaller with time.

Fig. 3 shows the percentage distribution of adaptive observation points for combined method, ensemble-spread method and true ensemble variance method. The percentage distribution has a similar shape, except that more adaptive observation points are picked from the "coastal regions" between data-dense and data-sparse region for the true ensemble variance method.

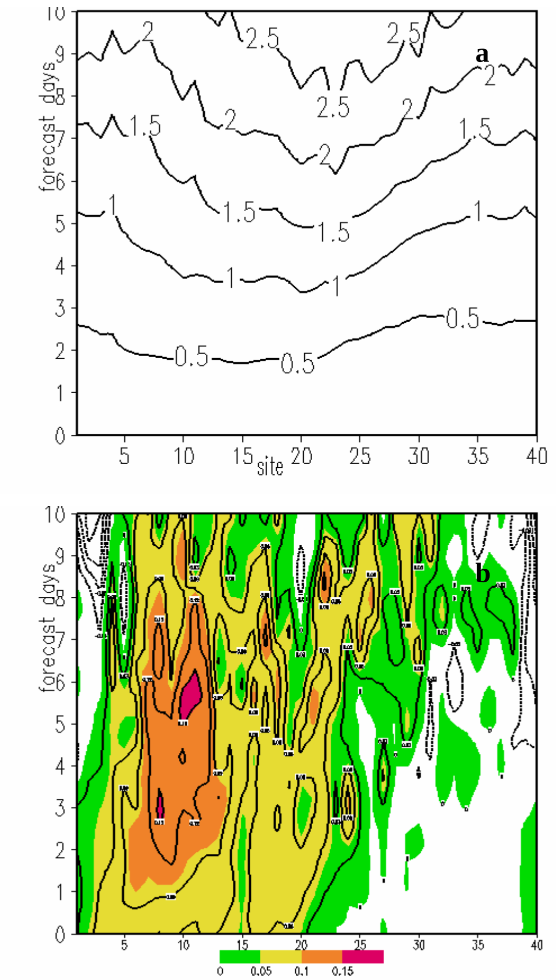


Fig. 2 Five-year-average forecast errors. (a) is the result from true ensemble variance method , (b) is the difference between the result of combined method and true ensemble variance method

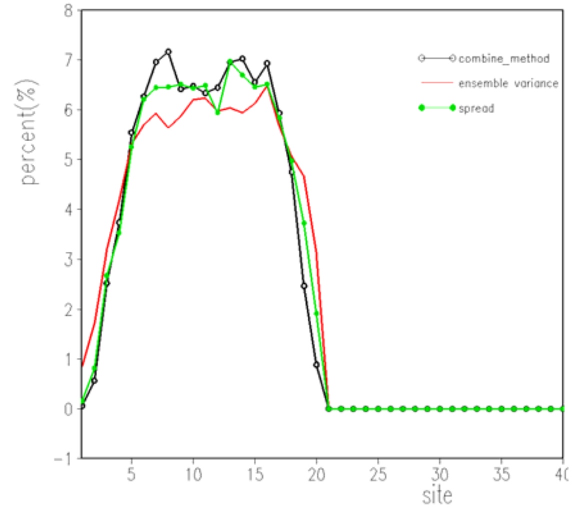


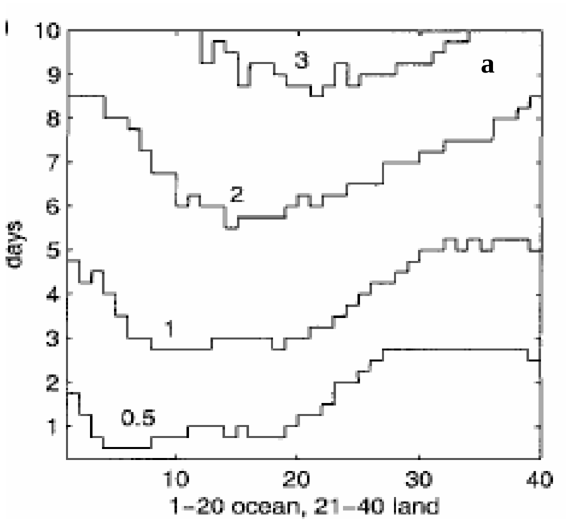
Fig.3. Adaptive observation sites distribution for combined method (black line), true ensemble variance method (red line) and ensemble spread method (green line) (Y-axis is the percentage of each point as adaptive observation point, X-axis is the grid point.

