P1.3 **OPTIMAL SAMPLING STRATEGIES FOR HAZARDOUS WEATHER DETECTION USING NETWORKS OF DYNAMICALLY ADAPTIVE DOPPLER RADARS**

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1. INTRODUCTION

The National Science Foundation Engineering Research Center for Collaborative Adaptive Sensing of the Atmosphere (CASA) is developing a revolutionary new paradigm for overcoming fundamental limitations in current radar technology such as the inability to sample the lower parts of the atmosphere (McLaughlin, 2005). CASA was created in fall 2003 and is led by the University of Massachusetts at Amherst with several partners including the University of Oklahoma, Colorado State University, and the University of Puerto Rico at Mayaguez. CASA is establishing a system of distributed, collaborative, and adaptive sensor (DCAS) networks that are unique because they can dynamically adjust their scanning strategies and other attributes collaboratively with other CASA radars to sense multiple atmospheric phenomena while at the same time meeting multiple end user needs.

In the first phase of its research program, CASA is placing test beds of small, inexpensive, low-power Doppler weather radars on existing infrastructures, such as cell phone towers, to test the DCAS concept. This network, called NetRad, will consist of four dual-polarization, mechanically-scanning Doppler radars that will be operating in central Oklahoma beginning in late winter of 2005. By 2008, the network is expected be expanded and include phased-array radars.

The DCAS networks are designed to overcome the fundamental limitations of current approaches to sensing and predicting atmospheric hazards. Distributed refers to the use of large numbers of solid-state radars that are spaced appropriately to overcome blockage due to the Earth's curvature, resolution degradation caused by beam spreading, and large temporal sampling intervals resulting from today's use of mechanically scanned antennas.

The radars can operate collaboratively by means of coordinated targeting of multiple radar beams based on atmospheric and hydrologic analysis tool such as detection, predicting, and tracking algorithms. By utilizing this collaboration, the system is able to determine needs and allocate resources such as radiated power, beam position, and polarization diversity towards regions of the atmosphere where a particular threat exists. The adaptive capabilities refer to the ability of the CASA radars and the associated computing and communications infrastructure to rapidly reconfigure in response to changing conditions in a manner that optimizes the response to competing end user demands. For example, this system could track tornadoes for public warning while simultaneously collecting information on the parent storm and providing quantitative precipitation estimates for input to hydrologic prediction models.

The objective of this paper is to develop a technique to apply, test, and analyze a sampling strategy to provide preliminary results of how to use the CASA radars to optimally adaptively sample the atmosphere. Although it may seem logical to simply scan a real tornado with as fine a temporal and spatial sampling as possible, that approach may be unnecessary and waste resources, especially when other phenomena are present simultaneously and competing for the same resource.

We approach this problem using an idealized, analytic vortex to serve as a proxy for a tornado. We then sample this flow field, as if it were being observed by one CASA radar, and use variational techniques to fit the pseudo-observations to the models of an idealized tornado vortex. We seek the minimum in the cost function, which defines the best (in a least squares sense) fit between model and pseudo-observations, across a variety of parameters including but not limited to azimuthal sampling interval, number of vertical levels sampled, distance of the radar from the vortex, and the number of radars available. This approach is similar to that used by Wood (1997) in the context of NEXRAD. In this paper, we evaluate the azimuthal sampling interval and the distance of the radar from the center of the vortex.

2. RANKINE COMBINED VORTEX APPROXIMATION

We use an approximation to the Rankine (1901) combined vortex (RCV) to prescribe the flow field for

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our simulated tornado. The RCV has a rotational velocity that increases linearly from zero at the center of the vortex to a maximum at the core radius. Beyond the core radius the rotational velocity decreases, with the velocity being inversely proportional to the distance from the rotation center. This traditional form of the RCV contains a cusp at the radius of maximum winds that represents a discontinuity (Figure 1). In order to obtain the Rankine combined vortex approximation (RCVA), we first consider the traditional RCV:

$$v = V_{\max} f(r) \quad (1)$$

where $V_{\max}$ is the maximum tangential wind and

$$f(r) = \begin{Bmatrix} \frac{r}{R}, & 0 \le r < R \\ \frac{R}{r}, & R \le r \end{Bmatrix} \quad (2)$$

where *r* is the radial distance to the vortex center and *R* is the core radius at which maximum tangential wind V_{max} occurs. A graph of this function (Figure 1) illustrates the discontinuity at the normalized radius of 1.