Method	**Time mean analysis RMS error**	**Operational possibilities**
True Ensemble variance	0.174	Impossible
Background ensemble spread	0.210	Possible
Local Pa method	0.211	Possible
Combined method	0.209	Possible
Random picking	0.480	Possible

Table 1 Summary of average analysis error for different methods

4. 2 Comparison with previous work (LE 98, HS 00, TU 04)

Several articles (LE98, HS00, TU04) have discussed adaptive strategies with this low-order system and the same land-ocean setup, and used the 10-day forecast as one standard to evaluate the adaptive observation strategies. Although the average 10-day forecast from our combined LETKF method is considerably better than the results obtained by LE98 and TU04, this difference cannot be attributed to the adaptive observation strategy only because LE98 and TU04 used a very simple assimilation method for the standard observation (direct insertion). HS00, on the other hand, performed the data assimilation with an Ensemble Kalman Filter with a number of ensemble members (1024) much larger than the dimensions of this model (40), so that the accuracy of the HS00 EnKF should be optimal and at least as good as the accuracy that can be attained with a 15-member LETKF. Therefore, it is reasonable to directly compare the result from our combined method with their singular vector approach. Fig. 4 compares the average 10-day forecast errors from the HS00 adaptive observations method based on singular vectors (Fig. 4a) and from the combined ensemble-based method (Fig. 4b). The initial difference is small over land, but over ocean the combined method errors are clearly smaller than those of the singular vector approach, and this advantage is maintained throughout the 10 days. Another advantage of the ensemble-based system is that it does not require the development of linear tangent and adjoint models.



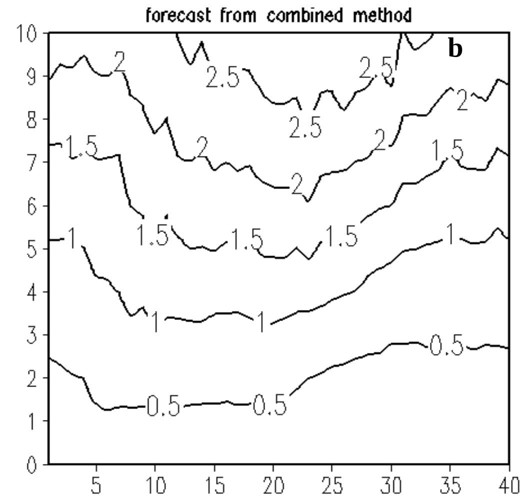


Fig. 4 (a) 10-day forecast RMS error from Hansen and Smith (2000). Singular vector adaptive observation strategy is used in this result. (b) 10-day forecast RMS error of combined method.

5. Discussion and conclusion

Several ensemble-based adaptive observation strategies were tested with a 15-member LETKF scheme in the Lorenz-40 variable model. These methods include ensemble-spread, local P^a method, and a combined method. Our EnKF-based adaptive observation method is related to the breeding of the analysis-forecast system method developed by TU04 in the sense that it not only provides the location with largest forecast error where the adaptive observation should be located, but also the optimal shape of the analysis corrections to the forecast which have to project on the local background error covariance. Using the same direct insertion data assimilation, TU04 obtained results better than those of LE98. Our results cannot be directly compared to theirs because we used a more advanced data assimilation method. However, they can be directly compared with those of HS00 who used a 1024-member EnKF for the data assimilation, and singular vectors to choose the adaptive observation location. Compared with singular vector method, the ensemble-based methods proposed here are clearly better, at least in this idealized model. The requirement of linear tangent and adjoint models make the singular vector method more difficult to implement. Considering the computational cost and accuracy, ensemble-based adaptive strategies are better.

The ensemble-spread method, local P^a method and combined method show similar results in this model. Ensemble-spread method has a low computational cost, but it doesn't explicitly consider the effect of the adaptive observation error. Local P^a method explicitly considers the observation error, but is much more expensive. A combined method has the advantage of both, reducing observation cost by selecting adaptive observation "candidates" first with the ensemble-spread method, and then using the local P^a method on these few "candidates".

Here, the same standard deviation error has been used for both the standard observation and adaptive observation, so the ensemble-spread method yield results similar to those of the local P^a method and the combined method even though it does not consider the possible effects of the observation error. However, adaptive observations usually have different observation errors with standard observation data set, such as adaptive observation with GPS dropsonde, the measured temperature error is only 0.2K, which is about 1K for standard observations, the local P^a and the combined methods can have an advantage under this condition. The combined method is better than the local P^a method for operational applications because it is less expensive.

We plan to compare these three schemes in the SPEEDY model, using realistic observation data sets, and considering observation error differences between standard observation and adaptive observation.

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