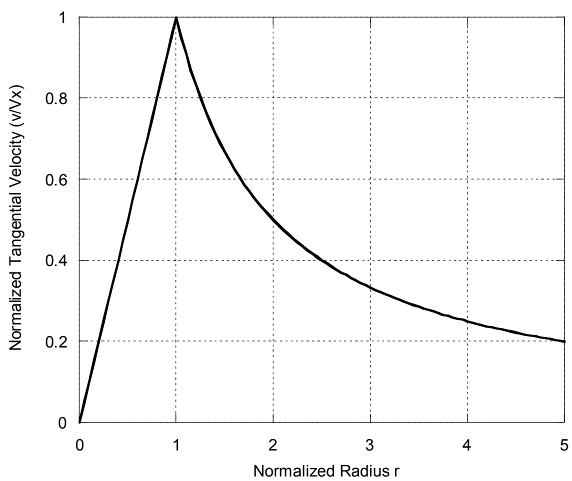

Figure 1: Tangential velocity in a Rankine combined vortex with cusp at a normalized radius of 1.

Because of the discontinuous nature of the first derivative of the RCV, difficulties in solving the cost function occur in the retrieval technique. In order to overcome this difficulty, a new function with no discontinuity in the core radius yet retains the features of the RCV can be introduced (White, personal communication):

$$\phi_n(r,R) = \frac{2nR^{2n-1}r}{(2n-1)R^{2n} + r^{2n}} \quad (3)$$

where n =1,2…. After testing various combinations of (3) with different values of n, the approximating function that best fits (2) and that is used herein as the RCVA is:

$$\hat{f}(r) = \frac{1}{2}[\phi_1(r) + \phi_2(r)] = \frac{\phi_1 + \phi_2}{2} \quad (4)$$

Figure 2 shows a comparison of the approximation's fit (labeled as "(phi1+phi2)/2") to the traditional RCV (labeled as "RCV or f(r)"), as well as the case where n =1 and n =2 for Equation (3).

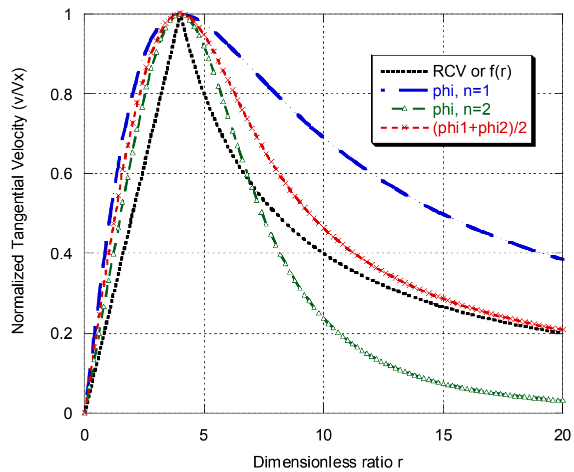

Figure 2: Comparison of RCV (Equation 2) to the φ_n (Equation 3) with varying values of n, and the RCVA (Equation 4).

In this paper, the RCVA idealized flow will serve as the model to which the flow generated by either φ_1 or φ_2 is fit using variational techniques. A similar approach was used by Wood (1997).

3. VARIATIONAL RETRIEVAL TECHNIQUE

In order to determine how to best observe the tornado proxy with CASA radars, a one dimensional analysis technique is used that is based on the variational principle used by Wood (1997). This variational principle optimally estimates the maximum tangential wind speed, V_{max}, and core radius R_{max} at which this wind speed occurs. Simulated Doppler velocities are generated by the combined NSSL Doppler radar simulation and retrieval technique; the velocities correspond to the RCVA idealized flow discussed in section 2 but are sampled in a manner that includes beam broadening, weighting along the beam center, etc. Simulation of the radar sampling process does not directly follow that of an actual radar because the mean Doppler velocity is calculated by averaging the Doppler velocity components within the effective radar beam. Normally, the radar pulses are averaged to produce a simulated mean Doppler velocity value. The program retrieves the maximum tangential velocity and the core radius of an axisymetric vortex from a single Doppler velocity signature of the vortex.

In order to use the variational technique, we must first calculate initial guesses of V_{max} and R_{max} that will be used to solve a set of nonlinear equations. By solving the set of nonlinear equations, we can then solve a set of linear equations using Gaussian elimination to get the retrieved values of V_{max} and R_{max}.

There are several steps in determining the initial guesses of V_{max} and R_{max} that serve as the input values to the cost function. These guesses, known as V_{rot} and R_a, must be close enough to the solution to give convergence and are computed from the Doppler velocity measurements as:

$$V_{rot} = \frac{\Delta V_d}{2} \tag{5}$$

where V_{rot} is the rotational velocity of the vortex and ΔV_d is the difference between the incoming (V_-) and outgoing velocity (V_+) peaks of the vortex and is defined as:

$$\Delta V_d = V_+ - V_- \tag{6}$$

The apparent radius, R_a, is given by:

$$R_a = \frac{D_a}{2} \tag{7}$$

where D_a is the apparent diameter and is the distance between the incoming and outgoing velocity peaks.

After specifying input values such as beamwidth and azimuthal interval, Doppler velocity values at range, elevation, and azimuth subpoints within the beamwidth volume and the two-way antenna pattern can be calculated. From this, the radial distance, *r*, from the circulation center to the subpoints of beamwidth volume center in Cartesian coordinates can be computed.

Using the radial distance *r*, the radial variation of the tangential component of the RCVA can be found by using $n = 1$ and $n = 2$:

$$\begin{aligned} v_{rcva} &= V_{max}\frac{\phi_1 + \phi_2}{2} \\ &= V_{max}\left(\frac{R_{max} r}{R_{max}^2 + r^2} + \frac{2R_{max}^3 r}{3R_{max}^4 + r^4}\right) \end{aligned} \tag{8}$$

With the tangential component, the mean Doppler velocity can be calculated within the beamwidth volume. Now the initial guesses for V_{max} and R_{max} are calculated.

These initial guesses will be used in the variational technique where Newton's Method is used to solve a set of nonlinear equations. Newton's Method essentially estimates the partial derivatives of the function by difference quotients, where a small change in the value of the variable is made called delta (Gerald and Wheatley, 1984). This small change in the value of the function is then divided by the change in the value of the variable; this process is done for each variable in each function. When using this technique, the initial guesses for the values of the variables must be close enough to the solution to give convergence. It is important to note also that delta should be small enough to give a realistic approximation to the partial derivative but not so small as to lead to an extreme amount of round-off.

The variational technique includes a cost function defined as:

$$J = \sum_{i=1}\left(V_i - \tilde{V}_i\right)^2 \tag{9}$$

where V_i is the modeled wind profile, $\tilde{V}_i$ is the observation, and i is the total number of observations. V_i is given by:

$$V_i = \frac{\sum_j G_j Z_j (v_{rcva}\cos\gamma\cos\phi)}{\sum_j G_j Z_j} \tag{10}$$

In (10), G is the Gaussian weighting function representing the shape of the radar beam, Z is reflectivity (assumed to be uniform in this paper), γ is the angle between the radar viewing direction at a target point and the tangential velocity and φ is the elevation angle. To determine the optimal estimate, the cost function J, which is a function of V_{max} and R_{max}, is minimized. A necessary condition for this minimization is:

$$\frac{\partial J}{\partial V_{max}} = 0 = 2\sum_i\left(V_i - \tilde{V}_i\right)\frac{\partial V_i}{\partial V_{max}} \tag{11}$$

$$\frac{\partial J}{\partial R_{max}} = 0 = 2\sum_i\left(V_i - \tilde{V}_i\right)\frac{\partial V_i}{\partial R_{max}} \tag{12}$$

From (11) and (12), we can see that two more partial derivatives are needed:

$$\frac{\partial V_i}{\partial R_{max}} = \frac{1}{2}\left(\frac{\partial \phi_1}{\partial R_{max}} + \frac{\partial \phi_2}{\partial R_{max}}\right)\cos\gamma\cos\varphi \tag{13}$$

$$\frac{\partial V_i}{\partial V_{max}} = \frac{1}{2}\left(\phi_1 + \phi_2\right)\cos\gamma\cos\varphi \tag{14}$$

Contained in (13) are two final partial derivatives that involve the RCVA discussed in section 2. Recall (2):

$$\phi_n = \frac{2nR^{2n-1}r}{(2n-1)R^{2n} + r^{2n}} \quad n = 1,2,...$$

Also recall that the best approximation is when $n = 1$ and $n = 2$, which results in:

$$\phi_1 = \frac{2Rr}{R^2 + r^2} \tag{15}$$

$$\phi_2 = \frac{4R^3 r}{3R^4 + r^4} \tag{16}$$

Therefore, with some simplications:

$$\frac{\partial \phi_1}{\partial R} = \frac{2r^3 - 2R^2 r}{(R^2 + r^2)^2} \tag{17}$$

$$\frac{\partial \phi_2}{\partial R} = \frac{(12R^2 r^5 - 12R^6 r)}{(3R^4 + r^4)^2} \tag{18}$$

The nonlinear equations were solved for the values of the computed partial derivatives of each function for each variable. The next step in retrieving V_{max} and R_{max} is to solve a set of linear equations using Gaussian elimination with partial pivoting. Because the set of linear equations are all augmented to the coefficient matrix and all the solutions are acquired at once, the set of equations can be solved with multiple right hand sides. The solutions are returned in the space of the augmentation columns (Gerald and Wheatley, 1984). From this routine, retrieved values of V_{max} and R_{max} are obtained.

4. DESCRIPTION OF EXPERIMENT

In this paper, we present results from prescribed changes in the parameters listed in Figure 3. We assume one virtual CASA radar is sampling the volume with uniform reflectivity across the vortex and that the data are perfect. The sampling will be done as though a mechanically scanning radar (MSR) and a phased array radar (PAR) is sampling. Here the MSR is assumed to be continuously scanning and takes into account beam smearing due to antenna rotation. Since the PAR can sample discretely by turning on and off elements on the antenna, no beam smearing is assumed.

Range (km)	2.5-30 in increments of 2.5
Azimuthal Sampling Interval (delta azimuth)	Mechanically scanning: 1°,2° Phased array: 1°,2°,4°
Vortex radius Rmax (km)	0.5,1.0,1.5
Vortex max wind speed Vmax (m/s)	40,60,80,105,130

Figure 3: Parameters varied throughout experiment and the ranges over which they are varied.

There are several input values to the retrieval program that must be considered. These include the model the observations will be generated from, an option for smoothing or no smoothing, and an option for noise or no noise added to the data. The effective beamwidth is another value inputted into the retrieval program. Effective beamwidth is the azimuthal broadening of a horizontally rotating beam at a given range, and depends on three radar parameters: antenna rotation rate, the time interval between pulses, and the number of pulses transmitted (Doviak and Zrnić, 1993, 193-197). For a radar antenna with a two degree beamwidth that collects data at two degrees azimuthal intervals, the effective beamwidth is computer to be 2.90°. Effective beamwidth was only used for the MSR and not the PAR since the latter is assumed to be stationary for our case. The elevation angle used in this paper was assumed to be zero degrees; we also assume that the simulated measurements are free of noise.

5. RESULTS

The results presented in this paper were obtained by varying the parameters listed in Figure 3 in the variational algorithm, leading to hundreds of experiments. These results help to illustrate that the amount of information collected can be maximized while using a minimum amount of resources. A few results are presented in Figures 4-7. Percentage error is used (relative error times 100) as a method of comparison for these results.

Figure 4 shows a comparison between a beamwidth of one degree and two degrees for a vortex with a radius of maximum winds (R_{max}) of 0.5km and 1km for the MSR. A smaller beamwidth produces a smaller percent error because its increase in beamwidth with range is smaller compared to the two degree beamwidth.

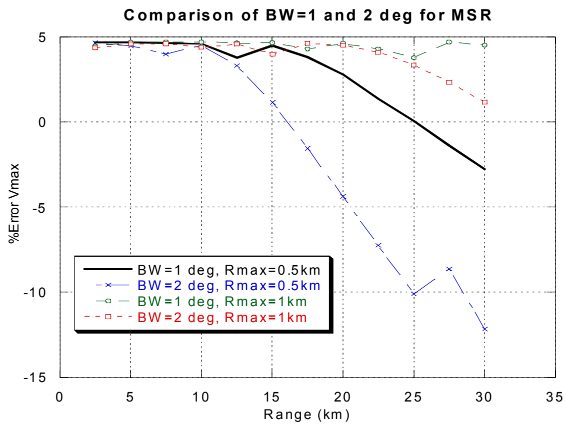

Figure 4: Different beamwidth comparison for the MSR for R_{max}=0.5km and 1km.

Figure 5 shows an example of PAR oversampling, in which the azimuthal sampling interval is one degree. Oversampling using an azimuthal sampling interval of one degree means that for a two degree beamwidth, the center of the beamwidth of a sample is one degree away from the center of the beamwidth of the preceding sample and there is overlapping in sampling. This is compared to the MSR, which has an azimuthal sampling interval of two degrees. Although the smallest percent error is at a longer range for the PAR than the MSR, the PAR overall percent error, especially at long ranges, is smaller.

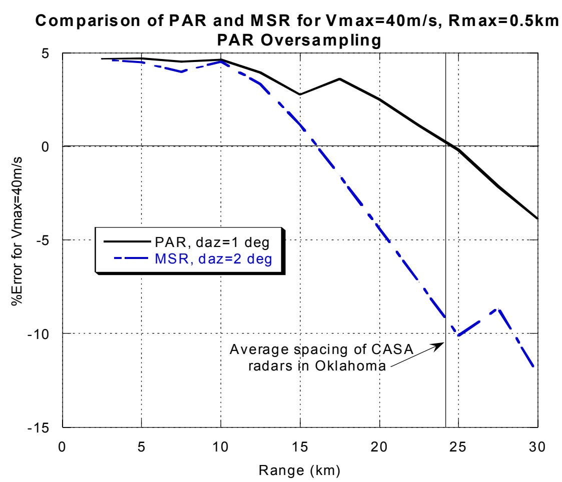

Figure 5: PAR oversampling (delta azimuth = one degree) compared to MSR sampling (delta azimuth = two degrees).

Figure 6 shows an example of PAR undersampling in which the azimuthal sampling interval is four degrees. Undersampling using an azimuthal sampling interval of four degrees means that for a two degree beamwidth, the center of the beamwidth of a sample is four degrees away from the center of the beamwidth of the preceding sample and there is no overlapping in sampling. This PAR sampling is compared to the MSR with an azimuthal sampling interval of two degrees. Using less resources by PAR undersampling shows this adaptive sampling method still results in a low percent error that decreases to near zero at a range of 22.5km. Beyond that range, the magnitude of the percent error increases, but this region will also be in another CASA radar's scanning area, helping to compensate for this decrease in retrieval capability.

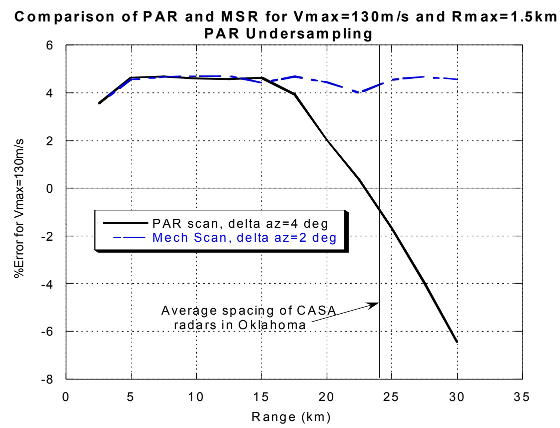

Figure 6: PAR undersampling (delta azimuth = 4 degrees) compared to MSR sampling (delta azimuth = two degrees).

In general, preliminary results for these CASA radar simulations show that using adaptive sampling techniques, such as undersampling, may be able to minimize the resources used while maintaining the goodness of fit.

6. FUTURE WORK

The next phase of this research will be to use several more parameters that can be changed also, including the number of radars adaptively sampling a vortex, adding translation to the tornado, the number of vortices, and possibly using non uniform reflectivity.

Other idealized flows will be used as well in the retrieval program, including the Burgers(1948)-Rott(1958) vortex (BRV). These flows will be used to create the flow as well as serve as the model to fit. The same parameters that are being changed for the RCVA case will be used for these flows as well. Metrics for optimization such as a cost function, probability density function, and information content will also be used.

7. CONCLUSIONS

CASA is developing a revolutionary new paradigm for overcoming fundamental limitations in current radar technology such as the inability to sample the lower parts of the atmosphere. To facilitate this, CASA is establishing a system of distributed, collaborative, and adaptive sensor (DCAS) networks that are unique because they can dynamically adjust their scanning strategies and other attributes collaboratively with other CASA radars to sense multiple atmospheric phenomena while at the same time meeting multiple end user needs.

The purpose of this paper is to show preliminary results of how to best use CASA radars and the DCAS idea in order to find the optimal sampling strategy for the radars. An optimal sampling strategy would maximize the amount of information that can be extracted from the atmosphere while using the minimum amount of resources to meet end user needs. Preliminary results for these CASA radar simulations show that using adaptive sampling techniques, such as undersampling, may be able to minimize the resources used while maintaining the goodness of fit.

8. References

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